PHASE COMPENSATION OF THERMAL DISTORTIONS OF LIGHT BEAMS IN THE PRESENCE OF HIGH-FREQUENCY PULSATIONS OF THE WIND VELOCITY

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The effectiveness of programmed and adaptive control of the phase of a light beam, propagating in a medium with large-scale nonuniformities whose freezing time is comparable to the convection time, is analyzed by means of a numerical experiment. A model of an elastic mirror, deformed by transverse forces and bending moments, is used as the actuator. The conditions under which adaptive control gives a higher light density in the observation plane than programmed correction are determined.

One of the basic effects accompanying the propagation of intense light beams in natural media is wind-induced refraction, which in the presence of pulsations of the wind velocity. Is manifested as random wandering and defocusing of the beam. A great deal of attention is now being devoted to the possibility of compensating these distortions with the help of controlling optical systems.[1,2] In the simplest formulation[3,4] the control of the phase of a light beam, propagating in a medium with large-scale nonuniformities, is studied in the quasistationary approximation. In this approximation it is assumed that the period of time during which the nonuniformities are "frozen" is longer than the time necessary to optimize the phase of the beam during the control process. Since such conditions are by no means always satisfied in the real atmosphere, it is of interest to investigate the effectiveness of dynamic phase correction in the case of high-frequency pulsations of the wind velocity. In this case transient processes engendered in the "beam-medium" system by both variations of the phase during control and a change in the realizations of the velocity field along the path are very important. In this paper these questions are analyzed in detail.

MATHEMATICAL MODEL OF THE OPTICAL SYSTEM

The description of the propagation of a light beam under conditions of nonstationary wind-induced refraction reduces to a system of differential equations for the complex amplitude of the electric field E and the temperature T of the medium[1,2,5]

$$2ik\,\frac{\partial E}{\partial z} = \Delta_{\perp}E + 2\,\frac{k^2}{n_0}\,\frac{\partial n}{\partial T}\,TE, \tag{1}$$

$$\frac{\partial T}{\partial t} + (\vec{V}\nabla_{\perp}) = \frac{\alpha I}{\rho C_p}, \qquad I = \frac{cn_0}{8\pi}\,EE^{*}, \tag{2}$$

where k is the wave number, n is the index refraction, ρC_p is the heat capacity per unit volume, α is the absorption coefficient of the medium, $\vec{V}$ is the wind velocity, and $\Delta_{\perp}$ and $\nabla_{\perp}$ are the Laplacian operator and the gradient with respect to the transverse coordinates x and y. The boundary condition for the field E is given at the point of entry into the atmosphere ($z = 0$):

$$E(x,\,y,\,0,\,t) = E_0(x,\,y)\,f(t)\,\exp(iU(x,\,y,\,t)), \tag{3}$$

where $E_0(x, y)$ is a Gaussian amplitude profile; $f(t)$ is the temporal envelope of the light pulse; and, $U(x, y, t)$ is the controlled wavefront, formed by the phase corrector (elastic mirror). For normal incidence on the mirror, whose deflection is $\omega(x, y, t)$, the wavefront of the reflected beam is given by

$$U(x,\,y,\,t) = 2k\,\omega(x,\,y,\,t). \tag{4}$$

In the thin-plate approximation the deflection $\omega(x, y, t)$ is described by a biharmonic equation[6] and depends on the controlling actions (the coordinates of the control $\vec{F}(t)$). In this work we employ a basic model of a corrector,[7] clamped at the center and deformed by transverse forces applied to four external rods. As shown in Ref. 8, moment-force control realized in this case makes it possible to reproduce the lowest optical modes with the mirror more accurately than in the case of purely force control.[7]

The degree of nonlinear distortions of the beam along the path is characterized by the parameter

$$R_V = \frac{2k^2a_0^3\alpha(\partial n/\partial T)}{n_0\rho C_p V n_0}\,I_0, \tag{5}$$

determined based on the average modulus of the wind velocity $V_0 = \langle V\rangle$. Here a_0 is the radius of the

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beam at the $1/e$ level and I_0 is the power density on the axis of the beam in the plane $z = 0$. The field concentration in the observation plane $z = z_0$ is estimated with the help of the focusing criterion

$$J_f(t) = \frac{1}{P_0} \iint \exp\left[-\frac{x^2 + y^2}{S_t^2}\right] I(x, y, z_0, t)\, dxdy, \quad (6)$$

where P_0 is the total power in the beam and S_t is the effective radius of the receiving aperture.

