MULTIELEMENT IMAGE CORRECTION SYSTEM

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The effectiveness of using an adaptive four-element segmented mirror, each element of which makes adjustments only for the tilts in two perpendicular directions, is investigated. An optical arrangement of the experiment is proposed. The wavefront sensor consists of four identical devices for measuring the position of the center of gravity of the image within each subaperture. The limiting resolution for such an optical system is analyzed.

In the last few years there have appeared a considerable number of works[1–6] on the important question of the application of the means and systems of adaptive optics for improving the quality of an optical image formed through the atmosphere. In these works, the theoretical and technical questions of constructing an optical adaptive telescope were solved to one extent or another.[2,5,6] A ground-based adaptive optical telescope with a quite large diameter (so that it would make sense to implement an expensive program for the development of an adaptive optical system) is required to eliminate the effect of atmospheric turbulence on the arriving optical radiation as well as the effects of an entire spectrum of mechanical perturbations of the elements of its support structure.

It is now recognized that it is desirable to use for the first loop of adaptive correction in telescopes a system for making adjustments (compensating) for displacements of the image, which, as a .rule, employs meters that measure the position of the center of gravity of the optical image formed. A number of telescope designs have now been realized in which the displacement of the center of gravity of the image is compensated. Actually the problem is to develop a highly accurate teleguide, which is a component of most telescopes.

The next step in making more complicated adaptive optical systems — correction of higher order aberrations of the phase front — already requires the development of specialized means: a wavefront sensor and a controllable active (adaptive) mirror. Two types of mirrors can be used: a segmented mirror, which performs zonal correction, and a flexible mirror, which performs modal correction of the aberrations of the phase front. In both cases, however, it is necessary to have a wavefront sensor that carries phase information.

In this paper it is shown that a four-element segmented mirror, each element of which makes adjustments only for the tilts in two perpendicular directions, is effective. It is obvious that the wavefront sensor in this case consists of four identical meters which measure the position of the center of gravity of the image within each subaperture.

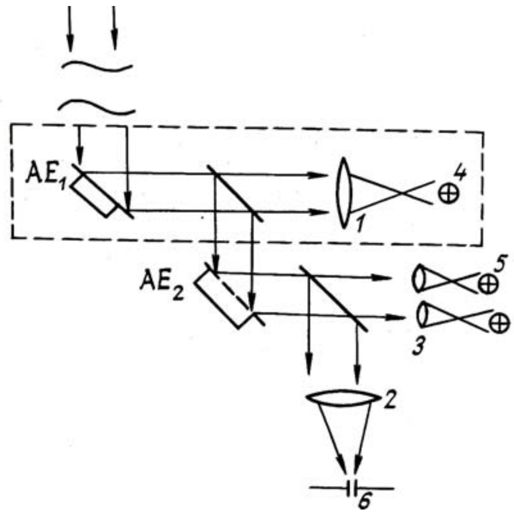

FIG. 1. Arrangement of the optical experiment: 1,2 — lenses; 3 — unit consisting of four lenses; 4 — coordinate photodetector; 5 — unit consisting of four coordinate-sensitive photodetectors; 6 — image photorecorder

An arrangement of the optical experiment is shown in Fig. 1. Here the first loop of adaptive correction of the displacement of the center of gravity of the image as a whole is made possible by the first adaptive element AE_1. If it is assumed that the aperture of the optical apparatus forming the image is a circle with radius R, then the phase of the wave after angular correction can be written in the form of the following expansion in Zernike polynomials $F_j(r/R)$:

$$S_1(r) = \sum_{j=4}^{\infty} a_j F_j(r/R), \quad (1)$$

where a_j are the expansion coefficients and in the sum (1) the first term $a_1F_1(r/R)$, which does not affect the phase fluctuations in the image, is dropped. At the second stage of image adaptation the local tilts of the wavefront within the subapertures are measured and

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they are corrected by the four-element segmented mirror AE_2. The arrangement of the subapertures and the meters, denoted by +, within a circle of radius R is shown in Fig. 2.

