EFFECT OF DIFFRACTION AT AN EMITTING APERTURE ON TIME OF LASER PULSE PROPAGATION IN A REFRACTIVE ATMOSPHERE

V.P. Aksenov, V.A. Banakh, and B.N. Chen

Institute of Atmospheric Optics, Siberian Branch of the Academy of Sciences of the USSR, Tomsk

Received June 2, 1990

We demonstrated that the time of arrival of a light pulse propagated in the refractive atmosphere depends on the mode light diffraction at the emitting aperture. The errors due to this effect in determining the distances to the objects using optical methods are estimated.

The development, of methods of measurements |and parameters of laser ranging systems (LRS) substantially reduced (from some meters to some Wntimeters) the errors in measuring the distances to such objects as artificial Earth's satellites.[1]

Further increase in the accuracy of LRS measurements required for solving the problems of geodesy, geodynamics, and geophysics can be achieved by using new laser sources due to compensation for atmospheric noise.[2]

One of the components of errors in determining the distances to an object is the atmospheric refraction caused by inhomogeneities of refractive index of air. The currently available methods for correcting the refraction errors are based on formulas of geometrical optics. They make it impossible to evaluate the effect of inhomogeneity of refractive index on laser pulse propagation under arbitrary diffraction conditions in the plane of the emitting aperture. It will be demonstrated below that this may have significant effect on the accuracy of determinating the distance to ar object.

The spatial distribution of the field in the emitting aperture plane of a laser source affecting the value of a time lag of a pulse propagating in a refractive medium is analyzed using the parabolic equation for a complex amplitude of wave.

Let a laser source $U_0(\rho)$, $\rho\{x, y\}$ be located in the plane $z' = z_0$. A wave pulse with a complex amplitude $U(z', \rho; t)$ propagates in a medium with the dielectric constant $\varepsilon(z', \rho)$ and is incident on a receiving lens with the amplitude transmittance $T(\rho)$ in the plane $z' = z$. In the receiving lens plane at the distance l from the lens a square–law detector is placed which records time of the wave pulse arrival.

We shall define this time as follows:

$$\tau = \frac{\int dt\ tW(l,\ t)}{\int dt\ W(l,\ t)}, \tag{1}$$

where $W(l,\ t) = \int d^2\rho\, U_t(l,\ \rho;\ t)\, U_t^*(l, \rho; t)$ is the wave pulse power recorded with the square-law detector, $U_t(l,\ \rho;\ t)$ is the complex amplitude of the wave pulse field in the plane l behind the receiving lens. As known,[3] $U_t(l,\ \rho;\ t)$ is related with the function $U(z,\ \rho;\ t)$ by means of the Debye relation. Hence, if the wave pulse is represented in the form

$$U(z,\ \rho;\ t) = \int_{-\infty}^{\infty} f(\omega)\ U(z,\ \rho;\ \omega)\ e^{i(\omega t + kz)} d\omega,$$

where $k = \omega/c$, ω is frequency, c is the velocity of light, and $f(\omega)$ is the temporal frequency spectrum of a pulse in the plane $z' = z_0$, then for τ, by analogy with Refs. 4 and 5, we shall derive

$$\tau = -iP_0^{-1}\int_{-\infty}^{\infty} d\omega \int d^2\rho \int d^2\rho' \int d^2R'\ \times$$
$$\times \exp\left[-\frac{(\rho - \rho_t)^2}{a_0^2} + \frac{ik}{2l}\left(1 - \frac{l}{F_t}\right)\rho' R' - \frac{ik}{l}\rho\rho'\right] \times$$
$$\times\ T(R' + \rho'/2)\ T(R' - \rho'/2)\ \times$$
$$\times \frac{\partial}{\partial \Delta\omega}\Big\{ f(\omega + \Delta\omega/2)\ f^*(\omega - \Delta\omega/2)\ \times$$
$$\times\ \Gamma(z,\ R',\ \rho';\ \omega,\ \Delta\omega)\ \exp(iz\Delta\omega/c)\Big\}\Big|_{\Delta\omega=0}, \tag{2}$$

where

$$P_0 = \int_{-\infty}^{\infty} d\omega \int d^2\rho \int d^2\rho' \int d^2R'\ \times$$
$$\times \exp\left[-\frac{(\rho - \rho_t)^2}{a_0^2} + \frac{ik}{2l}\left(1 - \frac{l}{F_t}\right)\rho' R' - \frac{ik}{l}\rho\rho'\right] \times$$
$$\times\ T(R' + \rho'/2)\ T(R' - \rho'/2)\ f(\omega)\ f^*(\omega)\ \times$$
$$\times\ \Gamma(z,\ R', \rho';\ \omega,\ 0),$$

