CALCULATION OF RESONANCE FUNCTIONS USED IN THE IMPACT THEORY OF BROADENING AND SHIFT OF THE SPECTRAL LINES

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The resonance functions of the Anderson–Tsao–Curnutte–Frost theory for the polarization part of the intermolecular potential have been calculated. Averaging of the resonance functions over Maxwell's distribution of the relative collision velocities has been carried out.

In order to calculate the absorption coefficients of atmospheric and polluting gases, we must know the parameters of the line shape, i.e., the half–width and the lineshift. Recently significant efforts have been directed to update the coefficients of broadening of the absorption lines of water vapor, methane, ozone, and carbon dioxide by a pressure of nitrogen, oxygen, and air.[1-4] The comparison of the calculated results with experimental data has shown a need for improvement of calculational methods, which take an account of high–order terms of the multipole expansion in the electrostatic potential, the polarization interactions, the repulsive forces, and the effect of the path curvature in the process of collisions. To take these factors into account, we must calculate new resonance functions.

Some of the resonance functions required to calculate the coefficients of broadening and lineshifts were obtained previously in Refs. 5–9. In this paper we present new functions for the induction and dispersion interactions and carry out the averaging of the resonance functions over Maxwell's distribution of the relative velocities.

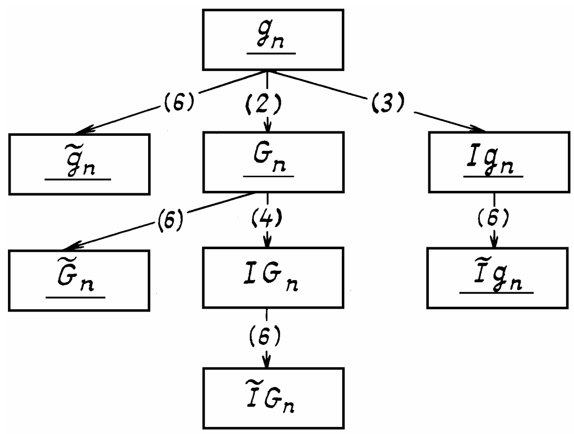

FIG. 1. The procedure for calculating the resonance functions. The formulas which were employed in the calculations are indicated in the parentheses; the functions, which can be represented analytically, are underlined.

The obtained results are applicable not only in the Anderson–Tsao–Curnutte–Frost method,[5,8] but they can also be used in other variants of the impact theory.[7]

1. The functions $g_n(\mathrm{k})$ $(n = 1, \dots, 7)$ under consideration and the functions $G_n(k)$ used in the methods of the "interruption approximation",[5] depend on the resonance parameter

$$k = \frac{2\pi cb}{v}\,\Delta E\,. \tag{1}$$

where b is the impact parameter, v is the relative collision velocity, and ΔE is the energy balance of the nonradiating transitions corresponding to the absorbing and buffer molecules. The functions $g_n(\mathrm{k})$ and $G_n(\mathrm{k})$ are related in the following way:

$$G_n(k) = k^{N-2}\int\limits_k^{\infty} \mathrm{d}k'\,\frac{g_n(k')}{k'^{N-1}}\,, \tag{2}$$

where $N = 10$ for $n = 1.2$ and $N = 12$ otherwise. The resonance functions of the shifts $Ig_n(\mathrm{k})$ and $IG_n(\mathrm{k})$ can be derived from $g_n(\mathrm{k})$ and $G_n(\mathrm{k})$ by means of Hilbert's transform[7]

$$Ig_n(k) = \frac{1}{\pi}\,P.V.\int\limits_{-\infty}^{\infty}\frac{g_n(k')}{k'-k}\,\mathrm{d}k'\,, \tag{3}$$

$$IG_n(k) = \frac{1}{\pi}\,P.V.\int\limits_{-\infty}^{\infty}\frac{G_n(k')}{k'-k}\,\mathrm{d}k'\,. \tag{4}$$

