## AIRBORNE LIDAR SENSING OF FOAM-COVERED SEA SURFACE: THE IMAGE ILLUMINANCE STRUCTURE

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In this paper we consider the spatial distribution of image illuminance for the case of airborne sensing of a foam-covered sea surface by a narrow laser beam. Analytical expressions are obtained for the mean illuminance for both the clear and turbid aerosol-loaded atmosphere. Sea-surface foam is demonstrated to affect the spatial distribution of the image illuminance.

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Lidar sensing is one of the promising approaches to studying the global ocean. Lidar techniques yield data on the global ocean level, on the parameters of ocean roughness, on optical properties of sea water, concentration of chlorophyll, on fish shoals (see, e.g., Refs. 1–3). One of the factors affecting the lidar signal is foam present on the sea surface.

The problem of the illuminance distribution in the image plane for a sea-surface-sensing lidar receiver was considered in Ref. 4. Below we consider the problem of the structure of the image illuminance during airborne sensing of a sea surface partially covered with foam.

Let us envisage foam-covered sea surface sounded by a lidar. Generally, one should consider the source [or transmitter] and receiver to be separated from each other and located at distances  $L_s$ and  $L_r$  from the surface being sounded, so that their optical axes form angles  $\theta_s$  and  $\theta_r$  with the normal to the calm sea surface. We assume that the sensing radiation wavelength lies in the IR range, where absorption by water is high. The principal contribution to the received signal is produced by light specularly from the air-water interface, while the radiation diffusely reflected by the water layer can be ignored. Let us also point out that the radiation wavelength is small compared with typical curvature radii and roughness heights of the sea surface.

Radiation scattered from a randomly rough surface acquires a large random phase shift at every point of the field.<sup>5</sup> Therefore, the average (over an ensemble of randomly rough surfaces) illuminance in the image  $\overline{E}(\vec{R})$  obtained when sensing a sea surface, partially covered with foam, will be determined as a mean of two extrema: foam totally absent from ( $\overline{E}_{sea}$ ), and foam completely covering ( $\overline{E}_{f}$ ) the sea surface

$$\overline{E}(\overrightarrow{R}) = (1 - W)\overline{E}_{aa}(\overrightarrow{R}) + W\overline{E}_{a}(\overrightarrow{R}),$$

where W is the fraction of the sea surface covered with foam and white-caps.

When obtaining the expression (1), it was assumed that the size and shape of foam-covered sea surface areas do not depend on elevations and declinations of the surface elements in a given realization. We employ two models of sea surface completely covered with foam: the model of a randomly rough surface with a locally Lambertian scattering phase function for its elements, and the model of a flat Lambertian surface.<sup>4,6–9</sup> For a flat Lambertian surface the value of  $\overline{E}_f(R)$  is known.<sup>10</sup> We will find

 $\overline{E}_{f}(\overline{R})$  for the model of a where randomly rough surface with a locally Lambertian scattering phase function of its elements.

Similarly to Ref. 11 we write out an expression to describe the illuminance in the image plane of the receiver lens. Consider a randomly uneven locally Lambertian surface S in the atmosphere (we assume the sensing angles  $\theta_s$ ,  $\theta_r$  to be sufficiently small so that mutual shading of the surface elements may be ignored). Its illuminance will then be

$$E_{f}(\vec{R}) = \frac{A}{\pi} \int d\vec{r} E_{ac}(\vec{r}) E_{rec}(\vec{r}, \vec{R}),$$
s
(2)

Here  $E_{ac}(\vec{r})$ ,  $E_r(\vec{r}, \vec{R})$  are the illuminances of a surface *S* in the atmosphere at point *r*, produced by actual and "fictitious" sources (the latter has parameters Identical to that of the receiver);<sup>10</sup> *A* is the albedo of a surface element covered with foam.