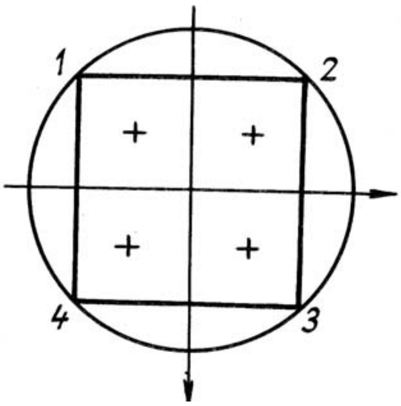

FIG. 2. Diagram of the arrangement of subapertures in a circle.

Assuming that the fluctuations of the intensity within each of the subapertures, which are numbered from one to four, is small, the following quantities are measured:

$$\alpha_{x_1} = \frac{\iint d^2r \left.\frac{\partial S_1}{\partial x}\right| (x_1, y_1)}{\iint d^2r},$$

$$\alpha_{y_1} = \frac{\iint d^2r \left.\frac{\partial S_1}{\partial y}\right| (x_1, y_1)}{\iint d^2r}, \tag{2}$$

These quantities are estimates of the increment to the phase, which are averaged within each subaperture. There are two possible limiting cases:

a) small subapertures, placed at the corners of a square; in this case in Eq. (2) the local tilts $\partial S_1/\partial x$ and $\partial S_1/\partial y$ are measured at four points;

b) large subapertures, comparable to the main aperture, when α_{xi} and α_{yj} (i, j = 1, 2, 3, and 4) are calculated from the formulas (2), where the integration is performed over the corresponding subaperture.

TABLE I.

Mode number	Radial power	Azimuthal frequency	Form of polynomial $F_j(r, \theta)$
1	0	0	I
2	1	1	$2r\cos\theta/R$
3	1	1	$2r\sin\theta/R$
4	2	0	$\sqrt{3}(2r^2/R^2 - 1)$
5	2	2	$\sqrt{6}r^2\sin2\theta/R^2$
6	2	2	$\sqrt{6}r^2\cos2\theta/R^2$

To determine which quantities will be measured, we shall study first the polynomials $F_j(r/R)$ and their derivatives (the Zernike polynomials up to degree six, inclusively, sire presented in Table I). The modes 4, 5, and 6 describe defocusing and astigmatism of the phase of the wavefront. We shall first study very small apertures (so that the aperture is significantly smaller than the coherence radius r_0). Then

$$\alpha_x = \frac{\partial S_1}{\partial x} = \frac{4\sqrt{3}}{R^2} a_4 x + \frac{2\sqrt{6}}{R^2} a_5 y + \frac{2\sqrt{6}}{R^2} a_6 x,$$

$$\alpha_y = \frac{\partial S_1}{\partial x} = \frac{4\sqrt{3}}{R^2} a_4 y + \frac{2\sqrt{6}}{R^2} a_5 x - \frac{2\sqrt{6}}{R^2} a_6 y, \tag{3}$$

We shall study a symmetric arrangement (Fig. 2) of subapertures on the sides of a square, inscribed in a circle of unit radius, with the coordinates

$$x_1 = -\frac{1}{2}, \quad y_1 = \frac{1}{2}; \quad x_3 = \frac{1}{2}, \quad y_3 = -\frac{1}{2};$$

$$x_2 = \frac{1}{2}, \quad y_2 = \frac{1}{2}; \quad x_4 = -\frac{1}{2}, \quad y_4 = -\frac{1}{2}.$$

Then

$$\begin{cases} \alpha_{x_1} = -\frac{2\sqrt{3}}{R^2} a_4 + \frac{\sqrt{6}}{R^2}(a_5 - a_6), \\ \alpha_{y_1} = \frac{2\sqrt{3}}{R^2} a_4 - \frac{\sqrt{6}}{R^2}(a_5 + a_6); \end{cases}$$

$$\begin{cases} \alpha_{x_2} = \frac{2\sqrt{3}}{R^2} a_4 + \frac{\sqrt{6}}{R^2}(a_5 + a_6), \\ \alpha_{y_2} = \frac{2\sqrt{3}}{R^2} a_4 + \frac{\sqrt{6}}{R^2}(a_5 - a_6); \end{cases}$$