The Cauchy principal value of the integral is given by the formula

$$P.V.\int\limits_a^b \dots = \lim_{\varepsilon\to 0}\left\{\int\limits_a^{k-\varepsilon}\dots + \int\limits_{k+\varepsilon}^{b}\dots\right\}. \tag{5}$$

Averaging over Maxwell's distribution of the relative velocities redefines the resonance functions[9]

$$\tilde{g}_n(k) = \frac{4}{\pi}\int\limits_{-\infty}^{\infty}|\mathbf{x}|\,\mathrm{e}^{-x^2} g_n\!\left(\frac{2k}{\sqrt{\pi}\,x}\right)\mathrm{d}x\,. \tag{6}$$

The functions $\tilde{G}_n(k)$, $\tilde{I}g_n(k)$, and $\tilde{I}G_n(k)$ can be found analogously.

The problem is to obtain starting from the well–known relations for $g_n(k)$ (see Ref. 7),:

a) the functions $G_n(k)$ based on Eq. (2),

b) the functions of the shifts $Ig_n(k)$ and $IG_n(k)$ based on Eqs. (3)–(5), and

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c) to perform averaging over Maxwell's distribution according to Eq. (6).

The procedure for calculating the resonance functions is schematically presented in Fig. 1.

2. The resonance functions of the polarization interactions have the form[7]

$$g_n(k) = e^{-2k} \sum_{m=0}^{L_n} a_m^{(n)} k^m . \qquad (7)$$

The functions $G_n(k)$, $Ig_n(k)$, and $IG_n(k)$ cannot be found in the literature. Substituting Eq. (7) into formulas (2)–(6) and employing some integrals from Ref. 10, after a number of transformations, we obtain

$$G_n(k) = e^{-2k} \sum_{m=0}^{L_n} a_m^{(n)} \times$$

$$\times \sum_{l=0}^{M_n} \frac{(-1)^l k^{m+l}}{(M_n + 1 - m)(M_n - m) \ldots (M_n - m - l)} =$$

$$= e^{-2k} \sum_{m=0}^{P_n} b_m^{(n)} k^m , \qquad (8)$$

$$Ig_n(k) = \frac{1}{\pi} \Bigg\{ e^{-2k} \operatorname{Ei}(2k) \sum_{m=0}^{L_n} a_m^{(n)} k^m -$$

$$- e^{2k} \operatorname{Ei}(-2k) \sum_{m=0}^{L_n} a_m^{(n)} (-k)^m +$$

$$+ \sum_{m=1}^{L_n} a_m^{(n)} \sum_{l=1}^{m} \frac{(l-1)!}{2^l} \left[(-1)^{m-l} - k^{m-l} \right] \Bigg\} =$$

$$= \frac{1}{\pi} \Bigg\{ e^{-2k} \operatorname{Ei}(2k) \sum_{m=0}^{L_n} a_m^{(n)} k^m -$$

$$- e^{2k} \operatorname{Ei}(-2k) \sum_{m=0}^{L_n} a_m^{(n)} (-k)^m + \sum_{m=0}^{R_n} c_m^{(n)} k^{2m+1} \Bigg\} , \qquad (9)$$

$$\tilde{g}_n(k) = \frac{8}{\pi} \sum_{m=0}^{L_n} a_m^{(n)} \left(\frac{2k}{\sqrt{\pi}}\right)^m \Bigg[\Gamma\left(\frac{-m+1}{2}\right) \frac{2k}{\sqrt{\pi}} \times$$

$$\times {}_0F_2\left(\frac{m+1}{2}, \frac{1}{2}; -\frac{4k^2}{\pi}\right) - \frac{1}{2} \Gamma\left(\frac{-m+2}{2}\right) \times$$

$$\times {}_0F_2\left(\frac{m}{4}, \frac{3}{2}; -\frac{4k^2}{\pi}\right) - (m-3)! \left(\frac{4k}{\sqrt{\pi}}\right)^{-m+2} \times$$

$$\times {}_1F_3\left(1; \frac{-m+3}{2}, \frac{-m+4}{2}, \frac{1}{2}; -\frac{4k^2}{\pi}\right) \Bigg] . \qquad (10)$$