Now, instead of integrating over the randomly uneven surface S in Eq. (2), we integrate over the surface  $S_0$  (which is a projection of S upon the Z = 0 plane) and average the value  $E_r(\vec{R})$  over the ensemble of surfaces (following a technique similar to that in Ref. 5). Using the expressions for illuminances from the actual and the "fictitious" atmospheric sources,<sup>10</sup> we then obtain the following expression for a narrow illuminating beam to describe the mean illuminance in the image plane of the receiver lens. It refers to the case of sensing a randomly uneven locally Lambertian surface in the aerosol-loaded atmosphere. (We assume for simplicity both the source and the detector are positioned in one and the same plane *XOZ*):

$$\begin{split} \overline{E}_{\mathbf{r}}(\vec{R}) &= \frac{Aa_{\mathbf{s}}a_{\mathbf{r}}}{L_{\mathbf{s}}^{2}L_{\mathbf{r}}^{2}} Q(C_{\mathbf{s}} + C_{\mathbf{r}})^{-1/2} \cdot p^{-1/2} \times \\ &\times \left[1 + 2\sigma^{2} \left[s - \frac{t^{2}}{p}\right]\right]^{-1/2} \cdot \exp\left\{-\left[R_{\mathbf{y}} \frac{L_{\mathbf{r}}}{F}\right]^{2} \times \right. \\ &\times (C_{\mathbf{s}}^{-1} + C_{\mathbf{r}}^{-1})^{-1} - \left[R_{\mathbf{x}} \frac{L_{\mathbf{r}}}{F \cos\theta_{\mathbf{r}}}\right]^{2} (C_{\mathbf{s}}^{-1} \cos^{-2}\theta_{\mathbf{s}} + \\ &+ C_{\mathbf{r}}^{-1} \cos^{-2}\theta_{\mathbf{r}})^{-1} (1 - \mu)\right\}, \end{split}$$

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where

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$$p = C_{s} \cos \theta_{s} + C_{r} \cos \theta_{r};$$

$$s = C_{s} \sin^{2}\theta_{s} + C_{r} \sin^{2}\theta_{r};$$

$$t = C_{s} \sin \theta_{s} \cos \theta_{s} + C_{r} \sin \theta_{r} \cos \theta_{r};$$

$$\mu = \frac{\left(\frac{\sin \theta_{r} - \frac{t}{p} \cos \theta_{r}}{2}\right)^{2} C_{r}^{2} \cos^{2} \theta_{r}}{(2\sigma^{2})^{-1} + s - \frac{t^{2}}{p}} \times \frac{(C_{r}^{-1} \cos^{-2} \theta_{r} + C_{s}^{-1} \cos^{-2} \theta_{s})}{(2\sigma^{2})^{-1} + s - \frac{t^{2}}{p}};$$

$$Q = a \exp\left(\frac{1}{2a}\right) \frac{1}{4} (\overline{\gamma_{x}^{2}} \overline{\gamma_{y}^{2}})^{-1/2} \sum_{k=0}^{\infty} \frac{a^{-k}}{k!} \left(\frac{\beta}{2}\right)^{2k} \times \frac{\sin \theta_{s} \sin \theta_{r}}{a^{-1/4}} \frac{\Gamma(2k+2)}{\Gamma(k+1)} \cdot W_{-k-3/4, k+3/4} \left(\frac{1}{a}\right)$$

$$- \sin \theta_{s} \sin \theta_{r} a^{-1/4} \frac{\Gamma(2k+3)}{\Gamma(k+2)} \cdot W_{-k-5/4, k+5/4} \left(\frac{1}{a}\right)$$

$$+ 2\cos \theta_{s} \cos \theta_{r} a^{-1/4} \frac{\Gamma(2k+1)}{\Gamma(k+1)} \cdot W_{-k-1/4, k+1/4} \left(\frac{1}{a}\right);$$

$$a = 4 \left(\frac{1}{\overline{\gamma_{x}^{2}}} + \frac{1}{\overline{\gamma_{y}^{2}}}\right)^{-1}; \Delta = \frac{1}{2\overline{\gamma_{x}^{2}}} - \frac{1}{2\overline{\gamma_{y}^{2}}}; \beta = \frac{\Delta a}{2}$$

