SYNTHESIS OF THE OPERATION ALGORITHM OF AN ADAPTIVE OPTICAL SYSTEM

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Received July 19, 1990

An adaptive optical system, in which adaptation is realized by regulating the filter gain, is synthesized. The resulting characteristics of the adaptive filter are determined.

The problem of synthesis of adaptive algorithms for processing light signals under conditions when ranging is performed in the presence of phase distortions with unknown properties, is extremely important. It should be noted that actually all adaptive algorithms, currently under development, for viewing through a turbulent medium under conditions of a priori uncertainty for phase-conjugated adaptive optical systems (AOS) have been obtained with the help of an adaptive Bayes approach by solving the equation of maximum likelihood.[1] However the solution of this equation presents great mathematical difficulties. This makes it necessary to seek simpler solutions which can be represented in a form that is convenient for implementation. Recurrence methods, which make it possible to obtain a solution in real time, are extremely promising in this respect.

In this paper an adaptive filtering algorithm, in which adaptation is realized directly by regulating the gain coefficients of a filter, is developed for the problem of obtaining an image, undistorted by a turbulent atmosphere, of a distant point object. The proposed algorithm is based on the moving summation technique,[2] and therefore it is quite simple. The main filtering unit is supplemented with an adaptation unit, which forms estimates of the gain coefficients of the filter.

Consider a phase-conjugate AOS, consisting of a Hartman, type sensor which senses distortions of the wavefront, a wavefront corrector, and an automatic control unit. Following Ref. 3, we shall describe the state vector z of the wavefront, characterizing the phase distortions, at discrete times by a stochastic difference equation

$$z(kT) = Az(kT - T) + \Gamma\eta(kT - T), \tag{1}$$

where A is the state matrix, Γ is the matrix for the perturbation-induced noise, η is a discrete "white" sequence with zero mean and correlation matrix $M\{\eta(iT)\eta^T(jT)\} = Q\delta_{ij}$ and Q is a symmetric nonnegative-definite matrix.

Then the output vector γ will be equal to

$$\gamma(kT) = Cz(kT), \tag{2}$$

where C is the output matrix.

The column vector $U_s(kT)$ of the output signals of the wavefront distortion sensor can be represented in discrete time in the form[3]

$$U_s(kT) = k_{s\alpha}[\gamma(kT) - \tilde{\gamma}(kT)] + U_f(kT) \tag{3}$$

where k is the slope of the response characteristic of the wavefront sensor; $\gamma(kT)$ is a column vector of tilts of the wavefront, incident on . the receiving aperture; $\tilde{\gamma}(kT) = C\hat{z}(kT / (kT - T)) = CA\hat{z}(kT - T)$ is the vector of displacements introduced by the corrector in the wavefront tilts; $U_f(kT)$ is a column vector of measurement errors, which are "white" sequences with zero mean and correlation matrix $M\{U_f(iT)U_f^T(jT)\} = R\delta_{ij}$; and R is a symmetric nonnegative-definite matrix.

We introduce into the analysis the figure of merit

$$J = \sum_{k=0}^{n} U_s(kT)^T R^{-1} U_s(kT) \tag{4}$$

If the matrices A, Γ, and C and the correlation matrices of the noise disturbances Q and the measurements R are completely known, then the optimal filter (based on the criterion that the functional (4) is minimum for the message (1) and the observation (3)) is a Kalman filter, which is described for this AOS by the relations (10) derived in Ref. 3.

In practice there arise situations when some parameters of the matrices A, Γ, C, Q, and R, characterizing the phase distortions, are unknown. In this case it is impossible to determine from the relations (10) of Ref. 3 the optimal gain matrix of the filter $K(kT)$, and some suboptimal gain matrix $K^*(kT)$ will be used in the filter. Then the estimates formed by the filter will also be suboptimal

$$\hat{z}(kT) = \hat{z}^*(kT/(kT - T)) + K^*(kT)\, U_s(kT);$$

$$\hat{z}^*(kT/(kT - T)) = A^*\hat{z}^*(kT - T), \tag{5}$$

where A^* is the analog of the matrix A in the optimal filter, but with different parameters.

We shall now pose the problem of finding the matrix $K^*(kT)$ such that the figure of merit (4) is minimized. We introduce the column vector $k = [k_{11} \ldots k_{mp}]$, whose components k_{ij} are elements of the matrix K^*. Then the matrix K^* can be regarded as a function of the vector k, i.e., $K^*(k)$.

We shall find an estimate $\hat{k}$ that minimizes the functional (4), under the constraint k = const. For this we write the Hamiltonian[4]

$$Z(k(kT),\ \lambda(kT + T)) = U_s^T(kT)R^{-1}U_s(kT) + \lambda^T(kT + T)k(kT), \tag{6}$$

where $\lambda(kT + T)$ are undetermined multipliers.

The canonical equations for the vectors λ and k in this case have the form

$$k(kT + T) = \frac{\partial Z(k(kT),\ \lambda(kT + T))}{\partial \lambda(kT + T)},$$

$$\lambda(kT) = \frac{\partial Z(k(kT),\ \lambda(kT + T))}{\partial k(kT)} = \lambda(kT + T) - \frac{\partial \hat{z}^{*T}(kT/(kT + T))}{\partial k} C^T R^{-1} U_s(kT + T) \tag{7}$$

with boundary conditions at the ends of the interval $[0,\ nT]$ $k(0) = k_0$, $\lambda(nt) = 0$.