TABLE I. The coefficients $a_m^{(n)}$ of formula (7).

g_n / a_m	a_0 a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
g_1	1 2	2	$\frac{4}{3}$	$\frac{13}{21}$	$\frac{4}{21}$	$\frac{2}{63}$	–	–	–	–
g_2	1 2	$\frac{88}{43}$	$\frac{184}{129}$	$\frac{1292}{1677}$	$\frac{580}{1677}$	$\frac{668}{5031}$	$\frac{24}{559}$	$\frac{6}{669}$	–	–
g_3	1 2	$\frac{46}{25}$	$\frac{76}{75}$	$\frac{9}{25}$	$\frac{2}{25}$	$\frac{2}{225}$	–	–	–	–
g_4	1 2	$\frac{872}{425}$	$\frac{1832}{1275}$	$\frac{964}{1275}$	$\frac{392}{1275}$	$\frac{364}{3825}$	$\frac{16}{765}$	$\frac{2}{765}$	–	–
g_5	1 2	$\frac{101}{50}$	$\frac{103}{75}$	$\frac{209}{300}$	$\frac{41}{150}$	$\frac{37}{459}$	$\frac{4}{225}$	$\frac{1}{450}$	–	–
g_6	1 2	$\frac{2694}{1325}$	$\frac{5564}{3975}$	$\frac{26332}{35775}$	$\frac{11336}{35775}$	$\frac{4184}{35775}$	$\frac{4088}{107325}$	$\frac{1186}{107325}$	$\frac{4}{1431}$	$\frac{4}{7155}$
g_7	1 2	$\frac{68}{35}$	$\frac{128}{105}$	$\frac{58}{105}$	$\frac{4}{21}$	$\frac{16}{315}$	$\frac{16}{1575}$	$\frac{2}{1575}$	–	–

The formulas analogous to Eqs. (9) and (10) can be obtained for the functions $IG_n(k)$ and $\tilde{G}_n(k)$ if we substitute $b_m^{(n)}$ for $a_m^{(n)}$ and $d_m^{(n)}$ for $c_m^{(n)}$. These coefficients are listed in Tables I–III. Here $\operatorname{Ei}(x)$ is the integral exponent, ${}_nF_m(\ldots)$ denote the hypergeometric functions,[11] $M_n = N - 3$ (see formula (2)), and P_n and R_n are the numbers of the last coefficients $b_m^{(n)}$ and $c_m^{(n)}$ in Tables II and III, respectively.

Thus, the resonance functions for the polarization interactions, including those averaged over the velocities, can be represented analytically, which simplifies the calculations of the broadening and lineshift coefficients in cases of the significant contribution of the induction and dispersion interactions.

TABLE II. The coefficients $b_m^{(n)}$ of formula (8).

G_n	b_m: b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
G_1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{14}$	$\frac{1}{63}$	–	–	–	–
G_2	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{133}{516}$	$\frac{47}{258}$	$\frac{227}{2236}$	$\frac{479}{10062}$	$\frac{21}{1118}$	$\frac{3}{559}$	–	–
G_3	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{9}{50}$	$\frac{7}{75}$	$\frac{13}{450}$	$\frac{1}{225}$	–	–	–	–
G_4	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{351}{1700}$	$\frac{373}{2550}$	$\frac{197}{2550}$	$\frac{13}{425}$	$\frac{13}{1530}$	$\frac{1}{765}$	–	–
G_5	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{81}{400}$	$\frac{83}{600}$	$\frac{7}{100}$	$\frac{2}{75}$	$\frac{13}{1800}$	$\frac{1}{900}$	–	–
G_6	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{541}{2650}$	$\frac{563}{3975}$	$\frac{8099}{107325}$	$\frac{3562}{107325}$	$\frac{1357}{107325}$	$\frac{458}{107325}$	$\frac{1}{795}$	$\frac{2}{7155}$
G_7	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{27}{140}$	$\frac{5}{42}$	$\frac{11}{210}$	$\frac{3}{175}$	$\frac{13}{3150}$	$\frac{1}{1575}$	–	–

