NEW METHOD FOR THE ANALYTICAL ESTIMATE OF THE LIGHT FIELDS IN A STOCHASTIC SCATTERING MEDIUM

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New method for the analytic solution of the transfer equation of the optical radiation in a stochastic scattering medium is developed in linear and quadratic approximations.

At present the model of a three–dimensional inhomogeneous scattering layer is known to be the most adequate model of the atmosphere. This model makes it possible to describe spatial inhomogeneity of atmospheric aerosol and stochastic structure of fogs and continuous cloudiness as well as the presence of cumulus clouds in the atmosphere. The theory of wave propagation in such media is being increasingly developed only in recent years. The Monte Carlo method is the most advanced numerical method for solving this problem.[1,2] As for the analytical methods, only the methods which are based on the perturbation theory and the small–angle approximation of the radiative transfer equation[3-8] have been developed. The methods of the perturbation theory[3-5] and the small–angle approximation[6,7] are applicable only for calculating light fields in stochastic media under conditions of weak small–scale fluctuations in the scattering indices. The method described in Ref. 8 despite its much wider field of application, has also some restrictions. The present paper describes the more general method (in comparison with the method described in Refs. 3–8) for the analytical solution of the radiative transfer equation in stochastic media which is based on a new approach to the perturbation theory for solving the stochastic radiative transfer equation.

We assume that a random field of the index of attenuation of optical radiation $\varepsilon(r)$ and differential index of light scattering $\sigma(\mathbf{r}, \gamma)$ at an angle γ is realized in a scattering layer. Here $\mathbf{r}$ is the radius vector of a point belonging to the layer. This layer is illuminated from above by radiation producing the brightness $B_0(\rho, \Omega\, t)$ on the upper plane boundary of the layer at time t, where $\boldsymbol{\rho}$ is the radius vector of a point belonging to the boundary, $\boldsymbol{\Omega}$ is the unit vector specifying the direction of observation of the brightness. The light field within the layer satisfies the stochastic radiative transfer equation

$$\left[\frac{1}{c}\frac{\partial}{\partial t} + \boldsymbol{\Omega}\cdot\nabla + \varepsilon(\mathbf{r})\right] I(\mathbf{r};\, \boldsymbol{\Omega};\, t) = \int_{4\pi} \sigma(\mathbf{r};\, \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}') I(\mathbf{r};\, \boldsymbol{\Omega}';\, t)\, d\Omega', \quad (1)$$

where c is the velocity of light in the medium, ∇ is the gradient operator, $I(\mathbf{r}, \boldsymbol{\Omega}, t)$ is the radiation brightness at the point $\mathbf{r}$ of the medium in the direction $\boldsymbol{\Omega}$ at time t. Let us denote the projection $\mathbf{r}$ onto the upper plane boundary of the layer by $\boldsymbol{\rho}$ and the length of the projection $\mathbf{r}$ onto the Z axis of the system of Cartesian coordinates which is directed downwards by z. The boundary conditions for Eq. (1) have the form

$$I(\rho;\, z = 0;\, \boldsymbol{\Omega};\, t) = B_0(\rho;\, \boldsymbol{\Omega};\, t) \text{ for } \mu > 0,$$

$$I(\rho;\, z = H;\, \boldsymbol{\Omega};\, t) = 0 \text{ for } \mu < 0,$$

where μ is the direction cosine of the vector $\boldsymbol{\Omega}$ with respect to the Z axis and H is the thickness of the layer. Because of linearity of the radiative transfer equation (1) the brightness $I(\mathbf{r}, \boldsymbol{\Omega}; t)$ is related to $B_0(\rho, \boldsymbol{\Omega}, t)$ via the linear functional

$$I(\mathbf{r};\, \boldsymbol{\Omega};\, t) = \int_0^t dt' \int d\rho' \int d\Omega' B_0(\rho';\, \boldsymbol{\Omega}';\, t') \times$$
$$\times\, G(\mathbf{r};\, \rho';\, \boldsymbol{\Omega};\, \boldsymbol{\Omega}';\, t;\, t'), \quad (2)$$

