

## AN INVESTIGATION OF INTENSITY FLUCTUATIONS OF REFLECTED RADIATION IN THE TURBULENT ATMOSPHERE BY THE MONTE CARLO METHOD

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*The results of numerical calculations of the propagation of reflected wave beams in the turbulent atmosphere are presented. The statistical characteristics of waves scattered by specular and corner-cube reflectors are analyzed.*

Adaptive correction of turbulent distortions of optical beams is performed, as a rule, using reference waves<sup>1</sup> which are obtained by scattering from special reflectors (beacons). Therefore, the need has arisen to study the statistical properties of signals in optical detection and ranging.<sup>1,2</sup> When the size of reflector is small, a nearly spherical reference wave is formed about the reflector with respect to the receiver. The properties of this wave have been studied quite well. In the case of a reflector of finite size, which cannot be considered as either a point or an infinite reflector, the situation is different. Significant difficulties arise when one attempts to calculate the statistical characteristics of waves scattered by reflectors of finite size (even specular reflectors). Replacing a real reflector with sharp edges by a reflector with smooth edges for which the reflectance follows a Gaussian law results in a dramatic difference between the calculated and measured characteristics;<sup>3</sup> however, it is often used in theoretical studies. To overcome these difficulties we propose to study the statistical characteristics of optical waves scattered by reflectors of substantially reduced size (reference beacons) by the Monte Carlo method, in which the forward and backward wave propagation is simulated by having the wave pass through a set of corresponding phase screens (to take diffraction into account).<sup>4</sup> If a supercomputer is available this approach is universal. It allows one not only to correctly take into account optical wave diffraction by a real reflector, but also to investigate the range of values of the turbulent parameters which are difficult for analytical calculations.<sup>6</sup>

We will describe the wave propagation to the reflector along the  $z$  axis with the help of the scalar parabolic approximation:

$$\frac{\partial U}{\partial z} = \frac{i}{2k} \left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2k^2 n_1 \right] \cdot U, \quad (1)$$

where  $U(z, \rho)$  is the complex amplitude of the wave. Its value on the boundary  $z = 0$  is

$$U_0(\rho) = \exp \left[ -\frac{\rho^2}{2a_0^2} - ik \cdot \frac{\rho^2}{2F} \right],$$

where  $x$  and  $y$  are the Cartesian coordinates in the plane perpendicular to the  $z$  axis,  $a_0$  is the effective radius,  $F$  is the radius of curvature of the phase front at the center of the emitting aperture,  $\rho^2 = x^2 + y^2$ ,  $k = 2\pi/\lambda$  is the wave number, and  $n_1$  are the relative fluctuations of the refractive index. We will take into account diffraction by the reflector by substituting the boundary condition at  $z = L$  (at the end of the propagation path)

$$U_{\text{ref}}(\rho, L) = V(\rho) \cdot U(\rho, L), \quad (2)$$

in the parabolic equation for the reflected wave

$$-2ik \frac{\partial U_{\text{ref}}}{\partial z} + \left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2k^2 n_1 \right] \cdot U_{\text{ref}} = 0. \quad (3)$$

The reflectance  $V(\rho)$  in the boundary condition (2) is a parameter of the reflector. So, for a real specular disk with diameter  $d$  and a corner-cube reflector with metallized reflecting surfaces the boundary conditions will be

$V(\rho, \rho') = A(\rho)\delta(\rho - \rho')$ ,  $A(\rho) = 1$ ,  $\rho \leq d$ ,  
in the case of the mirror, and

$V(\rho, \rho') = A(\rho)\delta(\rho + \rho')$ ,  $A(\rho) = 1$ ,  $\rho \leq d$ ,  
in the case of the corner-cube reflector.