TABLE III. The coefficients $c_m^{(n)}$ and $d_m^{(n)}$ of formula (9).

	c_m					d_m			
Ig_n	c_0	c_1	c_2	c_3	c_4	d_0	d_1	d_2	d_3
Ig_1	$\frac{133}{42}$	$\frac{46}{63}$	$\frac{2}{63}$	–	–	$\frac{8}{21}$	$\frac{5}{63}$	–	–
Ig_2	$\frac{13031}{3354}$	$\frac{5323}{5031}$	$\frac{803}{5031}$	$\frac{6}{559}$	–	$\frac{811}{617}$	$\frac{698}{5031}$	$\frac{12}{559}$	–
Ig_3	$\frac{13}{5}$	$\frac{91}{225}$	$\frac{2}{225}$	–	–	$\frac{11}{45}$	$\frac{7}{225}$	–	–
Ig_4	$\frac{617}{170}$	$\frac{3737}{3825}$	$\frac{409}{3825}$	$\frac{2}{765}$	–	$\frac{61}{170}$	$\frac{22}{225}$	$\frac{7}{765}$	–
Ig_5	$\frac{139}{40}$	$\frac{401}{405}$	$\frac{83}{900}$	$\frac{1}{450}$	–	$\frac{41}{120}$	$\frac{79}{900}$	$\frac{7}{900}$	–
Ig_6	$\frac{9491}{2385}$	$\frac{36307}{35775}$	$\frac{5168}{35775}$	$\frac{1366}{107325}$	$\frac{4}{7155}$	$\frac{8537}{21465}$	$\frac{3739}{35775}$	$\frac{1676}{107325}$	$\frac{2}{1431}$
Ig_7	$\frac{31}{30}$	$\frac{43}{63}$	$\frac{89}{1575}$	$\frac{2}{1575}$	–	$\frac{3}{10}$	$\frac{4}{63}$	$\frac{1}{225}$	–

3. The resonance functions of the shifts $Ig_n(k)$ and $IG_n(k)$ have "slow" asymptotics for large argument, which complicates the calculations. In order to obtain approximate formulas for large k, we represented the integral exponent by an asymptotic series

$$\mathrm{Ei}(x)\,\mathrm{e}^{-x} \simeq \sum_{m=0}^{\infty} m!/x^{m+1} . \tag{11}$$

With the use of Eqs. (11) and (9) we obtain

$$Ig_n(k) \simeq \frac{1}{\pi} \sum_{m=0} A_m^{(n)}\, k^{-(2m+1)} ,$$

$$IG_n(k) \simeq \frac{1}{\pi} \sum_{m=0} B_m^{(n)}\, k^{-(2m+1)} . \tag{12}$$

To make calculations for $k \geq 30$, it is sufficient to take the first three terms in the sum of Eq. (11) into account. The coefficients $A_m^{(n)}$ and $B_m^{(n)}$ are listed in Table IV.

4. Integral (6) has been calculated for all the resonance functions considered here as well as for the functions $f_n(k)$, $I_n(k)$, and $IF_n(k)$ ($n = 1, \ldots, 5$). The integration was performed numerically according to the trapezoidal rule (decrease of the integration step by a factor of 10 changes the result less than by 10^{-5}). The upper limit of integration can be set $k = 4$, since for larger k the integrand became negligible ($< 5\cdot 10^{-7}$) while $f_n\left(\frac{2k}{\sqrt{\pi x}}\right) < 2$.