where $G(\cdot)$ is the stochastic Green's function of Eq. (1). It is clear from Eq. (2) that for calculating the light fields produced by the arbitrary sources in the medium it is sufficient to know only the Green's function of Eq. (1). It is obvious that this function satisfies Eq. (1) with the boundary conditions

$$G(\rho;\, z = 0,\, \rho';\, \boldsymbol{\Omega};\, \boldsymbol{\Omega}';\, t;\, t') =$$

$$= \delta(\rho - \rho')\delta(\boldsymbol{\Omega} - \boldsymbol{\Omega}')\delta(t - t') \quad \text{for } \mu > 0,$$
$$G_0(\rho;\, z = H;\, \rho';\, \boldsymbol{\Omega};\, \boldsymbol{\Omega}';\, t;\, t') = 0 \quad \text{for } \mu < 0, \quad (3)$$

where $\delta(\cdot)$ is the delta function of Dirac. It can be easy seen that for the unscattered light

$$G_0(\mathbf{r};\, \rho';\, \boldsymbol{\Omega};\, \boldsymbol{\Omega}';\, t;\, t') = \exp\left[-\int_0^z \varepsilon(\rho' - \mathbf{a}u;\, u)\frac{du}{\mu_1}\right] \times$$

$$\times\, \delta(\rho - \rho' - \mathbf{a}z)\, \delta(\boldsymbol{\Omega} - \boldsymbol{\Omega}')\, \delta(t - t' - z/\mu_1 c), \quad (4)$$

where μ_1 is the direction cosine of the vector $\boldsymbol{\Omega}'$ with respect to the Z axis, $\boldsymbol{\alpha} = \boldsymbol{\Omega}'_\perp/\mu_1$ and $\boldsymbol{\Omega}'_\perp$ is the projection of $\boldsymbol{\Omega}'$ onto the upper boundary of the layer.

Let the Green's function $G(\cdot)$ be represented in the form of a sum of the Green's functions for the unscattered light $G_0(\cdot)$ and for the light multiply scattered in the medium $G^*(\cdot)$

$$G(\cdot) = G_0(\cdot) + G^*(\cdot). \quad (5)$$

The function $G^*(\cdot)$ satisfies the equation

$$LG^* = Q, \quad (6)$$

where the operator

$$L = \frac{1}{c}\frac{\partial}{\partial t} + \boldsymbol{\Omega}\cdot\nabla + \varepsilon(\mathbf{r}) - \int_{4\pi} d\Omega'\, \sigma(\mathbf{r};\, \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')(\cdot),$$

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$$Q = \sigma(\mathbf{r};\ \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}') \exp\left[-\int_0^z \varepsilon(\rho' - \mathbf{a}u;\ u)\frac{\mathrm{d}u}{\mu_1} \right] \times$$

$$\times\ \mathrm{d}(\rho - \rho' - \mathbf{a}z)\ \delta(t - t' - z/\mu_1 c)\ . \quad (7)$$

The operator L and the function Q can be represented by a sum of deterministic (L_0 and Q_0) and random (V and F) components

$$L = L_0 + V\ ,\quad Q = Q_0 + F\ ,$$

where

$$L_0 = \frac{1}{c}\frac{\partial}{\partial t} + \boldsymbol{\Omega}\cdot\nabla + \langle\varepsilon\rangle - \int_{4\pi} \mathrm{d}\Omega'\ \langle\sigma(\boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')\rangle(\cdot)\ ,$$

$$V = \tilde{\varepsilon}(r) - \int_{4\pi} \mathrm{d}\Omega'\ \tilde{\sigma}(\mathbf{r};\ \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')(\cdot)\ , \quad (8)$$

$\langle\varepsilon\rangle$ and $\langle\sigma(\boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')\rangle$ and $\tilde{\varepsilon}(\boldsymbol{r})$, and $\tilde{\sigma}(\boldsymbol{r};\ \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')$ are deterministic and random components of the attenuation index and differential light scattering index and the angular brackets denote mathematical averaging,

$$Q_0 = \left\langle \sigma(\mathrm{r};\Omega\cdot\Omega' \exp\left[-\int_0^z \varepsilon(\rho'-\mathrm{a}u;u)\frac{\mathrm{d}u}{\mu_1} \right] \right\rangle .$$

$$\delta(\rho - \rho' - \mathbf{a}z)\ \mathrm{d}(t - t' - z/\mu_1 c)\ , \quad (9)$$