$$\begin{cases} \alpha_{x_3} = \frac{2\sqrt{3}}{R^2} a_4 - \frac{\sqrt{6}}{R^2}(a_5 - a_6), \\ \alpha_{y_3} = -\frac{2\sqrt{3}}{R^2} a_4 + \frac{\sqrt{6}}{R^2}(a_5 + a_6); \end{cases}$$

$$\begin{cases} \alpha_{x_4} = -\frac{2\sqrt{3}}{R^2} a_4 - \frac{\sqrt{6}}{R^2}(a_5 + a_6), \\ \alpha_{y_4} = -\frac{2\sqrt{3}}{R^2} a_4 - \frac{\sqrt{6}}{R^2}(a_5 - a_6); \end{cases} \tag{4}$$

It is easy to see that the expressions for α_{xi} and α_{yi} (I = 1, 2, 3, and 4) have a definite symmetry.

Before proceeding with the calculations, we shall clarify the physical picture of what happens at different stages of adaptive correction according to the scheme in Fig. 1. We argue as follows: if the starting phase of the wave represents a plane, then the image formed by the optical system (equivalent lens) is an Airy diffraction pattern. The existence of phase aberrations spreads out .the. real image. After the first loop of correction with the help of AE_1 the position of the center of gravity of the image is stabilized, but the phase of the wave still contains astigmatism and defocusing aberrations. There is no doubt that these aberrations result in the appearance of local tilts within each subaperture. If only the defocusing of the phase front is taken into account, the magnitudes of these tilts, according to Eq. (4), are equal to

$$\begin{cases}\alpha_{x_1} = -2\sqrt{3}\dfrac{a_4}{R^2}, \\ \alpha_{y_1} = 2\sqrt{3}\dfrac{a_4}{R^2};\end{cases} \quad \begin{cases}\alpha_{x_2} = 2\sqrt{3}\dfrac{a_4}{R^2}, \\ \alpha_{y_2} = 2\sqrt{3}\dfrac{a_4}{R^2};\end{cases}$$

$$\begin{cases}\alpha_{x_3} = 2\sqrt{3}\dfrac{a_4}{R^2}, \\ \alpha_{y_3} = -2\sqrt{3}\dfrac{a_4}{R^2};\end{cases} \quad \begin{cases}\alpha_{x_4} = -2\sqrt{3}\dfrac{a_4}{R^2}, \\ \alpha_{y_4} = -2\sqrt{3}\dfrac{a_4}{R^2};\end{cases} \tag{5}$$

It is evident from Eq. (5) that α_1 and α_3, α_2 and α_4 always have opposite signs. This means that the tilts of each subaperture occur in pairs in antiphase, and therefore the corresponding points in the plane of the image are also shifted relative to the optical axis. We shall make numerical estimates of these displacements. From Eq. (1) we obtain the phase of the wave, giving rise to defocusing, for example, along the X axis, according to the formula

$$S_{1x} = 2\sqrt{3}\frac{a_4}{R^2}x^2,$$

corresponding to the slope angle

$$\theta_x = \frac{\partial S_1/\partial x}{k} = \frac{2\sqrt{3}\,a_4}{kR^2}.$$

The focal spots are displaced by the amount $|\Delta x| = F\sqrt{\langle\theta_x^2\rangle}$, where F is the focal length of the optical system; $\langle\theta_x^2\rangle = \dfrac{48\langle a_4^2\rangle}{k^2R^4}$ is the variance of the fluctuations of the slope angle of the wavefront; and, $\langle a_4^2\rangle$ is the variance of the fluctuations of defocusing of the wavefront. Using the data of Ref. 8, we have

$$\langle a_4^2\rangle = 2.32\cdot 10^{-2}(2R/r_0)^{5/3},$$

as that

$$|\Delta x| = F\sqrt{\langle\theta_x^2\rangle} \approx 1.88\,\frac{F}{kR}\,(R/r_0)^{5/3}. \tag{6}$$

It is easy to understand that when only the defocusing is taken into account the processing of the random tilts on each segment of the four-element mirror transforms its surface into a quadrangular prism, which reflects waves in four directions.

We shall analyze the effectiveness of the 1) propagation of radiation in vacuum (no turbulence); 2) an optical wave containing defocusing aberrations with corrected general tilts; and, 3) with correction of this defocusing with the help of a segmented four-element mirror.

