NUMERICAL SOLUTION OF A NONLINEAR EQUATION FOR A SOUND BEAM IN THE ATMOSPHERE

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A modified splitting algorithm for numerical simulation of the propagation of a high-power acoustic wave in the atmosphere is proposed. The algorithm takes into account diffraction and dissipation of energy. The calculations are compared with the published experimental results (Bochkarev et al. Propagation of Sound and Optical Wave in the Atmosphere).

Increasing the intensity of acoustic beams in acoustic[1,2] and radioacoustic[3] systems for sounding the meteorological parameters of the atmosphere in order to extend the operating range of the system results in am increase of the nonlinear absorption of the acoustic wave. In the process, the errors, for example, in the interpretation of the intensity of the signal that is received by the acoustic locator and carries information about the temperature and wind inhomogeneities of the atmosphere, increase, since the absorption of the signal emitted by the acoustic locator becomes strongly nonlinear. Methods for taking such errors in the sounding systems under consideration into account are either lacking[2] or they are only qualitative.[3] This is attributable primarily to the mathematical difficulties in taking into account simultaneously nonlinearity, diffraction, and dissipation in the theoretical analysis of the propagation of a high-power acoustic beam in the atmosphere. Serious technical difficulties also arise in testing the theoretical solutions obtained.

In this paper we develop a numerical method for solving this problem and we check the computational algorithm on specific experimental data.

The propagation of a high-power acoustic beam in the atmosphere can be described by the following equation in dimensionless variables[4]:

$$N\frac{\partial^2\rho^2}{\partial\theta^2} - \frac{\partial^2\rho}{\partial\theta\partial z} + B\Delta_\perp\rho = -M\frac{\partial^3\rho}{\partial\theta^3}. \qquad (1)$$

Here $\Delta_\perp = \frac{1}{R}\frac{\partial}{\partial R}\left[R\frac{\partial}{\partial R}\right]$ is the Laplacian operator in a plane normal to the direction of propagation; $R = (x^2 + y^2)^{0.5}a^{-1}$; $\theta = \omega\tau$; $\tau = t - xc^{-1}$; and, $z = xL_g^{-1}$. The coefficients $N = L_g(2L_p)^{-1}$, $B = (4\cdot\pi)^{-1}$, and $M = L_g\,\delta$ are the nonlinearity, diffraction, and dissipation parameters, respectively, where $L_g = a^2\lambda^{-1}$, $L_p = (\chi\omega A)^{-1}$; $\chi = (\gamma + 1)\cdot(2\rho_0 c)^{-1}$; $\gamma = C_pC_v^{-1}$; $\omega = 2\pi c\lambda^{-1}$, λ is the wavelength of the sound wave, δ is the sound attenuation coefficient; and ρ_0 is the density of the medium. Equation (1) was written for the normalized perturbation of the density of the medium $\rho = \rho^* A^{-1}$, where ρ^* is the density perturbation, $P = \rho^* c^2$, P is the sound pressure, c is the sound velocity, and A is the amplitude perturbation of the density of the medium on the axis of the radiator at $x = 0$.

No analytic methods for solving Eq. (1) analytically are currently available. Such problems are generally analyzed by solving Eq. (1) numerically with given initial and boundary conditions. An efficient approach is to Fourier transform Eq. (1). This converts the equation into a system of equations for the harmonics:

$$\frac{\partial\rho_m}{\partial z} + \frac{iB}{m}\Delta_\perp\rho_m + Mm^2\rho_m = imN\left[\rho^2\right]_m, \qquad (2)$$

where

$$\rho(\theta, R, z) = \sum_{m=-\infty}^{\infty}\rho_m\exp(im\theta),\quad \left[\rho^2\right]_m = \sum_{-\infty}^{\infty}\rho_\nu\cdot\rho_{m-\nu};$$

$m = \pm1, \pm2, \pm3, \ldots$ is the number of the harmonic; and, $\Delta = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$.

To solve each equation in the system (2) we used a modified splitting method developed by Konyaev[5] for wave-optics problems.