In the numerical solution of the equation of wave propagation along a path with reflection the turbulent perturbations are simulated by the same phase screens for both the forward and the backward transmission of the wave. The level of turbulence is modeled by the number of phase screens positioned along the path and the turbulent intensity of each individual phase screen. The perturbing effects of the simulated and natural media are considered to be equivalent if their mutual coherence functions along the forward propagation path are equal. As is well known, the mutual coherence

function  $\rho_c$  of the Gaussian beam and the coherence radius of the plane wave  $\rho_0$  are related by the expression

$$\frac{\rho_c}{\rho_0} = \frac{3(1 + z'^2) + 4qz'}{3 + z'^2 + qz'}, \quad z' = \frac{L}{ka_0^2}, \quad q = \frac{L}{k\rho_0^2}. \quad (4)$$

The relation between the parameter  $\beta_0^2 = 1.21C_n^2 k^{7/6} L^{11/6}$  and the coherence radius  $\rho_0$  is easily determined by solving Eq. (4):  $\rho_0 = (q/1.22)^{5/12}$ ,  $q = z'a_0^2 / \rho_0^2$ . Here  $C_n^2$  is the structure characteristic of the refractive index, which characterizes the turbulent propagation conditions in the continuous medium. The coherence radius for the forward propagation is calculated to the second significant digit beyond the decimal point. A collimated beam ( $F = 0$ ) with Fresnel number  $\Omega_r = ka_0^2 / 4L = 0.1$  and reflectors with the dimensions of one ( $\Omega_r = 1.7$ ) and two ( $\Omega_r = 6.8$ ) Fresnel zones were used in the trial calculations in the simulation.

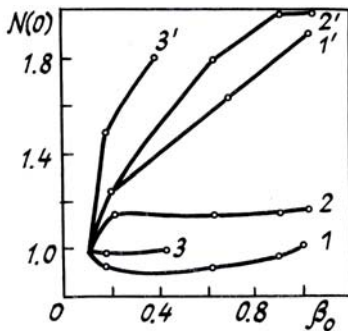


FIG. 1. Backscattering intensification coefficient on the beam axis  $N(0)$  as a function of the turbulent intensity on the propagation path  $\beta_0(L)$ . For a specular disk (curves 1, 2, 3) and for a corner-cube reflector (curves 1', 2', 3'): 1, 1' -  $\Omega_r = 1.7$ ; 2, 2' -  $\Omega_r = 6.8$ ; and 3 -  $\Omega_r = 0$ .

Figure 1 plots the intensification coefficient of the intensity fluctuations on the reflected beam axis  $N(0)$  as a function of the propagation conditions along the path  $\beta_0$ . This intensification coefficient is defined by the formula

$$N(R) = \langle I^R(x_0, R) \rangle / \langle I^R(x_0, R)_{unc} \rangle,$$

where  $\langle I^R(x_0, R)_{unc} \rangle$  is the average intensity over the path length  $2L$  (direct and reflected waves uncorrelated), and is a measure of the amplification of the intensity fluctuations due to turbulence along the propagation path. In addition, the propagating beam is limited by the diaphragm located at the distance  $L$  from the transmitter. It follows from the results given in Fig. 1 and comparison with the data given in Ref. 2 (for  $\beta_0 = 0.4$ ) that qualitative agreement holds in the case of weak turbulence; however, the quantitative discrepancies are large. An increase of the turbulent

intensity results in an insignificant change in the backscattering intensification coefficient on the reflected beam axis  $N(0)$  in the case of a specular disk, but in the case of a corner-cube reflector it leads to a monotonic increase of this coefficient by as much as a factor of two.

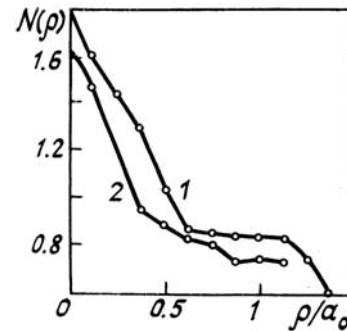


FIG. 2. The spatial distribution of the intensification effect  $N(\rho)$  in the image plane for the specular disk: 1)  $\Omega_r = 6.8$ , 2)  $\Omega_r = 1.7$ ,  $\beta_0 \cong 0.7$ .

The dependence of the backscattering intensification coefficient  $N(R)$  on the distance  $R$  from the center of the beam axis in the image plane is shown in Fig. 2. It can be seen from the figure that in the case of a collimated beam the intensification effect is localized in the immediate vicinity of the beam axis  $R \leq a_0$ , whereas outside of this region attenuation is observed, i.e., the redistribution of the intensity due to the double passage of the reflected radiation through the same inhomogeneities of the propagation medium.

