ON THE ACCURACY OF CALCULATION OF THE MEAN VALUES AND VARIANCES OF ATMOSPHERIC ABSOPTION AT 10.6 μm

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This paper discusses a technique for the approximate calculation of the mean values of the molecular absorption coefficient, atmospheric optical thicknesses, and the transmission function, and their standard deviations. Use of the statistical extrapolation technique is shown to make it possible to obtain more accurate values of the transmittance of the atmospheric column on the basis of measurements in the ground layer of the temperature and humidity.

Variations in the parameters of molecular absorption in the Earth's atmosphere (such as the absorption coefficient, the optical depth, and the transmittance) are governed by the variability of the following meteorological parameters: pressure, temperature, and water vapor content, which are subject to random spatial-temporal variations, for which reason a statistical approach may be taken to the characteristics of molecular absorption. Such an approach is widely used in meteorology in general. In such an approach one generally obtains the first and second moments of the above optical characteristics from the statistical data on the meteorological parameters.

References 1–3 described an approximate technique for calculating the mean values of the parameters of the molecular absorption and their standard deviations from an atmospheric model that includes the mean values of the air temperature, the concentrations of the various absorbing gases, and their standard deviations, together with the vertical correlation matrices for the air temperature, humidity, and ozone. Because of the limited nature of such statistical information. Refs. 1–3 employed the following approximations:

1) the molecular absorption coefficient was linearized with respect to temperature;

2) unit matrices were used for correlations between the concentrations of minor gas constituents (except for H_2O and O_3);

3) the probability distributions of the meteorological parameters were taken to be normal and lognormal.

The aim of the present study is to find how close the estimated average profiles of the absorption coefficient, the optical depth, and their covariance matrices, retrieved within such an approximation, relate to their actual profiles.

To answer this question, one would have to process the experimental data on the respective optical characteristics of the atmosphere statistically. However, one can only find experimental data on the integral characteristics of radiation extinction in the Earth's atmosphere in the literature. Moreover, even this information is hard to use since aerosol is always present in the real atmosphere, and it is very problematic to separate the molecular extinction from that due to aerosol. Therefore we did the following: we calculated the absorption coefficients and optical depth from a given set of vertical distributions of the pressure, temperature and humidity, and then subjected the obtained profiles to statistical processing.

The numerical model was configured for $\lambda = 10.591$ μm. The principal absorbing gases at this wavelength are H_2O and CO_2. The optical characteristics were calculated using the GEISA atlas as our database.[4] A comparison of the CO_2 spectral absorption parameters from Ref. 4 with the experimental data[5] showed good agreement. For example, the half-widths of lines associated with the transitions P(16)–P(24) coincided to within 1–3%, and the difference in intensities did not exceed 10%. To compute the water vapor continuum absorption we used the empirical formula suggested in Ref. 6.

Our meteorological vertical profiles were constructed from a 10-year series (1960–1970, July) of aerological sounding from the meteorological station "London". All in all, 65 profiles were selected. This number exceeds the minimum needed to obtain confident values of the first and second moments of the variable in question.[7]

Using these meteorological data we directly calculated 65 vertical profiles of the absorption coefficient and optical depth, and from them accurately evaluated the average vertical profiles of the absorption coefficient $\bar{\alpha}^e(H)$, optical depth $\bar{\tau}^e(H)$, and their standard deviations, $\bar{\sigma}^e_\alpha(H)$ and $\bar{\sigma}^e_\tau(H)$, as well as the covariance matrices $V^e_{\alpha\alpha}(H, H')$, $V^e_{\tau\tau}(H, H')$.

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Next, average pressures and temperatures were computed from the same data set in the altitude range from zero to 30 km, along with the temperature autocorrelation coefficients at different altitudes. The mean values of the humidity, their standard deviations, and the autocorrelation matrix were computed between 0 and 7 km, since data on humidity were available only up to a height of 7 km. Above 7 km the vertical profile of the humidity was extrapolated using the average zonal model[7] (midlatitude summer).

Using the thusly constructed meteorological model, approximate values of $\bar{\alpha}^a(H)$, $\sigma_\alpha^a(H)$, $\bar{\tau}^a(H)$, and $\sigma_\tau^a(H)$ were further calculated using the technique described in Refs. 1–3 together with the respective altitudinal covariance matrices $V_{\alpha\alpha}^a(H, H')$ and $V_{\tau\tau}^a(H, H')$.

The results of these calculations are presented in Table I.

TABLE I

H, km	Parameters of molecular absorption							
	$\bar{\alpha}^e$, km^{-1}	$\bar{\alpha}^a$, km^{-1}	$\bar{\sigma}^e$, km^{-1}	σ_α^a, km^{-1}	$\bar{\tau}^e$	$\bar{\tau}^a$	σ_τ^e	σ_τ^a
0	0.27	0.27	0.059	0.069	0	0	0	0
2	0.11	0.11	0.024	0.028	0.35	0.35	0.072	0.079
5	0.045	0.045	0.0073	0.008	0.56	0.55	0.11	0.12
7	0.027	0.027	0.0044	0.0039	0.62	0.62	0.12	0.12
10	0.013	0.013	0.0015	0.0016	0.68	0.68	0.12	0.12
20	0.010	0.010	0.0009	0.0011	0.79	0.78	0.11	0.12
30	0.015	0.014	0.0016	0.0019	0.91	0.90	0.11	0.12

It may be seen from this table that the average values of $\alpha(H)$ and $\tau(H)$, computed following the exact and approximate techniques, coincide within 1–2% in the 0–20 km range, and only at 30 km does the difference between $\bar{\alpha}^e(H)$ and $\bar{\alpha}^a(H)$ reach 6%. The standard deviations $\sigma_\alpha^e(H)$ and $\sigma_\alpha^a(H)$ differ by 10–15%, while the difference between $\sigma_\tau^e(H)$ and $\sigma_\tau^a(H)$ does not exceed 10%. Note that such accuracy is quite acceptable for quantitative optical models of the atmosphere.