Likewise, we shall seek the Green's function in the form of superpositions of the deterministic $\overset{*}{G_0}(\cdot)$ and random $\tilde{G}^*(\cdot)$ terms

$$G^*(\cdot) = G_0^*(\cdot) + G^*(\cdot)\ . \quad (10)$$

Equation (6) can be easy reduced to an equivalent system of two equations

$$L_0 G_0^* = Q_0 - \langle V\tilde{G}^*\rangle,\ L_0\tilde{G}^* = F + \langle V\tilde{G}^*\rangle - VG_0^* - V\tilde{G}^*. \quad (11)$$

Let us now examine the structure of the second equation of system (11). The right side of this equation can be rewritten in the form $Q - \langle Q\rangle + \langle VG\rangle - VG$. It is evident that the difference $Q - \langle Q\rangle$ describes fluctuations in the single scattered light and the difference $\langle VG\rangle - VG$ describes fluctuations in the multiply scattered light. The amplitude of these fluctuations can be easy estimated. The maximum value of Q is attained when the directly passing light arrive at the point $\{\rho;\ z\}$ without propagating through inhomogeneous media. The value of Q in this case is of the order of magnitude of $\sigma(\mathbf{r};\ \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')$, the mean value of Q is of the order of magnitude $\sigma(\mathbf{r};\ \boldsymbol{\Omega}'\boldsymbol{\Omega}')(1 - p_0)$, where p_0 is the absolute probability of the cloud presence. Thus, the amplitude of fluctuations of the term $Q - \langle Q\rangle$ is equal to $\sigma(\mathbf{r};\ \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')$.

As a rule, the scattering phase functions of natural media, i.e., the atmosphere and ocean, are strongly anisotropic. In this case the angular diagram of the function $G(\cdot)$ is much wider than that of $\sigma(\boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')$ and, therefore,

$$VG(\cdot) \simeq \tilde{\kappa}^*(\mathbf{r})G(\cdot)\ ,$$

where $\tilde{\kappa}^*$ is a random part of the effective index of light absorption by the medium.[9] Since the value $\kappa^* \ll \sigma(\mathbf{r};\ \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')$, it is obvious that in the right side of the second equation of system (11) the main contribution comes from the term F and, therefore, the operator V can be assumed to be small and the methods of perturbation theory can be used for solving this system of equations.

One of the key features of this approach should be noted. As is well known, the methods of perturbation theory are applicable only in the case in which a perturbing operator of the equation under study is small. In the conventional approaches[3–5] the operator which describes random fluctuations in the parameters of the medium is chosen for a small perturbing operator. For this reason, such approaches cannot be employed in principle in the case of strong fluctuations in the medium. In the given approach the operator which describes the effect of fluctuations in multiply scattered light in comparison with fluctuations in singly scattered light in system (11) is assumed to be small. Since this relation holds for both cases of weak and strong fluctuations, the approach we treat in this paper can be successfully used to describe wave propagation in the media in the case of strong fluctuations in scattering parameters as well.

Let us introduce an auxiliary parameter s, rewrite Eqs. (11) in the form

$$L_0 G_0^*) = Q_0 - s\langle V\tilde{G}^*\rangle\ ,$$

$$L_0\tilde{G}^* = F + s\langle V\tilde{G}^*\rangle - sVG_0^* - sV\tilde{G}^*\ , \quad (12)$$

and seek the function $\tilde{G}^*(\cdot)$ in the form of a series expansion in terms of powers of the parameter s

$$\tilde{G}^*(\cdot) = \sum_{\kappa=0}^{\kappa_0} \tilde{G}_k^*(\cdot)\ s^\kappa\ . \quad (13)$$