For the first case the calculations are trivial. We shall denote this distribution as $I_0(F, \rho)$:

$$I_0(F, \rho) = \Omega^2\exp\left\{-\rho^2\Omega^2/R^2\right\},$$

where $\Omega = kR^2/F$ and $A(\rho) = \exp(-\rho^2/2R^2)$ is the Gaussian aperture.

The distribution of the average intensity in the plane of the image, as a result of the defocusing owing to the nonuniformities of the medium (case 2) is given by the following integral:

$$\langle I(F, \rho)\rangle = \frac{k^2}{4\pi^2F^2}\iint d^4r_{1,2}\exp\left[-\frac{(r_1^2 + r_2^2)}{2R^2} + ik\rho\frac{(r_1 - r_2)}{F}\right]\left\langle\exp\left[2\sqrt{3}\frac{a_4}{R^2}\,i\,(r_1^2 - r_2^2)\right]\right\rangle, \tag{7}$$

where the expression in the brackets <...> corresponds to the term associated with the defocusing. From Refs. 7 and 8 we obtain

$$\left\langle\exp\left[i2\sqrt{3}\frac{a_4}{R^2}(r_1^2 - r_2^2)\right]\right\rangle = \exp\left[-6\frac{\langle a_4^2\rangle}{R^4}(r_1^2 - r_2^2)^2\right]. \tag{8}$$

substituting Eq. (8) into Eq. (7) and carrying out integration we arrive at the form

$$\langle I(F, \rho)\rangle = \frac{\Omega^2}{2R^2}\int_0^\infty dr\,r\,\frac{\exp(-r^2/4R^2)J_0(k\rho r/F)}{\left[1 + 24\langle a_4^2\rangle\dfrac{r^2}{R^2}\right]}. \tag{9}$$

The further calculations in Eq. (9) for the average intensity can be performed only for specific values of Ω and $\langle a_4^2\rangle$. To perform analytical calculations we shall transform from the distribution $\langle I\rangle$ to the corresponding angular spectrum

$$P(\kappa) = \iint d^2\rho\,\exp(i\kappa\rho)\langle I(F, \rho)\rangle. \tag{10}$$

Using the definition of the δ function

$$\iint d^2\rho \, \exp\left[i\kappa\rho + i\frac{k\rho\rho_1}{F}\right] = \frac{4\pi^2F^2}{k^2}\,\delta\left(\rho_1 + \frac{F}{k}\kappa\right), \quad (11)$$

and summing Eqs. (9), (10), and (11), we obtain

$$P(\kappa) = \frac{\pi R^2}{\left[1 + 24\dfrac{\langle a_4^2\rangle}{\Omega^2}\kappa^2R^2\right]}\exp\left[-\frac{\kappa^2R^2}{4\Omega^2}\right]. \quad (12)$$

The corresponding angular spectrum for vacuum, i. e., no defocusing, is

$$P_0(\kappa) = \pi R^2\exp\left(-\kappa^2R^2/4\Omega^2\right). \quad (13)$$

The relative behavior of the spectra (12) and (13) can be analyzed. Thus, in accordance with the law of conservation of energy

$$P_0(0) = P(0) = \iint d^2\rho\langle I(F, \rho)\rangle = \pi R^2.$$

The characteristic scale over which the vacuum value of the spectrum $P_0(\kappa)$ changes is determined by the exponential decay in Eq. (13) and corresponds to the frequency $\kappa \sim 2\Omega/R$, determining the angular size of the Airy disk. Because of the effect of defocusing the value of the spectrum $P(\kappa)$ for $\kappa = 2\Omega/R$ is less than the vacuum value. This relative decrease is

$$\frac{P(\kappa)}{P_0(\kappa)} = \left[1 + 24\frac{\langle a_4^2\rangle}{\Omega^2}\kappa^2R^2\right]^{-1}.$$

