OPTICAL SOUNDING BEAM PARAMETERS IN ABERRATING REFRACTION CHANNELS

I.P. Lukin

Institute of Atmospheric Optics,
Siberian Branch of the Academy of Sciences of the USSR, Tomsk
Received April 27, 1990

This paper presents a solution for the second-order mutual coherence function of optical sounding beam propagating through a refraction channel with small aberrations. Expressions are derived for the coordinates of the beam center of gravity and effective radius, and the distribution of the sounding beam intensity over its cross section. An analysis of the effects of aberrations of different orders in the channel on optical beam characteristics is made. It is shown that the aberration-free approximation can be used to describe a narrow (in comparison with the width of the refraction channel) sounding beam in a defocusing channel with small aberrations (e.g., with a Gaussian or Gaussian-like profile) only for sounding paths whose length does not exceed a few focal lengths of the refraction channel.

The data from gradient refraction measurements of variations of the regular dielectric constant (refractive index) in various media have found wide applications in science and industry.[1–3] One of their most interesting applications is optical sounding of cylindrical volumes of air with lateral parabolic gradient of the dielectric constant, known as "lens-type" refraction channels.[1–5] Techniques for paraxial sounding of such aberration-free refraction channels were considered in Refs. 4 and 5. It was shown there that the greatest amount of шnformation is contained in the wavefront curvature, the shift of the sounding beam center of gravity, and changes in the effective beam radius. Reference 6 considered the distorting effects of aberrations in a refraction channel upon the curvature of the sounding beam wavefront and upon the result of image refocusing behind the focusing lens. The present paper presents a theoretical study of the spatial intensity distribution in a sounding beam propagating close to the optical axis of the aberrating refraction channel. The domain of applicability in which the aberration-free approximation provides an adequate description of the beam is evaluated, including: the coordinates of its center of gravity, its effective radius, and the maximum intensity on the refraction channel optical axis.

Propagation of a sounding beam along the refraction channel with aberrations is described by the parabolic equation:

$$2ik\,\frac{\partial E(x,\ \rho)}{\partial x} + \Delta_{\perp}E(x,\ \rho) + k^2 G\left(\frac{\rho}{F_0}\right)E(x,\ \rho) = 0,$$

$$E(0,\ \rho) = E_0(\rho), \tag{1}$$

where $k = 2\pi/\lambda$; λ is the optical radiation wavelength; $\Delta_{\perp} = \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$ is the transverse Laplacian

$$G\left(\frac{\rho}{F_0}\right) = \varepsilon_2 + \alpha\left(\frac{\rho}{F_0}\right) + \left(\frac{\rho}{F_0}\right)^2 + \gamma\,\frac{\rho}{\rho}\left(\frac{\rho}{F_0}\right)^3 + \beta\left(\frac{\rho}{F_0}\right)^4;$$

ε_2 is the change in the medium dielectric constant along the optical axis of the refraction channel; F_0 is the refraction channel focal length; α, γ, and β are vector and scalar coefficients which characterize the aberrations in the refraction channel

$$\left(\gamma\,\frac{\rho}{\rho}\left(\frac{\rho}{F_0}\right) << 1,\ \beta\left(\frac{\rho}{F_0}\right)^2 << 1\right).$$

The equation for the second-order mutual coherence function of $E(x,\ \rho)$ in Eq. (1), $\Gamma_2(x,\ \rho_1,\ \rho_2) = E(x,\ \rho_1)E^*(x,\ \rho_2)$, has the form

$$ik\,\frac{\partial\Gamma_2(x,\ R,\ \rho)}{\partial x} + \frac{\partial^2\Gamma_2(x,\ R,\ \rho)}{\partial R\,\partial\rho} + \alpha\rho\,\frac{k^2}{2F_0}\Gamma_2(x,\ R,\ \rho) + \frac{k^2}{F_0^2}R\rho\Gamma_2(x,\ R,\ \rho) = \frac{k^2}{F_0^2}f(R,\ \rho)\Gamma_2(x,\ R,\ \rho).$$

Here

$$f(R,\ \rho) = -\frac{1}{2}\left\{\gamma\left[2R(R\rho) + R^2\rho + \frac{1}{4}\rho^2\rho\right] + \frac{\beta}{F_0}\left(4R^2 + \rho^2\right)R\rho\right\},$$

$R = 1/2\left(\rho_1 + \rho_2\right)$, and $\rho = \rho_1 - \rho_2$.