When $\kappa_0 = 0$, instead of system (12), we have two independent equations for $G_0^*(\cdot)$ and $\tilde{G}_0^*(\cdot)$:

$$L_0 G_0^* = Q_0 - \langle VL_0^{-1}F\rangle\ ,\ L_0\tilde{G}_0^* = F\ , \quad (14)$$

where L_0^{-1} is the operator inverse of L_0. When $\kappa = 1$, system (12) is reduced to three independent equations for G_0^*, $\tilde{G}_0^*$, and $\tilde{G}_1^*$:

$$(L_0 - \langle V L_0^{-1} V\rangle)\ G_0^* = Q_0 - \langle VL_0^{-1}F\rangle - \langle VL_0^{-1}\langle VL_0^{-1}F\rangle\rangle\ ,$$

$$L_0\tilde{G}_0^* = F\ , \quad (15)$$

$$L_0\tilde{G}_1^* = \langle VL_0^{-1}F\rangle - VG_0^* - VL_0^{-1}F\ .$$

Similar equations can be written for arbitrary κ_0. As can be seen from Eqs. (12), in calculating the mean value when $\kappa_0 = 0$, the terms linear in s are taken into account, and when $\kappa_0 = 1$, the quadratic terms are considered. For this reason the solutions of Eqs. (14) is called below the solutions of the radiative transfer equation in linear approximation while the solution of Eqs. (15)

– the solutions in quadratic approximation. The field of application of these equation is much wider than that of the method proposed in Ref. 8, since no restrictions are imposed on the relation between the horizontal scales of inhomogeneities and the width of the Green's function. The terms involving the operator V which enter into the equations for $G_0^*(\cdot)$ describe radiative interaction of inhomogeneities, e.g., clouds. The only restriction imposed on Eqs. (14) and (15) is the requirement of weak fluctuations in multiply scattered light in comparison with fluctuations in singly scattered radiation. It is important to note that in linear approximation the equations for G_0^* and $\tilde{G}_0^*$ are similar to the radiative transfer equation in the deterministic media illuminated by the sources with random brightness distribution. This fact enables one to use a large number of analytical methods for solving the radiative transfer equation in order to find the mean values and second moments of brightness.

As an illustration of the efficiency of the approach under discussion we find the mean brightness produced in a stochastic layer by an infinitely extended unidirectional stationary source. To solve the problem, we shall use a small–angle approximation of the radiative transfer equation which is valid for media with strongly anisotropic scattering phase function. Neglecting small terms of the order of s in the right side of Eqs. (14) and (15), which describe radiative interaction of inhomogeneities, for the mean values of brightness $B_0(z;\,\mathbf{\Omega})$ we have in linear approximation:

$$\mu_0\frac{dB_0}{dz}+\langle\varepsilon\rangle B_0=\frac{\langle\sigma^*\rangle}{4\pi}\int\limits_{4\pi} i_0(\Omega_\perp-\Omega_\perp' B_0(\Omega')d\,\Omega')+Q_0(z;\,\Omega), \quad (16)$$

where μ_0 is the direction cosine of the unit vector $\mathbf{\Omega}_0$ with respect to the Z axis which specifies the direction of illumination of the medium, $\langle\sigma^*\rangle$ is the mean value of the effective scattering index,[9] and $i_0(\gamma)$ is the scattering phase function of the medium.

To write the radiative transfer equation in quadratic approximation, we first of all find the result of action of the operator $\langle VL_0^{-1}V\rangle$ on the function $B_0(\cdot)$. Let the volume Green's function of the deterministic radiative transfer equation, found in small–angle approximation, be denoted by $\Gamma_0(\rho-\rho';\,\mathbf{\Omega}_\perp-\mathbf{\Omega}_\perp';\,z;\,z')$. Then

$$L_0^{-1}V\,B_0(\cdot)=\int dr'\tilde{\kappa}^*(r')\int\limits_{4\pi} d\Omega'\,B_0(\mathbf{\Omega}')\Gamma_0(\rho-\rho';\,\mathbf{\Omega}_\perp-\mathbf{\Omega}_\perp';\,z;\,z'), \quad (17)$$

$$\langle VL_0^{-1}V\rangle B_0(\cdot)=\int d\mathbf{r}'R_{\kappa\kappa}(\mathbf{r};\,\mathbf{r}')\int\limits_{4'} d\Omega'\,B_0(\Omega')\times$$