At the point $k = 0$ integrals (6) can be calculated exactly:

$$\tilde f_n(0) = \frac{8}{\pi} f_n(0) \int_0^{\infty} x\, \mathrm{e}^{-x^2} \mathrm{d}x = \frac{4}{\pi} f_n(0) = 1.273239\, f_n(0) ,$$

$$\tilde F_n(0) = \frac{4}{\pi} F_n(0) = \frac{4}{n\pi} , \tag{13}$$

$$\tilde{If}_n(0) = \tilde{IF}_n(0) = 0 .$$

$\tilde g_n(0)$, $\tilde{Ig}_l(0)$, $\tilde G_n(0)$, and $\tilde{IG}_n(0)$ can be written down analogousey.

Asymptotic estimates of the functions $\tilde{I}g_n(k)$ and $\tilde{I}G_n(k)$ are also found for large argument

$$\tilde{I}g_n(k) \simeq \sum_{m=0}^{L_n} \frac{\tilde{A}_m^{(n)}}{k^{2m+1}} \frac{(2m+1)!!}{2^{3m}},$$

$$\tilde{I}G_n(k) \simeq \sum_{m=0}^{L_n} \frac{\tilde{B}_m^{(n)}}{k^{2m+1}} \frac{(2m+1)!!}{2^{3m}}, \qquad (14)$$

anloogus formulas can be derived for the functions $\tilde{I}f_n(k)$ and $\tilde{I}F_n(k)$.

Note that the above given asymptotics implies that the functions averaged over the velocities are close in values for large k, since the first terms of the asymptotic series (12) and (14) which make the major contribution, coincide for $m = 0$.

The functions $\tilde{I}f_n(k)$, $\tilde{I}F_n(k)$ ($n = 1, \ldots, 3$), $\tilde{g}_1(k)$, $\tilde{I}g_1(k)$, $\tilde{G}_1(k)$, and $\tilde{I}G_1(k)$ which are most frequently used in calculations are listed in Table V. The values of the functions $\tilde{f}_n(k)$ and $\tilde{F}_n(k)$ havevbeen published by M. Cattani.[9]

TABLE IV. The coefficients $A_m^{(n)}$ and $B_m^{(n)}$ of formulas (12.)

	A_m				B_m		
Ig_n	A_0	A_1	A_2	A_3	B_0	B_1	B_2
Ig_1	6	$\frac{825}{28}$	429	–	$\frac{2}{3}$	$\frac{75}{28}$	33
Ig_2	$\frac{6384}{559}$	$\frac{68244}{559}$	$\frac{259479}{86}$	$\frac{19981125}{172}$	$\frac{2128}{1677}$	$\frac{6204}{559}$	$\frac{259479}{118}$
Ig_3	$\frac{231}{50}$	$\frac{429}{25}$	$\frac{429}{2}$	–	$\frac{21}{50}$	$\frac{33}{25}$	$\frac{143}{10}$
Ig_4	$\frac{3696}{425}$	$\frac{59631}{850}$	$\frac{50193}{34}$	$\frac{204165}{4}$	$\frac{336}{425}$	$\frac{4587}{850}$	$\frac{16731}{170}$
Ig_5	$\frac{1617}{200}$	$\frac{12441}{200}$	1287	$\frac{706095}{16}$	$\frac{147}{200}$	$\frac{957}{200}$	$\frac{429}{5}$
Ig_6	$\frac{19712}{1325}$	$\frac{312026}{1325}$	$\frac{2536391}{318}$	$\frac{42574545}{106}$	$\frac{1267}{1325}$	$\frac{10877}{1325}$	$\frac{410608}{2385}$
Ig_7	$\frac{33}{5}$	$\frac{429}{10}$	$\frac{11583}{14}$	–	$\frac{3}{5}$	$\frac{33}{10}$	$\frac{3861}{70}$

TABLE V. The nonadiabaticity functions averaged over Maxwell's velocity distribution.