We will use perturbation theory to find the solution of Eq. (2)

0235-6880/90/09 840-05 $02.00

$$\Gamma_2(x, R, \rho) \simeq \Gamma_2^{(0)}(x, R, \rho) + \Gamma_2^{(1)}(x, R, \rho),$$

where the function $\Gamma_2^{(0)}(x, R, \rho)$ is the solution of the equation

$$ik \frac{\partial \Gamma_2^{(0)}(x, R, \rho)}{\partial x} + \frac{\partial^2 \Gamma_2^{(0)}(x, R, \rho)}{\partial R\, \partial \rho} + \alpha\rho \frac{k^2}{2F_0} \Gamma_2^{(0)}(x, R, \rho) + \frac{k^2}{F_0^2} R\rho\Gamma_2^{(0)}(x, R, \rho) = 0 \qquad (3)$$

satisfying the boundary condition

$$\Gamma_2^{(0)}(0, R, \rho) = E_0(R + \rho/2)E_0^*(R - \rho/2),$$

and the function $\Gamma_2^{(1)}(x, R, \rho)$ is the solution of the equation

$$ik \frac{\partial \Gamma_2^{(1)}(x, R, \rho)}{\partial x} + \frac{\partial^2 \Gamma_2^{(1)}(x, R, \rho)}{\partial R \partial \rho} + \alpha\rho \frac{k^2}{2F_0} \Gamma_2^{(1)}(x, R, \rho) + \frac{k^2}{F_0^2} R\rho\Gamma_2^{(1)}(x, R, \rho) = \frac{k^2}{F_0^3} f(R, \rho)\Gamma_2^{(0)}(x, R, \rho) \qquad (4)$$

satisfying the boundary condition

$$\Gamma_2^{(1)}(0, R, \rho) = 0.$$

The solution of Eq. (3) is known,[3–5] and Eq. (4) is solved applying the method of characteristics to its Fourier transform. Thus the solution for the second-order mutual coherence function in the defocusing refraction channel with aberrations has the form

$$\begin{aligned}\Gamma_2(x, R, \rho) \simeq\ & \Gamma_2^{(0)}(x, R, \rho) - \\ & - \frac{ik}{4\pi^2 F_0^3} \int_0^\infty dx' \exp\left\{- i\, \frac{\alpha\rho}{2}\, k\, \mathrm{sh}\left[\frac{x' - x}{F_0}\right]\right\} \times \\ & \times \iint_{-\infty}^{\infty} d\kappa\, \exp\left\{i\kappa R + i\, \frac{\alpha\kappa F_0}{2}\left[1 - \mathrm{ch}\left(\frac{x' - x}{F_0}\right)\right]\right\} \times \\ & \times \iint_{-\infty}^{\infty} dR' f\left[R', \mathrm{ch}\left(\frac{x' - x}{F_0}\right)\rho + \frac{F_0}{k}\mathrm{sh}\left(\frac{x' - x}{F_0}\right)\kappa\right] \times \\ & \times \Gamma_2^{(0)}\left[x', R', \mathrm{ch}\left(\frac{x' - x}{F_0}\right)\rho + \frac{F_0}{k}\mathrm{sh}\left(\frac{x' - x}{F_0}\right)\kappa\right] \times \\ & \times \exp\left\{- i\, \frac{k}{F_0}\mathrm{sh}\left(\frac{x' - x}{F_0}\right)\rho R' - i\, \mathrm{ch}\left(\frac{x' - x}{F_0}\right)\kappa R'\right\}. \end{aligned} \qquad (5)$$

The coordinates of the center of gravity of the sounding beam $R_k(x)$ can be determined as follows:

$$R_k(x) = \frac{i}{\hat{\Gamma}_2(x, 0, 0)} \times \left.\frac{\partial \hat{\Gamma}_2(x, \kappa, 0)}{\partial \kappa}\right|_{\kappa=0} \qquad (6)$$

where

$$\hat{\Gamma}_2(x, \kappa, \rho) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} dR\Gamma_2(x, R, \rho)\exp\{- i\kappa R\}.$$

