THEORETICAL INVESTIGATION OF THE SPECTRAL BEHAVIOR OF THE OPTICAL RADIATION EXTINCTION COEFFICIENT OF A SYSTEM OF ORIENTED ICE PLATES

A.G. Borovoi, A.A. Popov, and O.V. Shefer

Institute of Atmospheric Optics,
Siberian Branch of the Academy of Sciences of the USSR, Tomsk
A.M. Gor'kii Mari Polytechnic Institute, Ioshkar–Ola
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An oscillating wavelength dependence is obtained in the IR for the extinction coefficient α of a system of oriented ice plates. Each curve α(λ) has a fine structure and certain specific features, which may be used to estimate the average size of the ice plates. We show that it is only the oriented ice plates which may cause a noticeable wavelength dependence of the extinction coefficient of a crystalline cloud.

We proposed to classify the atmospheric crystals according to the character of the optical radiation extinction in Ref. 1. The entire range of shapes of crystals can be divided into two groups. One group includes all the crystals lacking plane–parallel faces. The field scattered by such crystals within small scattering angles is completely determined by the diffraction field. As a result, their extinction efficiency is equal to 2, and their extinction cross section is equal to the doubled cross–sectional area of geometric shadow of the scatterer. The extinction coefficient of the studied volume containing such crystals is determined by their average size and number density, i.e., is independent of either the wavelength or the optical properties of crystal particles. In other words, the calculation of the extinction parameters for crystals from such a group is not difficult. The remaining group of crystals is more interesting in this respect. This group includes all polyhedral crystals, having at least a pair of plane–parallel faces. Previously, we have demonstrated[2] that the complete field scattered by such crystals is determined within small scattering angles as a coherent sum of the diffracted and scattered fields. Here by the scattered field we understand the electromagnetic field, into which the field of the refracted beams, leaving the crystal in the direction of the propagation of the incident wave, is converted. Since each beam is a part of the wavefront of the plane wave, one and the same formalism may be employed to determine the scattered and diffracted fields. As a rule, these fields are comparable to each other, if the crystals have plane–parallel faces. As a result, when both the geometric and optical parameters of the crystal change, its extinction efficiency oscillates about 2. It was demonstrated in Ref. 3 that the extinction efficiency may reach the limiting value of 4 for plate crystals. It is for the crystals of this shape that their optical properties mostly affect the extinction efficiency. Consequently, in this paper we chose the system of the oriented ice plates as the scattering medium to study theoretically the optical radiation extinction coefficient.

The relation for the cross section of polarized radiation extinction by a round plate has been derived in Ref. 4 within the framework of the method of physical optics

$$\sigma = 2S - \mathrm{Re}(B_{\parallel} + B_{\perp}) - \frac{I_2}{I_1}\mathrm{Re}(B_{\parallel} - B_{\perp})\cos 2\gamma + \frac{I_3}{I_1}\mathrm{Re}(B_{\parallel} - B_{\perp})\sin 2\gamma . \quad (1)$$

In addition to the doubled cross sectional area of the geometric shadow of the plate $2S$, several other terms enter in formula (1). Such additional terms are related to the polarized scattered field of the refracted beams, transmitted through the plate. The variable γ in the above formula is the angle of orientation of vectors $\mathbf{E}_1$ and $\mathbf{E}_2$ of the incident elliptically polarized wave with respect to the plane of incidence upon the plate base and I_1, I_2, and I_3 are the first three Stokes parameters of the incident wave. The amplitudes of the scattered fields $B_{\parallel}$ and $B_{\perp}$ of the refracted beams and the cross–sectional area of geometric shadow S of a plate of radius a and thickness d are given by the following relations:

$$B_{\parallel} = T_{\parallel}\tilde{T}_{\parallel}\sum_{j=1}^{J} S_j\, e^{i\delta_j} R_{\parallel}^{2(j-1)}, \quad (2)$$