k	$\tilde{I}f_1$	$\tilde{I}F_1$	$\tilde{I}f_2$	$\tilde{I}F_2$	$\tilde{I}f_3$	$\tilde{I}F_3$	$\tilde{g}_1$	$\tilde{I}g_1$	$\tilde{G}_1$	$\tilde{I}G_1$
0	0	0	0	0	0	0	1.2732	0	0.1592	0
0.1	0.0298	0.0806	0.0063	0.0061	0.0028	0.0014	1.2694	0.0672	0.1585	0.0096
0.2	0.0796	0.2025	0.0246	0.0228	0.0112	0.0056	1.2584	0.1329	0.1567	0.0189
0.3	0.1544	0.3199	0.0535	0.0472	0.0249	0.0122	1.2409	0.1958	0.1539	0.0276
0.4	0.2366	0.4214	0.0910	0.0766	0.0437	0.0212	1.2179	0.2551	0.1502	0.0358
0.5	0.3192	0.5047	0.1358	0.1088	0.0669	0.0321	1.1902	0.3105	0.1459	0.0432
0.6	0.3980	0.5703	0.1842	0.1421	0.0942	0.0445	1.1587	0.3615	0.1411	0.0499
0.7	0.4703	0.6203	0.2363	0.1751	0.1249	0.0581	1.1242	0.4080	0.1358	0.0559
0.8	0.5349	0.6568	0.2899	0.2069	0.1585	0.0727	1.0874	0.4501	0.1303	0.0612
0.9	0.5913	0.6819	0.3437	0.2369	0.1945	0.0878	1.0489	0.4879	0.1247	0.0658
1.0	0.6394	0.6976	0.3968	0.2646	0.2322	0.1032	1.0092	0.5214	0.1190	0.0697
1.1	0.6796	0.7057	0.4483	0.2898	0.2711	0.1186	0.9687	0.5508	0.1132	0.0730
1.2	0.7124	0.7076	0.4975	0.3123	0.3108	0.1339	0.9281	0.5764	0.1075	0.0758
1.3	0.7383	0.7045	0.5439	0.3322	0.3508	0.1489	0.8874	0.5983	0.1019	0.0781
1.4	0.7580	0.6975	0.5872	0.3495	0.3907	0.1633	0.8471	0.6169	0.0964	0.0799
1.5	0.7723	0.6874	0.6271	0.3642	0.4295	0.1772	0.8074	0.6324	0.0911	0.0813
1.6	0.7817	0.6751	0.6636	0.3765	0.4688	0.1903	0.7683	0.6450	0.0859	0.0823
1.7	0.7868	0.6610	0.6965	0.3866	0.5063	0.2027	0.7303	0.6549	0.0809	0.0829
1.8	0.7883	0.6456	0.7259	0.3946	0.5426	0.2143	0.6934	0.6624	0.0762	0.0833
1.9	0.7865	0.6294	0.7519	0.4007	0.5774	0.2250	0.6575	0.6678	0.0716	0.0834
2.0	0.7821	0.6126	0.7745	0.4051	0.6105	0.2348	0.6229	0.6711	0.0672	0.0833
2.1	0.7754	0.5955	0.7939	0.4079	0.6419	0.2438	0.5895	0.6726	0.0631	0.0829
2.2	0.7668	0.5784	0.8103	0.4094	0.6715	0.2519	0.5575	0.6725	0.0591	0.0824
2.3	0.7566	0.5613	0.8237	0.4095	0.6991	0.2591	0.5268	0.6710	0.0554	0.0817
2.4	0.7452	0.5445	0.8345	0.4096	0.7247	0.2655	0.4974	0.6682	0.0519	0.0809
2.5	0.7327	0.5279	0.8428	0.4067	0.7484	0.2711	0.4693	0.6643	0.0485	0.0800
2.6	0.7195	0.5117	0.8487	0.4039	0.7701	0.2759	0.4425	0.6594	0.0454	0.0790

TABLE V. (continued).