Let the Gaussian sounding beam propagate along the refraction channel close to its optical axis. Then, as was demonstrated in Refs. 4 and 5

$$\Gamma_2^{(0)}(x, R, 0) = E_0^2 \frac{a_0^2}{a^2(x)} \exp\left\{- \frac{[R - R_c(x)]^2}{a^2(x)}\right\}, \qquad (7)$$

where a_0^2 and $a^2(x)$ are the initial and the current radii of the sounding beam;[4,5] $R_c(x)$ is the displacement of the center of gravity of the sounding beam in the channel with linear ($\alpha \neq 0$) and parabolic inhomogeneities of the medium dielectric constant. Using Eqs. (5), (6), and (7) it can be shown that the shift of the center of gravity of the sounding beam in a refraction channel with aberrations is described by the expression

$$\begin{aligned} R_k(x) \simeq\ & R_c(x) - 2\gamma \frac{1}{F_0^2} \int_0^x dx' a^2(x')\, \mathrm{sh}\left[\frac{x' - x}{F_0}\right] \times \\ & \times \left[1 + \frac{R_c^2(x')}{a^2(x')}\right] - 4\beta \frac{1}{F_0^3} \int_0^x dx' a^2(x')\, \mathrm{sh}\left[\frac{x' - x}{F_0}\right] \\ & \times R_c(x')\left[1 + \frac{1}{2} \times \frac{R_c^2(x')}{a^2(x')}\right]. \end{aligned} \qquad (8)$$

For a wide (in the diffraction sense: $ka_0^2 / F_0 \gg 1$)) collimated beam propagating parallel to the channel optical axis (at $\alpha = 0$) we have

$$a(x') \simeq a_0 \mathrm{ch}\left[\frac{x'}{F_0}\right], \quad R_c(x') = R_0 \mathrm{ch}\left[\frac{x'}{F_0}\right]$$

where R_0 is the initial shift of the sounding beam center of gravity relative to the refraction channel axis,[4,5] so that expression (8) becomes

$$\begin{aligned} R_k(x) \simeq\ & R_0 \cdot f_1\left[\frac{x}{F_0}\right] + \gamma\left[1 + \frac{F_0^2}{a_0^2}\right]\left[\frac{a_0}{F_0}\right]^2 F_0 f_2\left[\frac{x}{F_0}\right] + \\ & + \beta\left[1 + \frac{1}{2}\frac{R_0^2}{a_0^2}\right]\left[\frac{a_0}{F_0}\right]^2 R_0 \cdot f_3\left[\frac{x}{F_0}\right], \end{aligned} \qquad (9)$$

where

$$f_1\left[\frac{x}{F_0}\right] = \mathrm{ch}\left[\frac{x}{F_0}\right];$$

$$f_2\left(\frac{x}{F_0}\right) = 2\left[\mathrm{sh}^2\left(\frac{x}{F_0}\right) + \frac{1}{3}\,\mathrm{sh}^4\left(\frac{x}{F_0}\right) - \frac{1}{3}\,\mathrm{ch}^4\left(\frac{x}{F_0}\right) + \right.$$

$$\left. + \frac{1}{3}\,\mathrm{ch}\left(\frac{x}{F_0}\right)\right];$$

$$f_3\left(\frac{x}{F_0}\right) = \frac{3}{2}\left(\frac{x}{F_0}\right)\mathrm{sh}\left(\frac{x}{F_0}\right) + \frac{3}{2}\,\mathrm{sh}^2\left(\frac{x}{F_0}\right)\mathrm{ch}\left(\frac{x}{F_0}\right) +$$

$$+ \mathrm{sh}^2\left(\frac{x}{F_0}\right)\mathrm{ch}^3\left(\frac{x}{F_0}\right) - \mathrm{ch}^5\left(\frac{x}{F_0}\right) + \mathrm{ch}\left(\frac{x}{F_0}\right)$$