$$B_{\perp} = T_{\perp}\tilde{T}_{\perp}\sum_{j=1}^{J} S_j\, e^{i\delta_j} R_{\perp}^{2(j-1)}, \quad (3)$$

$$S = (\pi\alpha^2 + 2ad\tan\beta)\cos\beta . \quad (4)$$

Here T and R are Fresnel's coefficients, δ_j is the relative run–on of the phase of the jth refracted beam transmitted $(2j - 1)$ times through the plate, S_j is the cross–sectional area of the jth beam, and β is the acute angle between the direction of the incident wave and the normal to the plate base. The superscript J in sums (2) and (3) determines the number of the refracted beams being formed. If $\beta \neq 0$, it always remains finite. For $\beta = 0$ the number of the refracted beams becomes infinite, and the problem of scattering is then symmetric. As a result, the formula for the extinction cross section takes the following form:

$$\sigma = 2\pi\alpha^2\left[1 - \mathrm{Re}\left(t\sum_{j=1}^{\infty} e^{i\delta_j}\, r^{\,j-1}\right)\right], \quad (5)$$

where

$$t = \frac{4\tilde{n}}{(\tilde{n}+1)^2}, \quad r = \left(\frac{\tilde{n}-1}{\tilde{n}+1}\right)^2, \quad (6)$$

$$\delta_j = kd[2(j-1)\,\tilde{n} - 1] . \quad (7)$$

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We have $\tilde{n} = n + i\kappa$ in Eqs. (6) and (7), where $\tilde{n}$ is the complex refractive index of a plate, and $k = 2\pi/\lambda$ is the wave number.

As could be expected, when the wave falls normally onto the plate base ($\beta = 0$), the extinction cross section is independent of the state of polarization of the incident field. Note in this connection that although the extinction cross section formally depends on the polarization properties of the incident wave, this effect is weakly manifested for $\beta \neq 0$ (see Ref. 4). Moreover, when either the geometric or optical properties of the plate vary, the behavior of the extinction cross sections, calculated from Eqs. (1) and (5), remain qualitatively unchanged, providing the angle β entering formula (1) does not exceed 45°. Thus, when speaking about determination of the general behavior of the integral extinction of optical radiation by the system of oriented plates, it stands to reason to choose for the integrable function the extinction cross section given by Eq. (5), which is more convenient for our analysis. If the extinction cross section is prescribed in a simpler form, such an integral characteristic of the scattering volume as the extinction coefficient is reduced to algebraic relation. In particular, the following formula has been derived for the extinction coefficient in Ref. 5:

$$\alpha = D\left[1 - \mathrm{Re}\left(t\sum_{j=1}^{\infty}\frac{r^{j-1}}{\left(1+(2j-1)\kappa k x_2 - i\left[(2j-1)n-1\right]k x_2\right)^{x_1+1}}\right)\right], \tag{8}$$

where

$$D = 2N\pi\bar{a}^2\,\frac{\mu + 2}{\mu + 1}\,. \tag{9}$$

In the derivation of formula (8) the distribution function of plates over their radii was prescribed by a gamma–function with the parameters a_m and μ. Remind that a_μ is the plate radius corresponding to the maximum of the distribution function and μ describes the steepness of the slopes of that maximum. The value of N in Eq. (9) is the number density of plates in the scattering volume and $\bar{a}$ is their average radius. Note that the parameter $\bar{a}$ is related to the parameters of the gamma–distribution a_m and μ in terms of $\bar{a} = a_m(\mu + 1)/\mu$. The value x_2 in Eq. (8) has the same dimensionality as $\bar{a}$ and a_m, i.e., is measured in micrometers, while x_1 is the dimensionless variable. Moreover, the parameters x_1 and x_2 are uniquely related to μ and a_m (see Ref. 5). Finally, in the derivation of Eq. (8) we accounted for the empirical relation between the thickness and the radius of a plate, given by the relation $d = A(2a)^\eta$, where $A = 2.020$, $\eta = 0.449$ (see Ref. 6). Note that the thickness d and the radius a in the latter formula should be given in micrometers.