a_0 is the effective radius of the square–law detector, ρ_t is the coordinate of the center of gravity of the intensity distribution in the plane of the receiving lens with the focal length F_t, $\Gamma(z, R, \rho; \omega, \Delta\omega) = U(z, R + \rho/2; \omega + \Delta\omega/2)U^*(z, R - \rho/2; \omega - \Delta\omega/2)$ is the two–frequency function of mutual coherence of the wave field falling in the lens. The medium dielectric constant is written[6] in the form:

$$\varepsilon(z',\ \rho) = 1 + \bar{\varepsilon}(z') + \mu(z')\rho x_0, \tag{3}$$

0235-6880/91/01 31–3 $02.00

where $\bar{\varepsilon}(z')$ is the deviation of the dielectric constant of a medium from unity when $|\rho| = 0$, $\mu(z')$ characterizes the gradient of the dielectric constant of a medium along the propagation path, and x_0 is the unit vector perpendicular to the oz' axis. The function $\Gamma(z, R, \rho; \omega, \Delta\omega)$ in the medium (3) for the narrow band ($\Delta\omega \ll \omega$) pulse, in the parabolic approximation, satisfies the equation

$$\left[\frac{\partial}{\partial z'} - \frac{i}{2}\frac{\Delta\omega}{c}\bar{\varepsilon}(z') - \frac{i}{k}\nabla_R\nabla_\rho + \frac{i}{2}\frac{\Delta\omega}{c}\left(\frac{1}{4}\Delta_R + \Delta_\rho\right) - \right.$$

$$\left. - \frac{i}{2}\mu(z')x_0\left[k\rho + \frac{\Delta\omega}{c}R\right]\right]\Gamma(z, R, \rho; \omega, \Delta\omega) = 0 \quad (4)$$

with the initial condition

$$\Gamma(0, R, \rho; \omega, \Delta\omega) = U_0(R + \rho/2)\, U_0^*(R - \rho/2),$$

where Δ_ρ is the transverse Laplacian and ∇_ρ is the transverse Hamiltonian.

Equation (4) can be solved using the Fourier transform over the variables R and ρ. We obtain

$$\Gamma(z, R, \rho; \omega, \Delta\omega)$$

$$= \left(\frac{k}{2\pi L}\right)^2 (1 - m^2)^{-1}\int d^2R' \int d^2\rho' \Gamma(0, R', \rho') \times$$

$$\times \exp\left[-\frac{ikL^3}{4}m(1 - m^2)\gamma - \frac{ik}{L}\varphi\frac{(R - R')^2}{1 - m^2} + \right.$$

$$+ \frac{ikL^3}{4}m(3 + m^2)\beta^2 - \frac{ikm}{4L}\frac{(\rho - \rho')^2}{1 - m^2} +$$

$$+ \frac{ik}{L}\frac{(R - R')(\rho - \rho')^2}{(1 - m^2)} + ikLmx_0(\beta R + \alpha R') +$$

$$\left. + \frac{ikL}{2}x_0(\alpha\rho' + \beta\rho)\right], \quad (5)$$

where

$$\beta = \int_0^1 d\xi\, \xi\, \mu(z_0 + \xi L),$$

$$\alpha = \int_0^1 d\xi (1 - \xi)\, \mu(z_0 + \xi L), \quad L = z - z_0,$$

$$\gamma = \int_0^1 d\xi (1 - \xi)^2\left[\int_0^1 dt\, \mu\left[z_0 + L(\xi + t(1 - \xi))\right]\right]^2,$$

$$m = \frac{1}{2}\frac{\Delta\omega}{\omega}.$$

Let us examine the limiting cases of field distribution at the emitting aperture: 1) plane wave $U_0(\rho) = 1$ and 2) spherical wave $U_0(\rho) = \delta(\rho)$. Then for a delta–pulse ($f(\omega) = 1$) from Eq. (2) taking Eq. (5) into account we derive for a plane wave, when the detector center is located at the point $\rho_{t,pl} = \frac{Ll}{2}(\alpha + \beta)\, x_0$,