$$\times\,\Gamma_0(\rho-\rho';\,\mathbf{\Omega}_\perp-\mathbf{\Omega}_\perp';\,z;\,z'),$$

where $R_{\kappa\kappa}(\mathbf{r};\,\mathbf{r}')=\langle\tilde{\kappa}^*(\mathbf{r})\tilde{\kappa}^*(\mathbf{r}')\rangle$ is the correlation function of the effective absorption index. In what follows, a random field is assumed to be horizontally homogeneous, and $R_{\kappa\kappa}(\mathbf{r};\,\mathbf{r}')=R_{\kappa\kappa}(\rho-\rho';\,z,\,z')$.

On the basis of Eqs. (15) and (17) and neglecting unimportant terms in the right side of Eq. (15) we obtain for $B_0(\cdot)$

$$\mu_0\frac{dB_0}{dz}-\int d\mathbf{r}'R_{\kappa\kappa}(\rho-\rho';\,z;\,z')\int\limits_{4\pi} d\Omega'\,\times$$

$$\times\,B_0(\mathbf{\Omega}')\,\Gamma_0(\rho-\rho';\,\mathbf{\Omega}_\perp-\mathbf{\Omega}_\perp';\,z;\,z')+\langle\varepsilon\rangle B_0=$$

$$=\frac{\langle\sigma^*\rangle}{4\pi}\int\limits_{4\pi} i_0(\mathbf{\Omega}_\perp-\mathbf{\Omega}_\perp' B_0(\mathbf{\Omega}')d\mathbf{\Omega}')+Q_0(\mathbf{z};\,\mathbf{\Omega}'). \quad (18)$$

We now introduce the Fourier transform of the function $B_0(\mathbf{\Omega})$

$$\hat{B}_0(\mathrm{p})=\int d\Omega_\perp B_0(\Omega)\,e^{-i\mathrm{p}\cdot\mathbf{\Omega}_\perp}. \quad (19)$$

The equation for $\hat{B}_0(\boldsymbol{p})$ follows from Eq. (18)

$$\mu_0\frac{d\hat{B}_0}{dz}+\left[\langle\varepsilon\rangle-\langle\sigma^*\rangle F(\mathbf{p})\right]\hat{B}_0-$$

$$-\int\limits_0^z d\boldsymbol{z}'\hat{B}_0(\boldsymbol{z}')\,L(\boldsymbol{z};\,\boldsymbol{z}')=\hat{Q}_0(\boldsymbol{z};\,\mathbf{p}), \quad (20)$$

where $\hat{Q}_0(z,\,\boldsymbol{p})$ is the Fourier transform of the function $Q_0(z;\,\mathbf{\Omega})$;

$$L(z;\,z')=\int\frac{d\omega}{4\pi^2}\hat{\Gamma}_0(\omega;\,\mathbf{p};\,\boldsymbol{z};\,\boldsymbol{z}')\,\hat{S}_\kappa(-\omega;\,\boldsymbol{z};\,\boldsymbol{z}'), \quad (21)$$

$\Gamma_0(\boldsymbol{\omega};\,\boldsymbol{\rho};\,z;\,z')$ is the spatial–angular spectrum of the Green's function of the deterministic radiative transfer equation;

$$\hat{S}_\kappa(\boldsymbol{\omega};\,\boldsymbol{z};\,z')=\int d\boldsymbol{\omega}\,R_{\kappa\kappa}(\omega;\,\boldsymbol{z};\,\boldsymbol{z}')\,e^{-i\omega\cdot\xi} \quad (22)$$

is the power spectrum of fluctuations in the effective absorption index;

$$F(\mathbf{p})=\frac{1}{2}\int\limits_0^{\pi/2}\gamma\,i_0(\gamma)\,J_0(p\gamma)\,d\gamma, \quad (23)$$

where $J_0(x)$ is the zero order Bessel function of the first kind. According to Refs. 9 and 10, we have

$$\hat{\Gamma}_0(\boldsymbol{\omega};\,\mathbf{p};\,z;\,z')=\exp\left\{-\left[\langle\varepsilon\rangle-\langle\sigma^*\rangle F(\mathbf{p})\right]\frac{(z-z')}{\mu_0}\right\}. \quad (24)$$