The envelope of the light pulse is modeled by the step function

$$f(t) = \begin{cases} 0 \text{ at } t < 0, \\ 1 \text{ at } t \geq 0. \end{cases} \quad (7)$$

For the time scale it is convenient to choose the averaged convection time $\tau_V = a_0/V_0$.

THE CONTROL ALGORITHMS AND THE MODEL OF THE MEDIUM

We shall study the propagation of a light beam on a horizontal path above a uniform underlying surface. Under such conditions the random wind-velocity field $\vec{V}(x, y, z, t)$ is statistically stationary. Since near the ground the vertical component of the velocity is on the average small compared with the horizontal component,[9] it can be assumed that the average velocity $\vec{V}_0 = \langle \vec{V} \rangle$ is parallel to the surface. In view of the fact that the components of the velocity perpendicular to the optical axis Z has the determining effect on the formation of the thermal lens in the light-beam channel we shall assume that $\vec{V}_0$ is the components of the velocity lying in the plane XOY. The pulsations of the wind velocity are associated with large-scale vortices, whose size L_V is much greater than the radius of the beam a_0. For this reason, the flow velocity can be assumed to constant across the beam, i.e., $\vec{V} = \vec{V}(z, t)$.

The estimates in Ref. 9 show that the maximum of the spectrum of fluctuations of the vertical component of the velocity lies in the range of frequencies ν in the interval from $0.1V_0/h \leq \nu \leq V_0/h$, where h is the altitude of the path. Thus the characteristic period of the pulsations $T_V = 1/\nu$ is related with the convection time τ_V by the relation $T_V \simeq h\tau_V/a_0$. Therefore on horizontal paths near the ground it is possible to have a regime of quite rapid pulsations of the velocity, during which the transient processes in the "beam-medium" system are very significant.

In the numerical model the random velocity field was rendered discrete in time and space.[4] The time dependence of the velocity was replaced by a collection of random functions $\vec{V}_1(z)$ whose average value is equal to $\vec{V}_0$. The spatial structure of $\vec{V}_1(z)$ was modeled by subdividing the path $0 \leq z \leq z_0$ into segments of length Δz of the order of the characteristic scale of the fluctuations L_V. The velocity of the medium was assumed to be constant within each segment and the velocities in neighboring segments were assumed to be statistically independent. When the model of the atmosphere was implemented on a computer the velocity of the medium in the segments was given in the form of vector sums of the regular component $\vec{V}_0$, parallel to the X-axis, and two fluctuation components δV_x and δV_y, which had a centered normal distribution. The regular component $\vec{V}_0$ was assumed to be constant for all z and t, while δV_x and δV_y were generated with the help of a random number generator. The standard deviations were identical for both fluctuation components and were chosen from the interval $0.1V_0 \leq \sigma_V \leq 0.5V_0$. The velocity pulsations were simulated by changing the realization $V_1(z)$ abruptly, and the period of the pulsations T_V was varied in the range $\tau_V \leq T_V \leq 50\tau_V$.

In this paper we compare the efficiencies of two algorithms for phase compensation of the thermal self-action: aperture sounding based on non-steady-state parameters of the light field in the medium[10] and programmed correction.[11] In the first algorithm the controls on the mirror are determined in accordance with the gradient procedure of "ascending a hump"

$$\vec{F}(t) = \vec{F}(t - \tau_d) + \alpha \text{ grad } J_f(t), \quad (8)$$

where τ_d is the delay time in the control system, α is the length of the gradient step, and the components of vector gradJ_f are calculated by making trial variations of the controls $\vec{F}$ with the parameters of the medium along the path help fixed.

In the programmed (*a priori*) correction algorithm the nonlinear phase increment is calculated by time-averaging the temperature field induced by the beam with no control

$$\langle T(x, y, z) \rangle = \frac{1}{\tau_*} \int_0^{\tau_*} T(x, y, z, t)\, dt.$$

In accordance with Ref. 11 the correcting phase $U(x, y)$ is determined as

$$U(x, y) = -\frac{k}{n_0} \frac{\partial n}{\partial T} \int_0^{z_0} \langle T(x, y, z) \rangle\, dz. \quad (9)$$

The method of least squares is used to approximate the profile found $U(x, y)$ by the surface of the lastic mirror.