$$\tau_{pl} = \frac{L}{c} + \frac{L}{2c}\int_0^1 d\xi\, \bar{\varepsilon}\,(z_0 + \xi L) + \frac{3L^3}{8c}(\gamma - \beta^2 + \alpha^2) \quad (6a)$$

and for the spherical wave, when the detector center is located at the point $\rho_{t,sph} = \frac{Ll}{2}\beta x_0$

$$\tau_{sph} = \frac{L}{c} + \frac{L}{2c}\int_0^1 d\xi\, \bar{\varepsilon}\,(z_0 + \xi L) + \frac{3L^3}{8c}(\gamma - \beta^2) -$$

$$- \frac{a_t^2}{cL}\frac{\left[1 + \frac{1}{2}\Omega_0\Omega_t\right]}{1 + \Omega_0\Omega_t}, \quad (6b)$$

where $\Omega_0 = \frac{ka_0^2}{L}$, $\Omega_t = \frac{ka_t^2}{L}$ and a_t is the effective radius of the receiving lens.

The last term in Eq. (6b) is always negligible as compared with the previous one. Taking this into account it follows from Eqs. (6a) and (6b)

$$\Delta\tau = \tau_{pl} - \tau_{sph} = \frac{3L^3}{8c}\alpha^2.$$

The revealed difference between propagation time of the plane and spherical wave pulses may also be explained clearly based on geometrical optics. The figure depicts trajectories of refracted rays from the sources of plane (ray *1*) and spherical (ray *2*) waves. Let the lengths of rays *1* and *2* be estimated. It is possible to show[7] that $\alpha_1 = \alpha_3$ and angle α_1 is one half as much as angle α_2 provided that the angles α_1 and α_2 are small and $\mu(z') = \text{const}$.

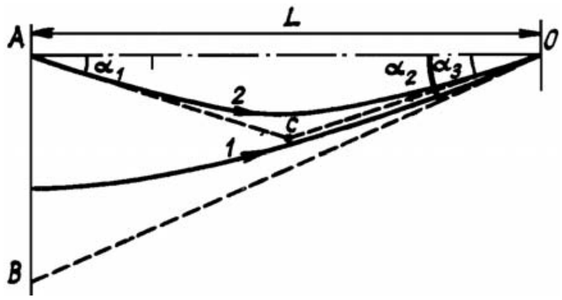

Fig. 1. Ray tracing geometry.

The length L_1 of ray 1 is estimated in terms of the tangent, segment to ray 1 at the point 0: $L_1 = OB$ and the length L_2 of ray 2 is estimated in terms of the tangent segments to ray 2 at the points A and O: $L_2 = AC + OC$. Geometrical plotting show that $AC = OC = L/\cos\alpha_1$ and $OB = L/\cos\alpha_2$. Hence,

$$\Delta L = L\left(\frac{1}{\cos\alpha_1} - \frac{1}{\cos\alpha_2}\right) \approx \frac{3}{2}L\alpha_1^2.$$

Let us estimate the order of magnitude $\Delta\tau$. As follows from Eq. (6), $\Delta\tau \approx 1$ ns for the path length $L = 100$ km and the angle of refraction $\kappa \approx 5'$ $\left(\kappa = \frac{L}{2}\alpha\right)$. This results in a difference in ranging $\Delta L = 0.3$ m. The resulting value ΔL substantially exceeds the nominal error in ranging with the third generation of the LRS that necessitates accounting for diffraction parameters of the LRS when refraction corrections are introduced.

REFERENCES

1. N.I. Kravchenko, V.M. Litovchenko, and A.V. Prokopov, Zarubezhnaya Radioelektronika, No. 5, **59** (1989).
2. J.B. Abshire and Ch.S. Gardner, IEEE Trans. **CE–23**. No. 4 (1985).
3. L.A. Chernov, *Waves in Randomly Inhomogeneous Media* (Nauka, Moscow, 1975), 172 pp.
4. K.C. Yeh and C.H. Liu, Radio Science **12**, No. 5, 671–680 (1977).
5. C.H. Liu and K.C. Yeh, IEEE Trans. Antennas and Propagation. **AP–26**, No. 4, 561 566 (1978).

6. V.V. Vinogradov, A.G. Kosterin, A.S. Medovikov, and A.I. Saichev, Izv. Vyssh. Uchebn. Zaved. SSSR, Ser. Radiofizika **28**, No. 10, 1227–1235 (1985).

7. V.A. Banakh and B.N. Chen, Opt. Atm. **1**, No. 2, 106–107 (1988).