Similarly, we can write the Fourier transform of Eq. (19) in linear approximation

$$\mu_0\frac{d\hat{B}_0}{dz}+\left[\langle\varepsilon\rangle-\langle\sigma^*\rangle F(\mathbf{p})\right]\hat{B}_0=\hat{Q}_0(\boldsymbol{z};\,\mathbf{p}), \quad (25)$$

It can be seen from comparison of Eqs. (20) and (25) that the linear approximation can be used for $\langle\kappa^*\rangle>\sigma_\kappa^2 l_{||}$ here $l_{||}$ is the longitudinal scale of inhomogeneities and σ_κ^2 is the variance of $\kappa^*(r)$. Equation (25) can be easy solved

$$\hat{B}_0=\int\limits_0^z\hat{Q}_0(\boldsymbol{\omega};\,\mathbf{p})\exp\left[-\kappa(p)(z-\boldsymbol{\omega})\right]d\boldsymbol{\omega}, \quad (26)$$

where $k(p) = <\varepsilon> - <\sigma^*>F(p)$. We failed to obtain rigorous solution of Eq. (20) in explicit form. For this reason, we present here several approximate solutions of this equation. To describe wave propagation in the case of small–scale fluctuations in the parameters of the medium along the Z axis with a longitudinal scale $l_{||} \ll H$, the model of the δ–correlated random process can be employed. Within the scope of this model

$$R_{\kappa\kappa}(\xi;\ z;\ z') = R^*_{\kappa\kappa}(\xi;\ z;)\ \delta(z - z')\ , \qquad (27)$$

where

$$R^*_{\kappa\kappa}(\xi;\ z;) = \int\limits_0^z R_{\kappa\kappa}(\xi;\ z;)\ z'\ dz'\ .$$

It can easy be seen that for model (27), $\hat{B}_0$ is also described by the relation similar to Eq. (26) in which $\kappa(p)$ must be replaced by the parameter

$$\kappa'(p) = \kappa(p) - R^*_{\kappa\kappa}(0;\ z)\ . \qquad (28)$$

We now obtain an approximate solution of Eq. (20) in another limiting case in which $l_{||} \gtrsim H$. Here the dependence of the function $L(z,\ z')$, entering into Eq. (20), on the variable z may be assumed to be weak. Introducing the notations

$$\Psi = \int\limits_0^z dz'\hat{B}_0(z')\ L(z;\ z')\ ,$$

we can write instead of Eq. (20), an equivalent system of equations

$$\mu_0 \frac{d\hat{B}_0}{dz} + \kappa(p)\hat{B}_0 - \Psi = \hat{Q}_0(z;\ p)\ ,$$

$$\frac{d\Psi}{dz} = \hat{B}_0\ L(z;\ z) + \int\limits_0^z dz'\hat{B}_0(z')\frac{dL(z;\ z')}{dz}\ .$$

In the last equation the term containing a derivative can be assumed to be negligible. In this case instead of the integro–differential equation (20), the second–order differential equation can be written:

$$\mu_0 \frac{d^2\hat{B}_0}{dz^2} + \kappa(p)\frac{d\hat{B}_0}{dz} - L(z;\ z)\ \hat{B}_0 = \frac{d\hat{Q}_0}{dz}(z;\ p)\ ,$$

which, in some cases, can be solved rigorously. In particular, for a random process $\kappa^*(r)$ stationary in z we have

$$\hat{B}_0 = C_1 \exp\left(-\frac{\kappa(p) - l}{2}z\right) + C_2 \exp\left(-\frac{\kappa(p)+l}{2}z\right) + \frac{2}{l} \times \int\limits_0^z d\hat{Q}_0(t)\ \exp\left(\frac{l}{2}\ \kappa(p)(t - z)\right)\ \mathrm{sh}\ \frac{1}{2}(z - t)\ , \qquad (29)$$

where the constants C_1 and C_2 can be found from the conditions $\hat{B}_0 = 0$ for $z = 0$ and $\hat{B}_0 \to 0$ as $z \to \infty$ and $l = (\kappa_p^2 + 4\sigma_\kappa^2)^{1/2}$.