For a transparent aerosol atmosphere<sup>10</sup> we have

$$a_{s} = \frac{P_{0}e^{-\tau_{1}}}{\pi\alpha_{s}^{2}}; \quad a_{r} = \frac{r_{r}^{2}}{r_{c}^{2}}e^{-\tau_{2}}; \quad C_{s} = (\alpha_{s}L_{s})^{-2};$$

$$C_{\mathbf{r}} = \left( \begin{array}{c} r_{\mathbf{c}} \\ \overline{F} \\ L_{\mathbf{r}} \end{array} \right)^{-2}; \quad \tau_{1} = \int_{0}^{L_{\mathbf{s}}} \sigma(z) dz; \quad \tau_{2} = \int_{0}^{L_{\mathbf{r}}} \sigma(z) dz.$$

To estimate  $a_{s,r}$  and  $C_{s,r}$  in an optically dense atmosphere we have<sup>10</sup>



Here  $2\alpha_s$  is the divergence angle of the source;  $P_0$  is the source output power;  $r_r$ ,  $r_c$  are the effective sizes of the receiving aperture and the circle of scattering for the receiving optical system;  $\sigma^2$ ,  $\overline{\gamma}_{xy}^2$  are the sea surface elevation and slope variances;  $\varepsilon(z)$  and  $\sigma(z)$  are the extinction and scattering coefficients of the medium; F is the focal lengths of the receiver lens;  $\langle \gamma^2(z) \rangle$  is the variance of the beam angular deviation in a single scattering event;  $\tilde{\sigma}$  is the effective scattering factor;  $\tilde{\sigma} = (1 - x_0)\sigma$ ,  $x_0$  is the isotropic part of the scattering phase function;  $^{10} W_{n,m}(z)$  is the Whittaker function;  $\Gamma(k)$  is the gamma-function.

The relationship (3) was obtained in the  $\beta \ll 1$  approximation, which is well satisfied for a wide range of sea wind roughness states.

Consider an extreme case: an isotropic randomly uneven surface ( $\overline{\gamma}_x^2 = \gamma_y^2 = \gamma_0^2$ ) and no atmosphere. The relationship (3) would then coincide with that from Ref. 11.

We now estimate the effect of foam on the illuminance structure in the image plane of the receiver lens. Using the relationships (1) and (3) and the results from Ref. 4, we obtain the following expression for the mean illuminance in the image generated during sensing of a partially foam-covered surface

$$\overline{E}(\overrightarrow{R}) \simeq a_1 \cdot a_2 \exp \left\{ - \left[ R_y \frac{L_r}{F} \right]^2 \left( C_0^{-1} + C_r^{-1} \right)^{-1} \right\} F(x),$$
(4)

where

$$F(x) = \left[ \exp\{-(x + \tilde{q})^2\} + \frac{a_3}{a_2} \exp\{-x^2\} \right]; \quad (5)$$

$$x = \left[ R_x \frac{L_r}{F} \right] b^{1/2}; \quad \tilde{q} = \delta b^{1/2}; \quad b = (1 - \mu) \cos^{-2}\theta_r \times (C_r^{-1} \cos^{-2}\theta_r + C_s^{-1} \cos^{-2}\theta_s)^{-1};$$

$$a_2 = (1 - \Psi) \frac{q^4 V^{20.125}}{q_x^4 (\overline{y_x^2} \, \overline{y_y^2})^{1/2}} \exp\left\{ \frac{q_x^2}{q_z^2 \, 2 \, \overline{y_z^2}} \right\};$$

$$a_1 = \frac{a_s a_r}{L_s^2 L_r^2} (C_s + C_r)^{-1/2} \left[ C_s \cos^2\theta_s + C_r \cos^2\theta_r + 2\sigma^2 C_s C_r \sin^2(\theta_s - \theta_r) \right]^{-1/2};$$

$$a_3 = \Psi \cdot A \cdot Q;$$

$$\tilde{\mu} = 1 + 2\sigma^2 C_s \sin\theta_s \sin(\theta_r - \theta_s) \cos^{-1}\theta_r;$$

$$q_x^2 = (\sin\theta_s + \sin\theta_r)^2; \quad q_z^2 = (\cos\theta_s + \cos\theta_r)^2;$$

$$q^2 = q_x^2 + q_z^2;$$

and  $V^2$  is the Fresnel coefficient for a clean sea surface devoid of foam.