$$(f_1,\ f_2,\ f_3 > 0).$$

Expressions (8) and (9) demonstrate that if the inputs from the linear and the quadratic components of the inhomogeneity in the medium dielectric constant to the sounding beam center of gravity are additive,[4,5] the aberrational components of the refraction channel profile make contributions which depend on $R_c(x)$. Odd-order aberrations make such contributions even for $R_c = 0$, while the even-order ones do not. If $\beta > 0$, then the shift in the direction of R_0 is larger than for the aberration-free profile, and vice versa. The intensity of *a* sounding pulse propagating along the optical axis of the refraction channel (at $\alpha = 0$) can be obtained from relation (5) if we substitute into it the aberration-free approximation of the second-order mutual coherence function:

$$\Gamma_2^{(0)}(x,\ R,\ \rho) = \frac{E_0^2 a_0^2}{a^2(x)}\exp\left\{-\frac{R^2 + \rho^2/4}{a^2(x)} - \frac{ik}{F_0}S(x)R\rho\right\},$$

where $S(x)$ is the beam wavefront curvature.[4,5] Calculations show that for a wide collimated beam, when $S(x) \simeq \mathrm{th}(x/F_c)$ (Refs. 4 and 5),

$$I(x,\ R) = \Gamma_2(x,\ R,\ 0) \simeq I_\sigma(x,\ R)\left\{1 - \gamma\,\frac{R}{F_0}\,\hat{f}_0\left(\frac{x}{F_0}\right) - \right.$$

$$- \gamma\,\frac{R}{F_0}\left(1 - \frac{1}{2}\,\mathrm{ch}^{-2}\left(\frac{x}{F_0}\right)\frac{R^2}{a_0^2}\right)\hat{f}_1\left(\frac{x}{F_0}\right) - \beta\left(\frac{a_0}{F_0}\right)^2 \times$$

$$\times \left(1 - \mathrm{ch}^{-2}\left(\frac{x}{F_0}\right)\frac{R^2}{a^2}\right)\hat{f}_2\left(\frac{x}{F_0}\right) + \beta\left(\frac{a_0}{F_0}\right)^2 \times$$

$$\left. \times \left(1 - 2\mathrm{ch}^{-2}\left(\frac{x}{F_0}\right)\frac{R^2}{a_0^2} + \frac{1}{2}\,\mathrm{ch}^{-4}\left(\frac{x}{F_0}\right)\frac{R^4}{a_0^4}\right)\hat{f}_3\left(\frac{x}{F_0}\right)\right\}, \qquad (10)$$

where

$$\hat{f}_0\left(\frac{x}{F_0}\right) = \mathrm{ch}^{-2}\left(\frac{x}{F_0}\right)\cdot f_2\left(\frac{x}{F_0}\right);$$

$$\hat{f}_1\left(\frac{x}{F_0}\right) = 6\left[\mathrm{ch}\left(\frac{x}{F_0}\right) - 1\right];$$

$$\hat{f}_2\left(\frac{x}{F_0}\right) = 4f_2\left(\frac{x}{F_0}\right);$$

$$\hat{f}_3\left(\frac{x}{F_0}\right) = 8\mathrm{ch}^2\left(\frac{x}{F_0}\right)\left[\mathrm{ch}\left(\frac{x}{F_0}\right) - 1\right];$$

$$I_\sigma(x,\ R) = \frac{E_0^2}{\mathrm{ch}^2\left(\frac{x}{F_0}\right)}\exp\left\{-\,\mathrm{ch}^{-2}\left(\frac{x}{F_0}\right)\frac{R^2}{a_0^2}\right\}$$

is the intensity distribution of the sounding beam in the aberration-free approximation.[4,5] In particular, on the optical axis of the refraction channel the expression for the sounding beam intensity acquires the simplest possible form

$$I(x,\ 0) \simeq E_0^2\mathrm{ch}^{-2}\left(\frac{x}{F_0}\right)\left\{1 - \beta\left(\frac{a_0}{F_0}\right)^2 f_1\left(\frac{x}{F_0}\right)\right\}, \qquad (11)$$

where

$$f_1\left(\frac{x}{F_0}\right) = 4f_2\left(\frac{x}{F_0}\right) - \hat{f}_3\left(\frac{x}{F_0}\right),\quad (f_1 > 0).$$

For $\beta > 0$ the sounding beam intensity along the channel optical axis is less than in the aberration-free case, while for $\beta < 0$ it is geater. Redistribution of intensity within the beam due to the third order aberration results in its increase along the vector γ, and to its decrease in the opposite direction, while on the channel axis the intensity remains the same as · in the aberration-free case.