It is also possible to perform analytical calculations of the broadening of the average intensity owing to defocusing from Eq. (9), expanding in a series the exponential term related with the defocusing. Then

$$\langle I(F, \rho)\rangle = \frac{k^2}{4\pi^2F^2}\iint d^4\rho_{1,2}\exp\left[-\frac{\rho_1^2}{4R^2} - \frac{\rho_2^2}{R^2} + i k\rho\rho_1/F\right]\exp\left[-24\frac{\langle a_4^2\rangle}{R^4}\rho_1^2\rho_2^2\right] \simeq$$

$$I_0(F, \rho)\left[1 - 96\langle a_4^2\rangle\left[1 - \frac{\rho^2}{R^2}\Omega^2\right]\right], \quad (14)$$

where

$$I_0(F, \rho) = \Omega^2\exp\left(-\rho^2\Omega^2/R^2\right), \quad (15)$$

The expression (14) is obtained from Eq. (9) if $24\langle a_4^2\rangle \ll 1$, so that Eq. (14) is of limited applicability.

The relative decrease in the intensity of radiation in the focal plane is

$$\frac{I_0(F, \rho) - \langle I(F, \rho)\rangle}{I_0(F, \rho)} = 96\langle a_4^2\rangle\left[1 - \frac{\Omega^2\rho^2}{R^2}\right]. \quad (16)$$

Next we shall study the distribution of the average intensity (case 3), using the scheme in Fig. 2. The distortions of the phase after correction of the general tilts of the image represent the defocusing

$$S_1(r) = \sqrt{3}\left[2\left(x^2 + y^2\right)/R^2 - 1\right].$$

It is obvious that $\iint \frac{\partial S_1}{\partial x}d^2r$ and $\iint \frac{\partial S_1}{\partial y}d^2r$, calculated over the entire aperture of effective radius R, are identically equal to zero. Let us assume that the local subapertures consist of four squares with side b, inscribed in a circle of radius R. The derivatives of the phase S_1 are equal to

$$\frac{\partial S_1}{\partial x} = 4\sqrt{3}\frac{a_4}{R^2}x, \qquad \frac{\partial S_1}{\partial y} = 4\sqrt{3}\frac{a_4}{R^2}y,$$

and according to Eq. (3) the corresponding tilts for the first subaperture are

$$\alpha_{x_1} = \frac{\iint d^2r \left.\dfrac{\partial S_1}{\partial x}\right|_{(x_1,\, y_1)}}{\iint d^2r} = -2\sqrt{3}\frac{a_4}{R^2}b,$$

$$\alpha_{y_1} = 2\sqrt{3}\frac{a_4}{R^2}b.$$

The correcting phase for each of the four zones of the mirror has the following form:

$$S_x = 2\sqrt{3}\frac{a_4}{R^2}bx, \qquad S_y = -2\sqrt{3}\frac{a_4}{R^2}bx.$$

As a result, when the local tilts are corrected by a four-element mirror (case 3) the distribution of the average intensity in the focal plane is equal to

$$\langle I_k(F, \rho)\rangle = \frac{k^2}{4\pi^2F^2}\iint d^4r_{1,2}\exp\left[-\frac{\left(r_1^2 + r_1^2\right)}{2R^2} + ik\rho\left(r_1 - r_2\right)/F\right]\left\langle \exp i\left\{2\sqrt{3}\frac{a_4}{R^2}\left[\left(r_1^2 - r_2^2\right) + b\left(x_1 - x_2\right) - b\left(y_1 - y_2\right)\right]\right\} + \exp i\left\{2\sqrt{3}\frac{a_4}{R^2}\left[\left(r_1^2 - r_2^2\right) - b\left(x_1 - x_2\right) - b\left(y_1 - y_2\right)\right]\right\} + \exp i\left\{2\sqrt{3}\frac{a_4}{R^2}\left[\left(r_1^2 - r_2^2\right) + \right.\right.$$

$$+ b(x_1 - x_2) + b(y_1 - y_2)\Big]\Big]\Big\} + \exp i\Big\{2\sqrt{3}\,\frac{a_4}{R^2}\Big[(r_1^2 - r_2^2) - b(x_1 - x_2) + b(y_1 - y_2)\Big]\Big]\Big\} \rangle , \tag{17}$$

where the brackets < ... > denote averaging over an ensemble of realizations of fluctuations of the phase in the received optical wave. After the integrals are calculated and the averaging is performed we obtain