The splitting procedure was applied to each equation in the system (2). It consisted of subdividing the axis of the evolution variable z (the longitudinal coordinate) into segments $\Delta z = z\cdot N_s^{-1}$ and replacing at each step Δz the Eqs. (2) with an equivalent system of equations

$$\begin{cases} \dfrac{\partial\rho_m}{\partial z} = -i\dfrac{B}{m}\Delta_\perp\rho_m, & \rho_m(0) = \rho_m, \\ \dfrac{\partial\rho'_m}{\partial z} = imN\left[\rho'^2\right]_m - Mm^2\rho'_m, & \rho'_m(0) = \rho_m\left[\dfrac{\Delta z}{2}\right], \\ \dfrac{\partial\rho''_m}{\partial z} = -i\dfrac{B}{m}\Delta_\perp\rho''_m, & \rho''_m(0) = \rho'_m(\Delta z). \end{cases} \qquad (3)$$

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In contrast to the familiar one-cycle scheme, the scheme described above is of second-order accuracy with respect to z, and it does not significantly complicate the computational algorithm and it increases the rate of convergence by an order of magnitude.

The first equation of the system (3), (this equation is of the parabolic type) was solved by a modified FFT method.

The modification consists of separating variables in the Laplacian operator and then using the one-dimensional FFT algorithms to solve the separate homogeneous equations. In doing so, the filtering function becomes one-dimensional $H(\kappa) = \exp(-Bm^{-1} \cdot \kappa^2\Delta z)$; this reduces the computer memory required and makes it possible to monitor the edge effects in the case of strong diffraction of the beam.

The diffraction for large values of the wave parameter z was calculated by the coordinate transformation method. The beam was assumed to be collimated up to $z = 1$. For $z > 1$ the beam was assumed to be divergent with geometric divergence $\delta p = L_g r^{-1}$, where r is the radius of the spherical wavefront. The adaptive computational algorithm follows the beam spread and stretches the coordinate grid at the rate of geometric divergence δ_p.

The absorption coefficient for each harmonics f_m studied, (f is the fundamental frequency and $m = 1 \ldots 10$) was calculated by the well-known standard formulas.[6]

The initial conditions for Eq. (2) were given in the form of a beam with a plateau-shaped amplitude distribution across the beam:
$\rho(\tau = 0) = \exp(x^n + y^n)\cos(i\theta)$, where $n = 8$.

The efficiency of the proposed procedure was checked by comparing the numerical results with the experimental data reported by Bochkarev, et al.[1] For this, the most representative dependence of the sound pressure on the electric power of the acoustic source, measured on the axis of the beam at a distance of 30 m from the radiator with the fundamental frequency $f = 3.5$ kHz, was selected from Ref. 1. To compare the calculations with the experiment the values of the electric power were converted into the sound pressure P_0 whose values corresponded to the exact solution of the diffraction problem for a plane rectangular radiating aperture with given dimensions.[7] The reference point for the sound pressure was taken to be the point corresponding to the minimum electric power (100 W) supplied to the radiator.

The result of the comparison is shown in Fig. 1, where the sound pressure P of a real radiator[1] (curve 2) and the sound pressure calculated taking into account the nonlinearity, diffraction, and dissipation of sound (curve 1) are plotted as a function of the pressure P_0 corresponding to the exact solution of the diffraction problem at 30 m from the source for $f = 3.5$ kHz. The insignificant discrepancy between the calculations and the measurements for large values of P_0 is connected with the computational and measurement errors as well as other factors. One reason for the discrepancy is that the nonlinear distortions of the acoustic wave at $z = 0$ were neglected; in Ref. 1 no information about these distortions is given and for such systems they are quite significant (~ 15%). Another reason is that in Eq. (1) atmospheric turbulence is neglected.

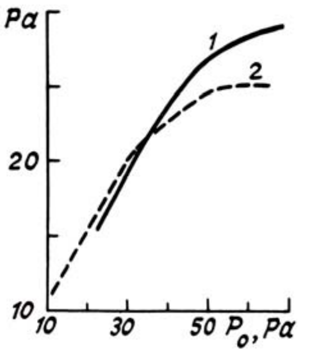

FIG. 1.

Figure 1 also demonstrates the saturation effect, which can be used as an a criterion for optimization of the power of the probe beam. The nonlinear absorption of sound, according to Fig. 1, amounted to ~ 10 dB at a peak power of the radiator.

The favorable comparison the absolute computed and measured values of the pressure allows us to recommend the proposed procedure for solving problems in nonlinear atmospheric acoustics.

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