RESULTS OF NUMERICAL ANALYSIS

It is convenient to perform the preliminary investigations of the effectiveness of programmed correction in the presence of pulsations of the wind velocity on the basis of the quasistationary approximation (neglecting the transient processes accompanying a change in realization); this is valid for $T_V \geq 50\tau_V$. In this case large amounts of computer time are not required and the statistical analysis of the parameters of the beam can be performed in the observation plane based on a sufficiently large number of realizations M (in this paper $M \sim 100$).

The effectiveness of programmed control in the case of moderate nonlinearity of the medium is illustrated in Fig. 1. One can see that the correction performed with respect to the averaged parameters of the medium, determined for a quite large number of realizations $M_{cor} \simeq 60$, gives a focusing criterion that is 80 to 90% higher than in the case when there is no control, and in addition the average gain is virtually independent of the average amplitude of the velocity pulsations σ_V. At the same time, if in determining the phase correction the parameters of the medium are averaged over a small number of realizations M_{cor}, then programmed correction is less effective (Fig. 2). As expected, for small values of the statistical spread in the energy parameters of the beam Increases, since the characteristics of the medium are not determined reliably in this case.

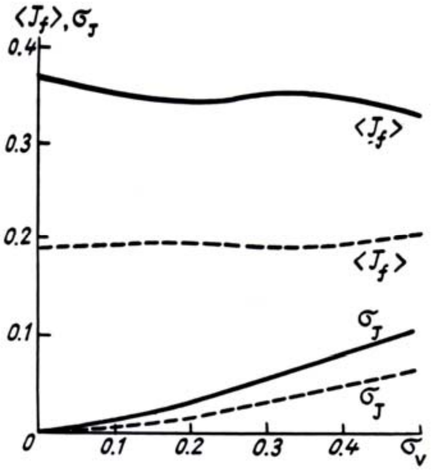

FIG. 1. The average valve $\langle J_f \rangle$ and standard deviation σ_J of the focusing criterion as a function of the amplitude of the velocity fluctuations σ_V in the absence of control (dashed lines) and in the case of programmed correction (solid lines). The averaging was performed over 100 realizations. Conditions of propagation: $R_V = -20$, $z_0 = 0.5ka_0^2$.

There are a number of peculiarities in adaptive correction in a medium in which there are pulsations of the wind velocity. In particular, when the velocity realizations change the nonlinear thermal lens induced on path by the bean can change substantially, so that a search for the optimal control coordinates must be made anew for each concrete realization. In addition, phase optimization must be performed over the time T_V during which the nonuniformities are "frozen". Thus the presence of pulsations of the wind velocity imposes stringent requirements on the speed of the control algorithms.

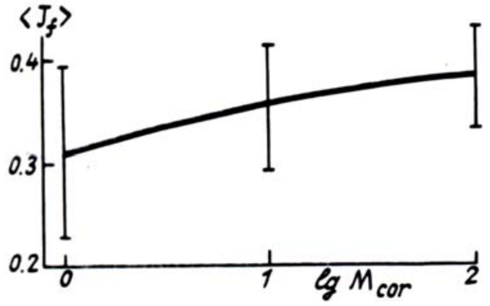

FIG. 2. The average valve $\langle J_f \rangle$ of the focusing criterion as a function of the number of realizations M_{cor} for which the averaging is performed in the process of programmed correction. The statistical spread σ_J is indicated at several points by vertical bars. The conditions of propagation are: $R_V = -20$, $z_0 = 0.5ka_0^2$, and $\sigma_V = 0.3V_0$.

The most promising algorithm in such problems is the aperture-sounding algorithm based on the transient parameters of the light field in the medium. As shown in Ref. 10, this algorithm is indeed fast, but its stability depends on the length a of the gradient step. The optimal step length α_{opt}, in turn, is very sensitive to the effective nonlinearity on the path, which in the presence of wind-velocity pulsations is highly variable. The problem of choosing a is simplified by transferring from zonal control (control based on separate drives) to nodal control, when the actions on the mirror art combined so as to be able to perform separate sounding with respect to the lowest modes of the wavefront: tilts, defocusing, and astigmatism. Numerical experiments show that modal control makes it possible to increase the stability of aperture sounding with respect to fluctuations of the nonlinearity parameter along the path.