$$\langle I_k(F,\ \rho)\rangle = \frac{k^2}{2\pi^2F^2}\iint d^4r_{1,2}\exp\left[-\frac{r_1^2}{4R^2} - \frac{r_2^2}{R^2} + ik\rho r_1/F\right]\exp\left[-24\frac{\langle a_4^2\rangle}{R^4}r_1^2r_2^2\right]\times$$
$$\times\left\{\exp\left[-6\frac{\langle a_4^2\rangle}{R^4}b^2(x_1 - y_1)^2\right]\times \mathrm{ch}24\frac{\langle a_4^2\rangle}{R^4}(r_1r_2)b(x_1 - y_1) + \exp\left[-6\frac{\langle a_4^2\rangle}{R^4}b^2(x_1 + y_1)^2\right]\mathrm{ch}24\frac{\langle a_4^2\rangle}{R^4}(r_1r_2)b(x_1 + y_1)\right\} , \tag{18}$$

where $r_1 = (x_1,\ y_1)$.

One possible further simplification of the last expression is to transform to the spectrum $P_c(\kappa)$. Uncomplicated calculations make it possible to transform the quantity in the braces in Eq. (18) to the form

$$P_c(\kappa) = \iint d^2\rho\ \exp(i\kappa\rho)\langle I_c(F,\ \rho)\rangle =$$
$$= \frac{1}{2}\exp\left\{-\frac{\kappa^2R^2}{4\Omega^2} - 6\frac{\langle a_4^2\rangle}{\Omega^2}b^2\kappa^2\right\}\times$$
$$\times\left[\exp\left(12\frac{\langle a_4^2\rangle}{\Omega^2}b^2\kappa_x\kappa_y\right)\iint d^2r\ \exp\left[-\frac{r^2}{R^2}\left(1 + 24\frac{\langle a_4^2\rangle}{\Omega^2}R^2\kappa^2\right)\right]\mathrm{ch}\left(24\frac{\langle a_4^2\rangle}{\Omega^2}(\kappa r)b(\kappa_x - \kappa_y)\right) +\right.$$
$$\left.+ \exp\left[-12\frac{\langle a_4^2\rangle}{\Omega^2}b^2\kappa_x\kappa_y\right]\iint d^2r\ \exp\left[-\frac{r^2}{R^2}\left(1 + 24\frac{\langle a_4^2\rangle}{\Omega^2}R^2\kappa^2\right)\right]\mathrm{ch}\left(24\frac{\langle a_4^2\rangle}{\Omega^2}(\kappa r)b(\kappa_x + \kappa_y)\right)\right] . \tag{19}$$

Here the index "c" in the spectrum P(κ) denotes the corrected spectrum.

Expanding the function cos$h$ in a series and retaining the first two terms, we obtain

$$P_c(\kappa) = \frac{\pi R^2}{\left[1 + 24\frac{\langle a_4^2\rangle}{\Omega^2}R^2\kappa^2\right]}\exp\left\{-\frac{\kappa^2R^2}{4\Omega^2} - 6\frac{\langle a_4^2\rangle}{\Omega^2}b^2\kappa^2\right\}\times$$
$$\times\ \mathrm{ch}12\frac{\langle a_4^2\rangle}{\Omega^2}b^2\kappa_x\kappa_y\left\{1 + \frac{48\cdot24\langle a_4^2\rangle^2\kappa^2b^2/\Omega^4}{\left[1 + 24\frac{\langle a_4^2\rangle}{\Omega^2}R^2\kappa^2\right]}\times\right.$$
$$\left.\times\left[R^2\kappa^2\mathrm{ch}\ 12\frac{\langle a_4^2\rangle}{\Omega^2}b^2\kappa_x\kappa_y + 2\kappa_x\kappa_yR^2\mathrm{sh}\ 12\frac{\langle a_4^2\rangle}{\Omega^2}b^2\kappa_x\kappa_y\right]\right\},$$
$$\kappa = (\kappa_x,\ \kappa_y). \tag{20}$$

Since the characteristic drop of $P_0(\kappa)$ is $\kappa R \sim 2\Omega$, $\kappa \sim 2\Omega/R$, $\kappa_x \cong \sqrt{2\Omega/R}$ and $\kappa_y = \sqrt{2\Omega/R}$, the decrease in the spectrum owing to defocusing is equal numerically to

$$\left.\frac{P(\kappa)}{P_0(\kappa)}\right|_{\kappa = 2\Omega/R} = \frac{1}{\left[1 + 96\langle a_4^2\rangle\right]} .$$