It has been shown in Ref. 5 that the contribution of each of the terms of the infinite sum in Eq. (8) is negligible in the visible. As a result, the extinction coefficient α is independent of the wavelength, so that it is given by the formula

$$\alpha = D\,. \tag{10}$$

Formula (10) takes exactly the same form as for the crystals lacking plane–parallel faces. However, now this formula is interpreted differently. The point is that a complete mutual compensation of the refraction beams occurs in the system of crystals with plane–parallel faces in the visible. In other words, when integrating the rapidly oscillating function, the contribution to the integral comes only from its regular part, whose role plays the diffraction field. Such an interpretation makes it possible to explain from a common viewpoint both the neutral behavior of the extinction coefficient in the visible and its noticeable wavelength dependence in the IR. Indeed the period of oscillations of the extinction efficiency in the visible is so small that the function of distribution of plates over the radii may be considered constant during this period. As a result, all deviations of this factor from its average value, determined by the diffraction field, are completely compensated during each oscillation period. The situation changes when we go over to the IR. In this range the period of oscillations increases many times and becomes so long, that the distribution function may be noticeably changed during this period. As a result, the complete mutual compensation of scattered fields of the refraction beams did not take place in the scattering volume. Therefore, the uncompensated increment to the diffraction field in the IR always exists whose value depends, in particular, on the wavelength.

It has been demonstrated in Ref. 5 when calculating the infinite sum in Eq. (8) in the IR, it is sufficient to take only the first term into account, because all other terms are negligible in comparison with it. When calculating the extinction coefficient α in the IR, the latter statement is equivalent to taking account, along with the total diffraction field, of the scattered fields from those refraction beams, which have passed through the crystal only once. In this case the formula for the extinction coefficient takes the form

$$\alpha = D\left[1 - \mathrm{Re}\left(\frac{t}{\left(1 + \kappa k x_2 - i(n-1)k x_2\right)^{x_1+1}}\right)\right]. \tag{11}$$

Formulas (8) and (11) have been derived by approximating the integral representation of the extinction coefficient with an error of not more than 2.5%. Such an estimate is valid for Eq. (11) in the IR, and for Eq. (8) – over the entire wavelength range.

TABLE I. The parameters x_1 and x_2 entering into the approximation formula for the extinction coefficient.

$\bar{a}$, μm	μ					
	1	2	3	4	5	6
50	17.17	22.13	27.08	32.05	37.02	41.99
	1.18	0.86	0.68	0.56	0.47	0.41
60	17.17	22.13	27.08	32.04	37.02	41.99
	1.28	0.93	0.73	0.61	0.52	0.45
70	17.17	22.13	27.08	32.04	37.02	41.99
	1.37	1.00	0.79	0.65	0.55	0.48
80	17.17	22.13	27.08	32.05	37.01	41.98
	1.46	1.06	0.83	0.69	0.59	0.51
90	17.17	22.13	27.08	32.05	37.02	41.99
	1.54	1.12	0.88	0.73	0.62	0.54
100	17.17	22.13	27.08	32.04	37.01	41.99
	1.61	1.17	0.92	0.76	0.65	0.56
150	17.17	22.13	27.08	32.05	37.02	41.98
	1.93	1.41	1.11	0.91	0.78	0.68
200	17.17	22.13	27.08	32.05	37.02	41.99
	2.20	2.20	1.60	1.26	1.04	0.77
250	17.17	22.13	27.08	32.05	37.02	41.98
	2.43	1.77	1.39	1.15	0.98	0.85

The values x_1 and x_2, entering into formula (11), depend solely on the parameters of the distribution $\overline{a}$ and μ.[5] This makes it possible to precalculate x_1 and x_2 for any admissible values of $\overline{a}$ and μ. Table I presents the calculated values of x_1 and x_2, corresponding to certain values of $\overline{a}$ and μ, actually observed for the ice plates. Note that the intermediate values of x_1 and x_2, which are not listed in Table I, may be obtained by interpolation.