k	$\widetilde{If_1}$	$\widetilde{IF_1}$	$\widetilde{If_2}$	$\widetilde{IF_2}$	$\widetilde{If_3}$	$\widetilde{IF_3}$	$\tilde{g}_1$	$\widetilde{Ig_1}$	$\tilde{G}_1$	$\widetilde{IG_1}$
2.7	0.7056	0.4959	0.8525	0.4004	0.7899	0.2800	0.4170	0.6536	0.0424	0.0779
2.8	0.6913	0.4807	0.8543	0.3963	0.8077	0.2834	0.3927	0.6471	0.0396	0.0767
2.9	0.6767	0.4659	0.8543	0.3917	0.8236	0.2861	0.3697	0.6399	0.0370	0.0755
3.0	0.6619	0.4516	0.8527	0.3865	0.8378	0.2882	0.3479	0.6322	0.0345	0.0742
3.1	0.6471	0.4378	0.8496	0.3811	0.8501	0.2897	0.3272	0.6240	0.0322	0.0730
3.2	0.6323	0.4246	0.8453	0.3753	0.8607	0.2906	0.3076	0.6154	0.0301	0.0716
3.3	0.6175	0.4119	0.8397	0.3692	0.8697	0.2911	0.2891	0.6065	0.0280	0.0703
3.4	0.6030	0.3996	0.8331	0.3630	0.8772	0.2910	0.2716	0.5973	0.0261	0.0690
3.5	0.5886	0.3879	0.8257	0.3567	0.8831	0.2906	0.2550	0.5879	0.0244	0.0677
3.6	0.5745	0.3767	0.8174	0.3502	0.8877	0.2898	0.2394	0.5784	0.0227	0.0663
3.7	0.5606	0.3660	0.8084	0.3437	0.8909	0.2886	0.2247	0.5688	0.0212	0.0650
3.8	0.5471	0.3557	0.7988	0.3371	0.8929	0.2870	0.2108	0.5591	0.0197	0.0637
3.9	0.5338	0.3458	0.7887	0.3306	0.8937	0.2852	0.1978	0.5494	0.0183	0.0624
4.0	0.5210	0.3364	0.7782	0.3240	0.8934	0.2832	0.1855	0.5397	0.0171	0.0611
4.1	0.5084	0.3273	0.7674	0.3176	0.8921	0.2808	0.1739	0.5301	0.0159	0.0599
4.2	0.4962	0.3187	0.7563	0.3111	0.8899	0.2793	0.1630	0.5205	0.0148	0.0586
4.3	0.4844	0.3104	0.7449	0.3048	0.8868	0.2756	0.1528	0.5109	0.0138	0.0574
4.4	0.4729	0.3025	0.7334	0.2986	0.8829	0.2727	0.1432	0.5015	0.0128	0.0562
4.5	0.4618	0.2949	0.7218	0.2924	0.8783	0.2697	0.1342	0.4922	0.0119	0.0550
4.6	0.4511	0.2877	0.7101	0.2864	0.8730	0.2666	0.1257	0.4830	0.0111	0.0539
4.7	0.4406	0.2807	0.6984	0.2805	0.8671	0.2634	0.1177	0.4640	0.0103	0.0528
4.8	0.4306	0.2741	0.6867	0.2747	0.8607	0.2601	0.1102	0.4651	0.0096	0.0517
4.9	0.4208	0.2627	0.6751	0.2690	0.8537	0.2567	0.1032	0.4564	0.0089	0.0506
5.0	0.4114	0.2615	0.6635	0.2635	0.8463	0.2532	0.0967	0.4478	0.0083	0.0496
5.1	0.4023	0.2557	0.6520	0.2581	0.8385	0.2497	0.0905	0.4384	0.0077	0.0486
5.2	0.3936	0.2500	0.6406	0.2529	0.8304	0.2462	0.0847	0.4312	0.0072	0.0476
5.3	0.3851	0.2446	0.6294	0.2477	0.8220	0.2427	0.0793	0.4231	0.0067	0.0467
5.4	0.3769	0.2394	0.6183	0.2428	0.8132	0.2392	0.0742	0.4153	0.0062	0.0458