Correspondingly the increase in the spectrum due to correction (at the frequency $\kappa = 2\Omega/R$) is given by the expression

$$F(\kappa) = \left.\frac{P_k(\kappa)}{P(\kappa)}\right|_{\kappa = 2\Omega/R} \cong \exp\left[-24\frac{\langle a_4^2\rangle}{R^2}b^2\right]\times$$
$$\times\ \mathrm{ch}\left(48\frac{\langle a_4^2\rangle}{\Omega^2}b^2\right)\left\{1 + \frac{8\cdot24\cdot24\langle a_4^2\rangle^2b^2/R^2}{1 + 96\langle a_4^2\rangle}\times\right.$$
$$\left.\left[\mathrm{ch}\left(48\frac{\langle a_4^2\rangle}{R^2}b^2\right) + \mathrm{sh}\left(48\frac{\langle a_4^2\rangle}{R^2}b^2\right)\right]\right\} . \tag{21}$$

The effect of the adaptive correction on eliminating the defocusing can be obtained in the form of numbers with the help of Eq. (21); in addition, the parameters of the problem in this case will be $\langle a_4^2\rangle$, Ω, and b/R.

The angular resolution R can be formulated as the ratio of the average intensity at the center of the image and its total energy. The total energyin the image is equal to $P(\kappa = 0) = \pi R^2$. The limiting resolution (for vacuum) $R_0 = \dfrac{k^2R^2}{\pi F^2}$ is determined by the parameters of the optical system (k, R, F). Under conditions of defocusing

$$R = \frac{\Omega}{2\pi R^2}\int_0^\infty dx\,x\,\frac{\exp(-x^2/4)}{(1 + 24\langle a_4^2\rangle x^2)} .$$

Thus the resolution is found to depend on the parameter $\langle a_4^2 \rangle$, which characterizes the intensity of the turbulence. For $96\langle a_4^2 \rangle = 7.07(R/r_0)^{5/3} \ll 1$ the values of the resolution are summarized in Table II. In the opposite case ($96\langle a_4^2 \rangle \gg 2$) the resolution is given by the formula

$$R/R_0 \simeq \frac{-\ln\left[1.78/96\langle a_4^2 \rangle\right]}{48\langle a_4^2 \rangle} \exp\left[1/96\langle a_4^2 \rangle\right] .$$

TABLE II.

R/r_0	$96\langle a_4^2 \rangle$	R/R_0
0.03	0.0205	0.98
0.05	0.058	0.95
0.07	0.084	0.93
0.10	0.152	0.89
0.12	0.206	0.83
0.15	0.300	0.718

An examination of the distribution of the average intensity shows that the relative decrease of the intensity on the axis due to defocusing only (for $R/r_0 = 1$) is equal to a factor of 2.5, and the corresponding spreading is approximately a factor of 1.6. We shall calculate the relative increase in the values of the angular spectrum $F(\kappa) = P_c(\kappa)/P(\kappa)$ from Eq. (21) for the cutoff frequency of the spectrum $P_0(\kappa)$ ($\kappa = 2\Omega/R$). We shall use subapertures $b = R/2$. It is understandable that the narrowing of the spectrum $P(\kappa)$ compared with $P_0(\kappa)$ results correspondingly in spreading of the distribution $\langle I(F, \rho) \rangle$.