Thus the problem of minimizing the functional (4) has been reduced to a two-point boundary-value problem, whose solution by the method of invariant embedding[5] gives the following equations for estimating the gain coefficients of the filter:

$$\hat{k}(kT + T) = \hat{k}(kT) + P(kT + T)\frac{\partial \hat{z}^{*T}(kT + T))}{\partial k} C^T R^{-1} U_s(kT + T);$$

$$P^{-1}(kT + T) = P^{-1}(kT) + \frac{\partial \hat{z}^{*T}(kT + T))}{\partial k} C^T R^{-1} C \left[\frac{\partial \hat{z}^{*T}(kT + T))}{\partial k}\right]^T - U_s^T(kT) R^{-1} C \frac{\partial}{\partial k}\frac{\partial \hat{z}^{*T}(kT + T)}{\partial k}. \tag{8}$$

It should be noted that in many problems the second derivatives in Eq. (8) have virtually no effect on the operation of the adaptation loop, so that they can be dropped. The first derivative appearing in Eq. (8) is obtained by differentiating Eq. (5) with respect to k. We have

$$\frac{\partial \hat{z}^{*T}(kT + T))}{\partial k} = \left\{ \frac{\partial \hat{z}^{*T}(kT)}{\partial k}\left[1 - C^T K^*(kT) \right] + U_s(kT)\frac{\partial K^*}{\partial k} \right\} A^{*T}. \tag{9}$$

Combining Eqs. (5), (8), and (9), we obtain the equations describing the operation of'an adaptive filter

$$\hat{z}^*((kT + T)/kT) = A^*\hat{z}^*(kT),$$

$$\hat{z}^*((kT + T) = \hat{z}^*((kT + T)/kT) + \hat{K}^*(kT + T)\, U_s(kT + T),$$

$$\hat{K}^*(kT + T) = \hat{K}^*(kT) + P(kT + T)\frac{\partial \hat{z}^{*T}(kT + T))}{\partial k} C^T R^{-1} U_s(kT + T),$$

$$P^{-1}(kT + T) = P^{-1}(kT) + \frac{\partial \hat{z}^{*T}(kT + T))}{\partial k} C^T R^{-1} C \left[\frac{\partial \hat{z}^{*T}(kT + T))}{\partial k}\right]^T,$$

$$\frac{\partial \hat{z}^{*T}(kT + T))}{\partial K^*} = \left\{ \frac{\partial \hat{z}^{*T}(kT)}{\partial K}\left[1 - C^T \hat{K}^*(kT) \right] + U_s(kT) \right\} A^{*T}. \tag{10}$$

A block diagram of the adaptive filter is presented in Fig. 1.

It follows from the equations of an adaptive filter (10) that the filter consists of two units: the main filtering unit, described by the first and second relations in Eq. (10), and an adaptation unit, described by the third, fourth, and fifth relations in Eq. (10). The estimates of the gain coefficients $\hat{K}^*(kT)$, formed by the adaptation block, are employed in the main filter together with the true values of the gain coefficients.

The proposed approach to synthesis of the AOS is completely justified, if the unknown parameters are the matrices of the variances of the measurement noise and the excitation noise or the matrix Γ for excitation noise, since these matrices affect only the magnitude of the gain coefficients of the filter. The situation is somewhat more complicated in the case when the state matrix A is uncertain, since this matrix determines not only the gain coefficients of the filter, but also the corresponding matrix A^* in the filter. For this reason, by regulating only the gain coefficients of the filter it is impossible completely to eliminate the effect of the uncertainty of the matrix A. However such regulation does make it possible to compensate partially for the effect of this uncertainty, and in most practical situations this may be sufficient, since the matrix A is formed based on some known statistical data.

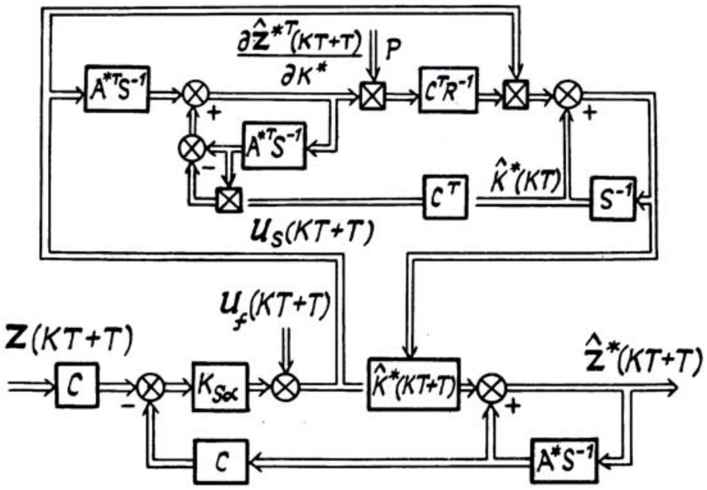

FIG. 1. Block diagram of the adaptive filter.

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