Figure 1 presents the dependences of the refraction index n and the absorption index κ on the wavelength λ. The curves $n = n(\lambda)$ and $\kappa = \kappa(\lambda)$ are plotted according to the data published in Ref. 7. Note first of all that the values of the κ increase in the IR reaching 10^{-1} and even exceeding this value at certain wavelengths. As a result, the values of κ and $n - 1$ are comparable to each other, practically over the entire IR range. This means that the absorption index κ should strongly affect the spectral behavior of the extinction coefficient α. It is quite easy to find from analysis of Eq. (11) for α that with an increase of κ, the amplitude of the "refraction" term decreases. The effect of κ becomes particularly strong in the vicinity of the minima in the function $n = n(\lambda)$. The minima of the refraction index n, corresponding to the wavelengths of 2.9 and 10.9 μm, can be interpreted as a result of resonance interaction of optical radiation with the crystalline lattice of ice.[7] The effect of the refraction index on the spectral behavior of the extinction coefficient is as follows. With n not only the amplitude of the "refraction" term decreases, but also its sign alternates periodically. In other words it is because of the refraction index $n(\lambda)$ that the dependence $\alpha = \alpha(\lambda)$ acquires an oscillating character.

With increase of the average radius $\overline{a}$ of the ice plates the value of x_2 increases too, which, in its turn, leads to decrease of the relative amplitude of oscillations of the dependence $\alpha = \alpha(\lambda)$. This can be easy seen from the comparison of the curves $\alpha(\lambda)$ in Figs. 2 and 3, plotted for the small and large ice plates. As could be expected, the curves for small plates (Fig. 2) exhibit a finer structure. Moreover, each of them also has certain specific features, based on which one may estimate the average size of the ice plates. In particular, the position of the minimum in each curve, shown by the arrow, depends on average size of ice plates.

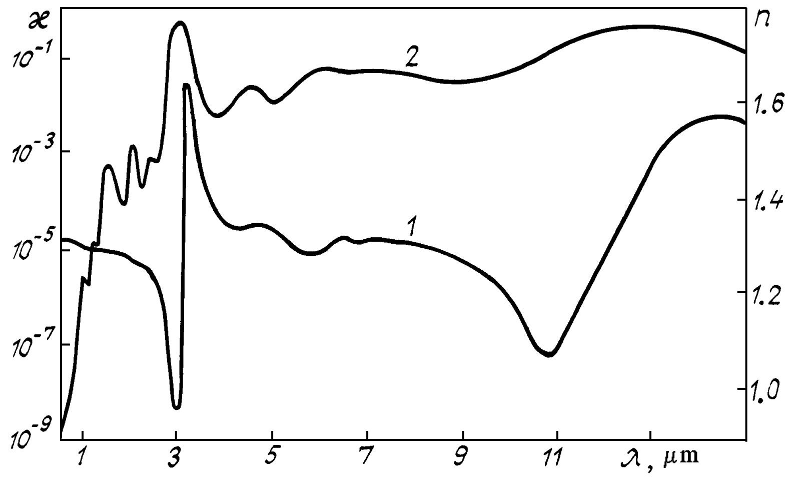

FIG. 1. The real and imaginary parts of the complex index of refraction of ice vs. wavelength: 1) $n = n(\lambda)$ and 2) $\kappa = \kappa(\lambda)$.