5.5	0.3690	0.2344	0.6074	0.2379	0.8043	0.2356	0.0694	0.4076	0.0058	0.0449
5.6	0.3614	0.2296	0.5966	0.2332	0.7951	0.2321	0.0650	0.4001	0.0054	0.0440
5.7	0.3540	0.2250	0.5860	0.2286	0.7758	0.2285	0.0608	0.3927	0.0050	0.0431
5.8	0.3469	0.2206	0.5756	0.2241	0.7764	0.2250	0.0569	0.3856	0.0047	0.0423
5.9	0.3400	0.2163	0.5654	0.2198	0.7669	0.2216	0.0533	0.3786	0.0043	0.0415
6.0	0.3334	0.2122	0.5555	0.2156	0.7573	0.2181	0.0498	0.3718	0.0040	0.0407
6.5	0.3034	0.1937	0.5086	0.1964	0.7088	0.2015	0.0357	0.3403	0.0028	0.0372
7.0	0.2788	0.1782	0.4670	0.1798	0.6612	0.1861	0.0256	0.3127	0.0020	0.0341
7.5	0.2565	0.1651	0.4303	0.1656	0.6159	0.1721	0.0183	0.2886	0.0014	0.0315
8.0	0.2380	0.1537	0.3980	0.1533	0.5737	0.1595	0.0131	0.2676	0.0010	0.0292
8.5	0.2220	0.1439	0.3698	0.1426	0.5349	0.1483	0.0094	0.2491	0.0007	0.0272
9.0	0.2080	0.1353	0.3449	0.1334	0.4996	0.1383	0.0067	0.2394	0.0005	0.0254
9.5	0.1958	0.1277	0.3231	0.1252	0.4677	0.1294	0.0048	0.2187	0.0003	0.0239
10.0	0.1849	0.1209	0.3037	0.1180	0.4390	0.1214	0.0035	0.2060	0.0002	0.0226
11.0	0.1666	0.1093	0.2713	0.1059	0.3898	0.1081	0.0018	0.1847	0.0001	0.0203
12.0	0.1517	0.0999	0.2452	0.0961	0.3500	0.0974	0.0009	0.1674	0.0001	0.0184
13.0	0.1393	0.0919	0.2239	0.0881	0.3174	0.0886	0.0005	0.1532	0.0	0.0169
14.0	0.1289	0.0852	0.2062	0.0813	0.2905	0.0814	0.0003	0.1413	0.0	0.0156
15.0	0.1199	0.0794	0.1911	0.0756	0.2679	0.0753	0.0001	0.1312	0.0	0.0145
16.0	0.1122	0.0793	0.1783	0.0706	0.2488	0.0701	0.0001	0.1225	0.0	0.0135
17.0	0.1054	0.0699	0.1671	0.0662	0.2323	0.0656	0.0	0.1149	0.0	0.0127
18.0	0.0994	0.0659	0.1572	0.0624	0.2180	0.0616	0.0	0.1082	0.0	0.0120
19.0	0.0940	0.0624	0.1485	0.0590	0.2055	0.0582	0.0	0.1023	0.0	0.0113
20.0	0.0892	0.0592	0.1408	0.0560	0.1944	0.0551	0.0	0.0970	0.0	0.0107
22.5	0.0791	0.0526	0.1246	0.0496	0.1714	0.0487	0.0	0.0859	0.0	0.0095
25.0	0.0711	0.0473	0.1118	0.0446	0.1535	0.0436	0.0	0.0771	0.0	0.0086
27.5	0.0646	0.0430	0.1014	0.0404	0.1390	0.0396	0.0	0.0700	0.0	0.0078
30.0	0.0592	0.0394	0.0928	0.0370	0.1270	0.0362	0.0	0.0641	0.0	0.0071

*Note. The functions of $\tilde{f}_1$, $\tilde{f}_2$, $\tilde{f}_3$, $\tilde{F}_1$, $\tilde{F}_2$, $\tilde{F}_3$ .. are diven in Ref. 9.

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