Knowing the intensity distribution across the beam, one may compute its initial characteristics: the optical radiation flux, the coordinates of the center of gravity, and the effective beam size (radius). Integrating expression (10), one can show that the optical radiation flux remains unchanged:

$$P = \iint_{-\infty}^{\infty} dR\,I(x,\ R) = \pi a_0^2 E_0^2,$$

The expression for the beam center of gravity

$$R_k = \frac{1}{P}\iint_{-\infty}^{\infty} dR\ R\,I(x,\ R) \simeq \gamma\left(\frac{a_0}{F_0}\right)^2 F_0\cdot f_2\left(\frac{x}{F_0}\right)$$

Coincides with relation (9) at Rc = 0, and the effective beam radius is then determined by the following expression[3]:

$$a_{\mathrm{eff}}^2(x) = \frac{1}{P}\iint_{-\infty}^{\infty} dR\ R^2 I(x,\ R) \simeq$$

$$\simeq a^2(x)\left\{1 + \beta\left(\frac{a_0}{F_0}\right)^2 f_a\left(\frac{x}{F_0}\right)\right\}, \qquad (12)$$

where

$$f_a\left(\frac{x}{F_0}\right) = 4f_2\left(\frac{x}{F_0}\right)\quad (f_a > 0).$$

It follows from Eq. (12) that the effective beam radius changes under the impact of only the even-order terms in the power-series expansion of the gradient of the channel dielectric constant.

The results obtained above enable one to evaluate the applicability of the aberration-free approximation to the case when a sounding beam, narrow in comparison with the refraction channel width, propagates through a defocusing refraction channel with low aberrations. Analyzing Eqs. (9)–(12), one can see that the effect of such aberrations can be neglected if the following conditions are satisfied:

$$4\beta\left(\frac{a_0}{F_0}\right)^2 f_2\left(\frac{x}{F_0}\right) < 1,$$

$$\beta\left(1+\frac{1}{2}\frac{R_0^2}{a_0^2}\right)\left(\frac{a_0}{F_0}\right)^2 \frac{f_3\left(\frac{x}{F_0}\right)}{f_1\left(\frac{x}{F_0}\right)} < 1, \qquad (13)$$

$$\gamma\left(1+\frac{R_0^2}{a_0^2}\right)\left(\frac{a_0}{F_0}\right)^2 \frac{F_0}{R_0}\frac{f_2\left(\frac{x}{F_0}\right)}{f_1\left(\frac{x}{F_0}\right)} < 1.$$

Consider, for example, a symmetric Gaussian profile of the medium dielectric constant in the refraction channel.[7] Conditions (13), in this case, take on the following form:

$$\left(\frac{a_0}{a_k}\right)^2 f_2\left(\frac{x}{F_0}\right) < 1,$$

$$\left(1+\frac{1}{2}\frac{R_0^2}{a_0^2}\right)\left(\frac{a_0}{a_k}\right)^2 \frac{f_3\left(\frac{x}{F_0}\right)}{f_1\left(\frac{x}{F_0}\right)} < 1, \qquad (14)$$

Here a_k is the radius of the refraction channel ($a_k \gg a_0$). Calculations show that conditions (14) can be satisfied for $(a_k/a_0) \sim 10^{-1}$ if $x/F_0 < 1$ and $(a_k/a_0) \sim 10^{-2}$ if $x/F_0 < 3$.

To summarize: the present study, together with Refs. 6 and 7, demonstrates that the aberration-free approximation may be used to describe the parameters of a narrow sounding pulse propagating in a defocusing weakly-aberrating channel (e.g., with a Gaussian or Gaussian-like profile). However, it is applicable to short paths only, not longer than a few focal lengths of the refraction channel (F_0).

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