The angular structure of the light field can be determined by using the inverse Fourier transform. The illumination in the medium $E = \mu_0\hat{B}_0(0)$ and the coefficient of diffuse transmission of the scattered radiation $T = \hat{B}_0(0)$ can be found based on Eqs. (26) and (29) in a rather a simple way.

We may write, by way of example, the relation for the diffuse transmittance for the model of a cloud layer described in Refs. 1 and 11. Within the scope of this model

$$\hat{Q}_0(z;\ 0) = \sum_{j=1}^{0} D_j\, p_0\sigma_0 \exp(-\lambda fz/\mu_0)\ , \qquad (30)$$

where

$$D_1 = \frac{\Lambda_2 - \sigma_0}{\Lambda_2 - \Lambda_1}\ ,\ D_2 = \frac{\sigma_0 - \Lambda_1}{\Lambda_2 - \Lambda_1}\ ,$$

$$\Lambda_{1,2} = \frac{1}{2}\left(\sigma_0 + A \pm \sqrt{(\sigma_0 + A)^2 - 4Ap_0\sigma_0}\right)\ ,$$

$$A = A_*\sin\vartheta_0(\sin\varphi_0 + \cos\varphi_0)\ ,$$

A_*^{-1} is the characteristic scale inhomogeneities of a cloudy medium, σ_0 is the scattering index of a cloud, p_0 is the absolute probability of the cloud presence, ϑ_0 and φ_0 are the polar and azimuthal angles of illumination. For model (30) the solution of Eq. (29) has the form

$$T = \sum_{j=1}^{2} \frac{4\Lambda_j\, D_j\, p_0\sigma_0}{(\kappa_0^* - 2\Lambda_j)^2 - l^2} \times \left[\exp\left(-\frac{\kappa_0^* + l}{2\mu_0}H\right) - \exp\left(-\frac{\Lambda_j}{\mu_0}H\right)\right]\ , \qquad (31)$$

where $\kappa_0^* = p_0\sigma_0\Phi$ and Φ is the fraction of the backscattered light in the singly scattered light.

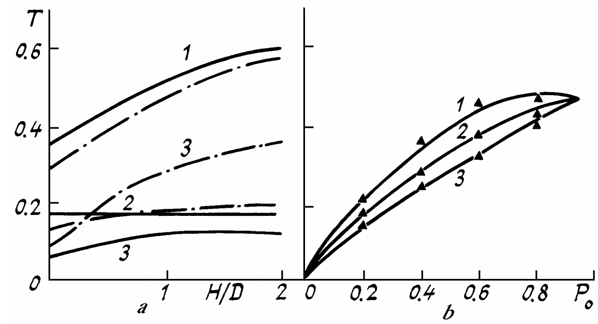

FIG. 1. Different methods for calculations of the transmittance coefficient T: a) for $A = 1.65\ (p_0 - 0.5)^2 + 1.04/D$, *where D is the specific size of a cloud,* $\sigma_0H = 5.0$, $\theta_0 = 60°$, $\varphi_0 = 0$ *(1),* $\sigma_0H = 30$, $\theta_0 = \varphi_0 = 0$ *(2),* $\theta_0 = 60°$, $\varphi_0 = 0$ *(3);*
b) for $A_* = (2.32p_0^2 - 0.92\ p_0 + 1.69)/D$, $\theta_0 = 30°$, $\varphi_0 = 45°$, $\sigma_0H = 15$, $H/D = 0.25$ *(1),* 0.50 *(2),* 1.0 *(3). Solid curve is the values obtained based on Eq. (31) and dot–dashed curves and triangles – based on the Monte Carlo method.*

Figure 1 shows the results of comparison between the values of T obtained from formula (31) and by the Monte Carlo method.[1,12] As can be seen from Fig. 1 the data are substantially different only for optically thick clouds at inclined illumination. The reason for this is a low accuracy of small–angle approximation for optically thick scattering layers.

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