Employing the result from Refs. 4 and 10 we have for the model of foam in the form of a flat Lambertian surface

$$\overline{E}(\overline{R}) \simeq \overline{a}_1 \cdot \overline{a}_2 \exp\left\{-\left[R_y \frac{L_r}{F}\right]^2 (C_s^{-1} + C_r^{-1})^{-1}\right\} \overline{F}(x), (6)$$

where

$$\overline{F}(x) = \left[ \exp\{-(x + \tilde{q})^2\} + \frac{\bar{a}_3}{\bar{a}_2} \exp\{-\frac{x^2}{1-\mu}\} \right].$$
(7)  
$$\overline{a}_3 = W \cdot A \cdot \cos\theta_s \cos\theta_r; \quad \overline{a}_1 = a_1(\sigma = 0);$$
$$\overline{a}_2 = a_2 \cdot (1 + 2\sigma^2 C_s C_r \cdot \sin^2(\theta_s - \theta_r))$$
$$\times (C_s \cos^2 \theta_s + C_r \cos^2 \theta_r)^{-1}).$$

The relationships (4), (6) are obtained for a narrow illuminating beam  $\alpha_s^2 \Box \overline{\gamma}_{x,v}^2$ .

Figure 1 presents computation results for illuminance distribution in the image plane of the receiver lens.

These calculations were conducted for the two models of foam described by relationships (5) and (7) for the following values of the parameters entering them:  $\theta_s = \theta_r = 30^\circ$ ;  $L_s = L_r = 1000$  m;  $\alpha_s = 0.0087$ ;  $r_c/F = 5 \cdot 10^{-5}$ ;  $\mu_s = \mu_r = 0$  (curves 1, 3);  $\mu_s = \mu_r = 0.003$  (curves 2, 4); U = 10 m/s (curves 1, 2), U = 22 m/s (curves 3, 4).



FIG. 1. Spatial distribution of illuminance in the image.

The values  $\overline{\gamma}_x^2$  and  $\overline{\gamma}_y^2$  were calculated from the relationships of Cox and Munk,<sup>12</sup> where the following relations were used for W and  $\sigma$ :  $W = 0.009 \ U^3 - 0.3296 \ U^2 + 4.549 \ U - 21.33$ ;  $\sigma = 0.016 U^2$ , where U is the surface wind speed, m/s.

It is seen from Fig. 1 that both the atmosphere and foam present at the sea surface significantly affect the spatial distribution of image illuminance. In the absence of foam the distribution of image illuminance is asymmetrical relative to the center of the image plane. The latter feature is physically explained by different reflection conditions at different points of the illuminated area. During monostatic sensing this leads to maximum energy coming to the receiver not from the central part of the area, but from one displaced toward the lidar (remaining within the illuminated area, however). At higher source divergences this effect strengthens. More foam at the sea surface (resulting from higher wind speeds U) makes the illuminance distribution more symmetrical with respect to the center of the image plane (foam patches scattered chaotically across the receiver field of view contribute to the illuminance component, symmetrical with respect to the image center). Atmospheric turbidity (at higher beam divergences) leads to the opposite effect; the shift in the illuminance distribution the image plane increases.

The choice of foam model affects the spatial distribution of the illuminance only slightly. Graphs for both models are merged in Fig. 1.

The relationships obtained in this paper can be employed for correcting the atmospheric and sea roughness effects in the signals recorded by lidars sensing the sea surface.

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