With a series of uncomplicated calculations Eq. (20) can be written in the form

$$P_c(\kappa) = \frac{1}{2} \exp\left[-\kappa^2R^2/4\Omega^2 - 6\langle a_4^2 \rangle b^2\kappa^2/\Omega^2\right] \times$$
$$\times \left\{\exp\left[12\frac{\langle a_4^2 \rangle}{\Omega^2} b^2\kappa_x\kappa_y\right] \iint d^2r \times\right.$$
$$\times \exp\left[-\frac{r^2}{R^2}\left[1 + 24\frac{\langle a_4^2 \rangle}{\Omega^2}R^2\kappa^2\right]\right] \mathrm{ch}24\frac{\langle a_4^2 \rangle}{\Omega^2}(\kappa r) b(\kappa_x - \kappa_y) +$$
$$+ \exp\left[-12\frac{\langle a_4^2 \rangle}{\Omega^2} b^2\kappa_x\kappa_y\right] \iint d^2r \exp\left[-\frac{r^2}{R^2} \times\right.$$
$$\left.\times\left[1 + 24\frac{\langle a_4^2 \rangle}{\Omega^2}R^2\kappa^2\right]\right] \mathrm{ch}24\frac{\langle a_4^2 \rangle}{\Omega^2} \kappa r\, b(\kappa_x + \kappa_y)\Bigg\},$$
$$\kappa = (\kappa_x, \kappa_y). \tag{22}$$

Analysis of this last expression shows that the correct spectrum $F_c(\kappa)$ is symmetric relative to the axes κ_x and κ_y:

$$F_c(\kappa_x, 0) = F_c(0, \kappa_y) = \exp\left[-\frac{\kappa_x^2R^2}{4\Omega^2} - 6\frac{\langle a_4^2 \rangle b^2\kappa_x^2}{\Omega^2}\right] \times$$
$$\times \iint d^2r \exp\left[-\frac{r^2}{R^2}\left[1 + 24\frac{\langle a_4^2 \rangle}{\Omega^2}R^2\kappa_x^2\right]\right] \times$$
$$\times \mathrm{ch}24\frac{\langle a_4^2 \rangle}{\Omega}(\kappa_x x) b\kappa_x.$$

At the same time it turns out that the spectrum reaches a maximum on the' diagonals ($\kappa_x = \kappa_y$):

$$F_c(\kappa_x, \kappa_x) = \frac{1}{2} \exp\left[-\frac{\kappa_x^2R^2}{4\Omega^2} - 6\frac{\langle a_4^2 \rangle b^2\kappa_x^2}{\Omega^2}\right] \times$$
$$\times \iint d^2r \exp\left[-\frac{r^2}{R^2}\left[1 + 24\frac{\langle a_4^2 \rangle}{\Omega^2}R^2\kappa_x^2\right]\right] \times$$
$$\times \left\{\exp\left[12\frac{\langle a_4^2 \rangle}{\Omega^2} b^2\kappa_x^2\right] + \exp\left[-12\frac{\langle a_4^2 \rangle}{\Omega^2} b^2\kappa_x^2\right] \times\right.$$
$$\left.\times \mathrm{ch}48\frac{\langle a_4^2 \rangle}{\Omega^2}\kappa_x x b\kappa_x\right\}. \tag{23}$$

We shall find the ratio of the spectra using the series expansion of Eq. (23):

$$K(\kappa_x) = \frac{F_c(\kappa_x, \kappa_x)}{F(\kappa_x, \kappa_x)} = \frac{1}{2}\exp\left[6\frac{\langle a_4^2 \rangle b^2\kappa_x^2}{\Omega^2}\right] +$$
$$+ \frac{1}{2}\exp\left[-18\frac{\langle a_4^2 \rangle b^2\kappa_x^2}{\Omega^2}\right] +$$
$$+ \left\{1 + \frac{1}{4}\frac{\left[48\frac{\langle a_4^2 \rangle b^2\kappa_x^2}{\Omega^2}\right]^2}{\left[1 + 24\frac{\langle a_4^2 \rangle b^2\kappa_x^2}{\Omega^2}\right]}R^2 + \ldots\right\},$$

with $b = R/2$ and $\kappa_x^2 = 2\Omega/R$.

It is obvious that the effect of an increase in the value of the spectrum along the diagonals (which results in a corresponding decrease of the width of the distribution of the average intensity), owing to the action of the four-element adaptive corrector, will increase as the ratioR/r0 increases. Thus for $R/r_0 = 5$

$K(\kappa)\big|_{\kappa=2\Omega/R} = 12.6$, and $K = 4.64$ for $R/r_0 = 4$. If the distribution of the average intensity with correction is presented, then it acquires a characteristic four-lobe form, and in addition the smallest spreading is observed along the diagonals.

Thus we believe that it would be useful to adopt the method of image correction in a telescope, described here, using technically simple apparatus.

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