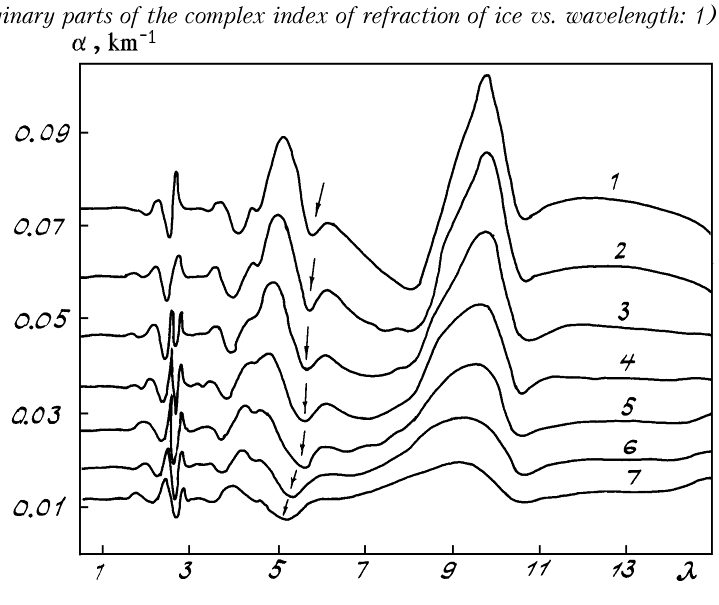

FIG. 2. Extinction coefficients of small ice plates vs. wavelength for $N = 1$ l^{-1} and $\mu = 5$: 1) $\overline{a} = 100$, 2) 90, 3) 80, 4) 70, 5) 60, 6) 50, and 7) 40 μm.

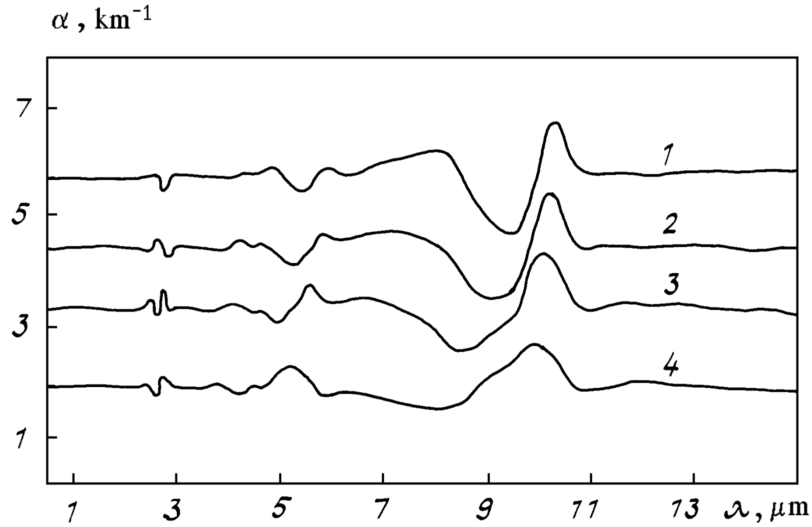

FIG. 3. Extinction coefficients of large ice plates vs. wavelength for $\mu = 5$: *1)* $N = 12.5\ l^{-1}$ *and* $\bar{a} = 250\ \mu m$, *2)* $N = 15\ l^{-1}$ *and* $\bar{a} = 200\ \mu m$, *3)* $N = 20\ l^{-1}$ *and* $\bar{a} = 150\ \mu m$, *and 4)* $N = 25\ l^{-1}$ *and* $\bar{a} = 100\ \mu m$.

Increasing the wavelength λ equivalent to decreasing the parameter x_2. Hence, with λ the amplitudes of oscillations of the extinction coefficient should also increase. This is easy to see analyzing the same curves shown in Figs. 2 and 3. However within the wavelength range 11 – 15μm the extinction coefficient has almost neutral spectral behavior. This is explained by the advanced increase in the values of $n - 1$ and κ in comparison with the wavelength λ. As a result, the oscillation amplitudes rapidly diminish.