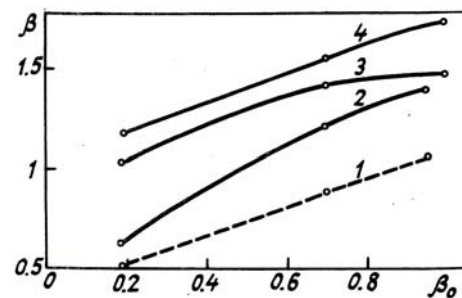


FIG. 3. The rms values of the intensity fluctuations as a function of the level of turbulence for the specular disk (2, 4 -  $\Omega_r = 1.7$ ;  $\Omega_r = 6.8$ ) and for the corner-cube reflector (1, 3).

Figure 3 presents the rms values of the intensity fluctuations on the beam axis  $\beta(0)$  as functions of the level of turbulence  $\beta_0$  on the propagation path. As can be seen, in these two cases the fluctuations of the radiation reflected from a corner-cube reflector are less than from a specular, disk of the same diameter both for weak and strong turbulence. The dynamics of variation of the level of the intensity fluctuations of a narrow beam with close to diffraction-limited parameters reflected from the specular disk was studied in Ref. 5 as a function of  $\beta_0$  by the Huygens-Kirchhof

method generalized for the case of a smoothly inhomogeneous turbulent atmosphere. It was assumed there that the reflectance of the specular disk  $V(\rho)$  as a function of distance to the center of the disk  $\rho$  followed a Gaussian distribution. Comparison of these results shows that the Huygens-Kirchhof method leads to air underestimate of the intensity fluctuations for  $0.18 \leq \beta_0 \leq 1.0$ .

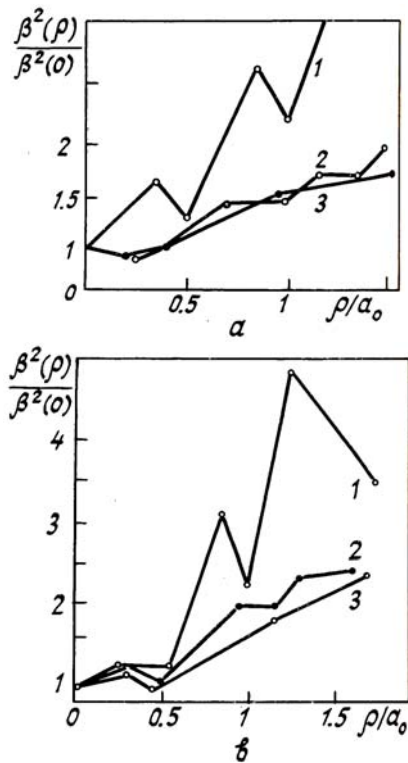


FIG. 4. The spatial distribution of the variance of the intensity fluctuations  $\beta^2(\rho)/\beta^2(0)$  (a) for the specular disk and (b) for the corner-cube reflector for different levels of the turbulent intensity: 1)  $\beta_0 = 0.18$ , 2)  $\beta_0 = 0.7$ , and 3)  $\beta_0 = 1.07$ ,  $\Omega_r = 1.7$ .

Figure 4 shows the dependence of the variance of the intensity fluctuations in the image plane on the distance from the beam axis for different levels of turbulence for a plane mirror (Fig. 4a) and a corner-cube reflector with dimensions equal to one Fresnel zone (Fig. 4b). It can be seen from the figures that the variance of the fluctuations varies nonmonotonically due to the effect of diffraction on the reflector. The diffraction pattern becomes more and more smeared out as the turbulent intensity  $\beta_0$  increases, and for  $\beta_0 = 1$  the increase of the variance with distance from the beam axis became smoother. The same behavior of the variance of the fluctuations takes place when the size of reflectors is equal to two Fresnel zones.

The results presented are a good example of the possibilities of the proposed method for studying the specific characteristics of reflected waves in different adaptive and ranging systems under conditions which are as close as possible to the real ones, while other methods of calculation do not give reliable quantitative data.

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