It is also of interest to compare the approximate and exact statistical characteristics of molecular absorption in a forecast problem. It has been suggested[1] that the linear statistical extrapolation technique be used for adjusting the optical depth $\tau(H)$ in the layer 0–H:

$$\hat{\tau}(H) = \bar{\tau}(H) + \frac{R_{\alpha\tau}(0, H)\,\sigma_\tau(H)}{\sigma_\alpha(0)}\left[\hat{\alpha}(0) - \bar{\alpha}(0)\right], \tag{1}$$

where $R_{\alpha\tau}(0, H)$ is the correlation coefficient for $\alpha(0)$ and $\tau(H)$ in that layer; $\sigma_\tau(H)$ and $\sigma_\alpha(0)$ are the standard deviations of the optical depth of the layer 0–H and the absorption coefficient at the level $H = 0$ km, respectively; $\hat{\alpha}(0)$ is the value of the absorption coefficient at $H = 0$ km, either measured or computed from the measured meteorological parameters. Lacking data on $\hat{\alpha}(0)$, the mean value $\bar{\tau}$ is employed as an estimate of $\hat{\tau}$. If operational information on the meteorological parameters (including the temperature, pressure, and concentrations of absorbing gases at the surface) is available, the values of the optical depth may be adjusted, whereby the error of such an adjustment is given by the formula

$$\delta_\tau^a(H) = \sigma_\tau^a(H)\sqrt{1 - R_{\alpha\tau}^2(0, H)} \tag{2}$$

The values of σ_τ^a and $R_{\alpha\tau}(0, H)$ in formula (2) are computed using the approximate technique described above, so that the extrapolation error $\delta_\tau^a(H)$ is only approximately known.

The next step consisted in testing the accuracy of the calculations of δ_τ. To this end, we computed the extrapolated expectation value $\hat{\tau}_1(H)$ in the layer 0–7 km for each of 65 values of $\hat{\alpha}_1(0)$ taken at ground-level (see formula (1)), and also the error

$$\delta_\tau^e = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[\hat{\tau}_i(H) - \tau_i(H)\right]^2} \tag{3}$$

where $\tau_1(H)$ is the optical depth for the ith realization of the meteorological parameters. Table II presents the results of such calculations. The maximum deviations of the actual optical depths from their averages and also from the extrapolated values are also presented in Table II. It follows from Table II that $\delta_\tau^a(H) > \delta_\tau^e(H)$, i.e., the "error tube" calculated using formula (2) is wider than the actual one. Taking the value $\delta_\tau^a(H)$ as our estimate causes us to exceed the actual error, so that the probability of the extrapolated value $\hat{\tau}(H)$ falling in the interval $[\bar{\tau}(H) - \delta(H), \bar{\tau}(H) + \delta_\tau(H)]$ is increased.

TABLE II.

Model	Parameters of molecular absorption (0—7 km)					
	$\bar{\tau}^a$	σ_τ^a	δ_τ^e	δ_τ^a	$\max\lvert\bar{\tau} - \tau_1\rvert$	$\max\lvert\hat{\tau} - \tau_1\rvert$
station "London"	0.62	0.12	0.071	0.087	0.29	0.17
Average zonal model	0.71	0.23	0.084	0.14	0.29	0.22

In conclusion let us consider the question of the adequacy of the average zonal model for forecasting molecular absorption characteristics in the area of London. The values of $\bar{\alpha}^a(0)$, $\sigma_\alpha^a(0)$, $\bar{\tau}^a(H)$, and $\sigma_\tau^a(H)$ were computed for the average zonal model (middle altitude summer) using the approximate technique. Further, the values of $\hat{\alpha}_1(0)$ together with the (extrapolated] expectation values $\hat{\tau}_1(H)$ and the extrapolation errors $\delta_\tau^a(H)$ were computed for the London area using formula (2), and the values of $\delta_\tau^e(H)$ — using formula (3). These results are given in Table II. Their analysis yields the following conclusions:

1) the average optical depths differ by 20%;

2) the rms error $\sigma_\tau(H)$ has increased by almost a factor of two, the coefficient of variation $\sigma_\tau(H) / \bar{\tau}(H)$ is about 30%. This fact is explained by the wider geographical region, covering, besides London, also various quasihomogeneous areas;[7]

3) the extrapolation error $\delta_\tau^a(H)$, computed by formula (2) has significantly increased, and exceeds the value of $\delta_\tau^e(H)$ by almost a factor of 1.5.

The second and third conclusions above are not unexpected and vividly illustrate the deterioration of the accuracy of the optical model for geographical areas wider than those for which it was constructed.

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