The degree of concentration of the size of ice plates around their average value is determined by the parameter μ for the gamma–distribution. Smaller values of μ correspond to a weaker change of the distribution function and hence to more complete compensation for the scattered fields of the refracted beams. As a result in the case of a mildly sloping distribution function the amplitudes of oscillations in the $\alpha = \alpha(\lambda)$ curve diminish and its fine structure is smoothed. This can be seen by comparing the spectral behavior of the curves $\alpha(\lambda)$ in Fig. 4, plotted for various μ. As for the large plates whose size is only weakly concentrated around the average value, note that, except for narrow spectral regions (see curve *1*), the corresponding extinction coefficient has neutral behavior in the IR.

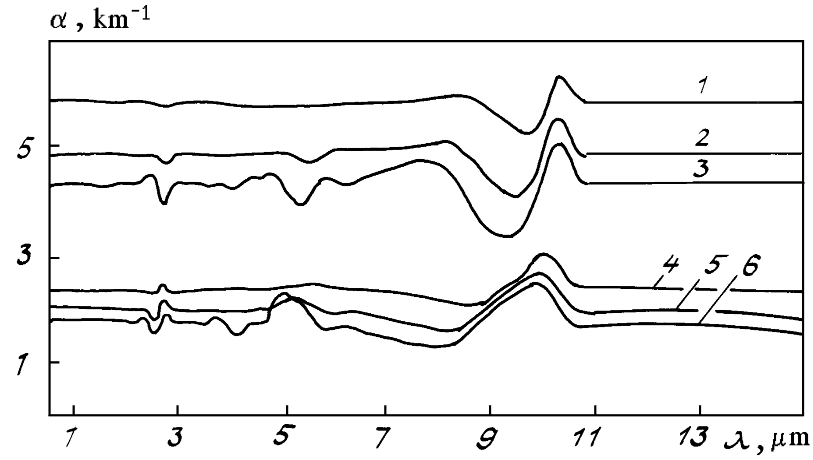

FIG. 4. Extinction coefficients of ice plates vs. wavelength for various values of μ: *curves 1, 2, and 3 correspond to* $\mu = 1$, 3, *and* 8, *respectively;* $\bar{a} = 250\ \mu m$, *and,* $N = 10\ l^{-1}$; *curves 4, 5, and 6 correspond to* $\mu = 1$, 3, and 8; $\bar{a} = 100\ \mu m$; *and* $N = 25\ l^{-1}$.

A noticeable dependence of the extinction coefficient on the wavelength in the IR is a result of oscillations of the extinction efficiency. For plate crystals the range of the possible values of the extinction efficiency is (0, 4). For any other form of crystals with plane–parallel faces this range is much narrower. For example, for a hexagonal column,[3] we have found that its extinction efficiency oscillates approximately from 1 to 3. Moreover, for an overwhelming number of combinations of the geometric and optical parameters of a hexagonal column the extinction efficiency falls within the interval (1.3, 2.7). It should also be taken into account that even if such hexagonal columns are coaxially oriented within the scattering volume, each of them may occupy any position determined by rotation of the column about its axis. After partial averaging of the light scattering characteristics of the system of such crystals, the range of variation in the extinction efficiency will become even narrower. It is quite clear that the low amplitude of oscillations of the scattering efficiency govern low variation in the extinction coefficient vs wavelength. In other words, for a system of a needle–like crystals the spectral behavior of the extinction coefficient should be close to neutral, even when all such crystals are oriented. A neutral behavior of the extinction coefficient should also be observed in the IR for the chaotically oriented crystals. Indeed, because of their aerodynamic properties, crystals with chaotic orientation has minimal and maximal dimensions that differ only slightly. The extinction efficiency of such plates may only slightly deviate from its asymptotic value of 2. When we average over all possible orientations, such deviations are mutually compensated. Thus, if one finds during the observations of Ci clouds in the IR that their extinction coefficient is wavelength dependent, such a dependence would most probably be a result of the presence of oriented ice plates in the scattering volume.

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