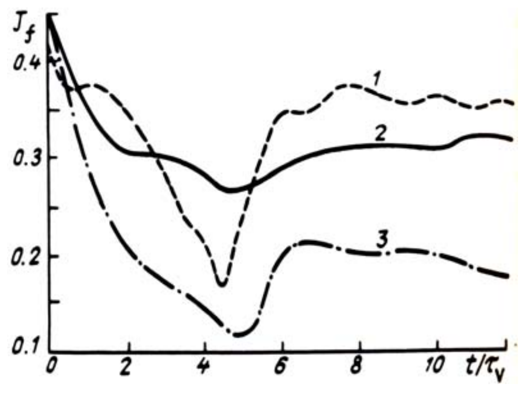

FIG. 3. The focusing criterion J_f as a function of time in the case of programmed control (curve 1) and adaptive control (curve 2) as well as in the absence of control (curve 3) in the presence of wind-velocity pulsations: $\sigma_V = 0.3V_0$, $T_V = 2\tau_V$, $R_V = -20$, $z_0 = 0.5ka_0^2$.

In Fig. 3 the process of dynamic compensation of nonstationary thermal defocusing with the period $T_V = 2\tau_V$, during with the realizations change, is compared with the *a priori* correction, whose phase was calculated with $\tau_{av} \simeq 50\tau_V$.

The time-averaged values of the focusing criterion $\langle J_f \rangle$ are presented in Fig. 4 as a function of the average nonlinearity of the medium on the path $\langle |R_V| \rangle$ for *a priori* and adaptive correction. It is obvious that the effectiveness of adaptive control relative to programmed control increases as beam power (the parameter $|R_V|$) increases. Increasing the period T_V during which the pulsations are "frozen" also results in an increase of the relative effectiveness of the adaptive correction (Fig. 4b). For $T_V \geq (2.5 - 3)\tau_V$ the concentration of the field in the observation plane, given by the adaptive- correction algorithm, is steadily higher than the concentration achieved using programmed control based on the average parameters of the medium.

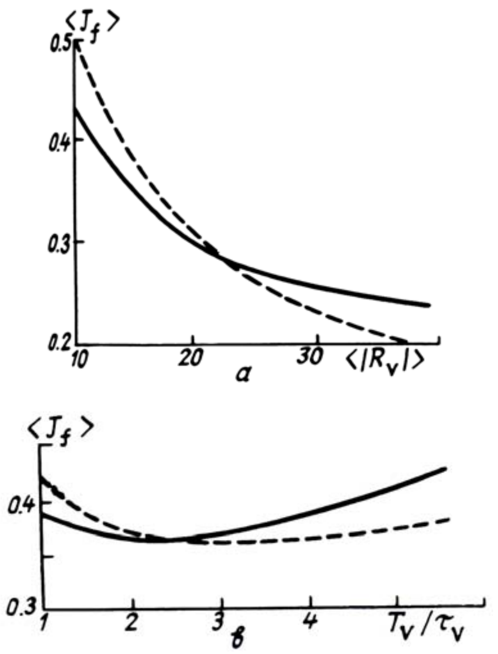

FIG. 4. The average valves of the focusing criterion versus the average nonlinearity of the medium (a — $T_V = 2\tau_V$) and the period of the wind-velocity pulsations (b — $\langle R_V \rangle = -20$) with programmed (dashed lines) and adaptive *(solid lines) control of the beam. The path length $z_0 = 0.5ka_0^2$.*

Comparing the result of modeling of adaptive and *a priori* correction leads to the following conclusions. In the case of weak or moderate average nonlinearity of the medium ($|R_V| \leq 20$) or with high-frequency pulsations of the velocity with the period $T_V \leq 2\tau_V$ programmed correction is the most effective method for compensating thermal distortions. To obtain reliable information about the conditions of propagation of a beam the parameters of the medium must be averaged a priori over not less than 30–50 realizations of the wind velocity. As the power of the radiation and the period during which the velocity pulsations are "frozen" increase the aperture-sounding algorithm becomes relatively more efficient and it becomes preferable for $|R_V| > 20$ or $T_V \geq 2\tau_V$.

In the range of parameters studied here adaptive correction also has the advantage that it does not require *a priori* knowledge about the propagation medium, since information about the medium is acquired directly during the control process.

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