PROPAGATION AND SCATTERING OF SOUND WAVES IN THE TURBULENT MEDIA (ATMOSPHERE OR OCEAN)

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Statistical characteristics of the sound waves, propagating through the moving media with random sound speed, density, and velocity inhomogeneities, are calculated in the Born approximation using the geometric–acoustics approximation and the methods of smooth perturbation and parabolic equation as well as theory of multiple scattering. A new method for remote sounding of the atmospheric humidity fluctuations is proposed.

Theory of the sound wave propagation through the media with random fluctuations of sound speed c has been well developed in literature. However, only fragmentary results have been obtained describing the effect of fluctuations in the medium density ρ and in the velocity of the moving medium $\mathbf{v}$ on the sound field. In this paper we develop theory of the sound wave propagation through the media with randomly inhomogeneous c, ρ, and $\mathbf{v}$. In ample detail the results of this theory have been presented in Ref. 1.

Note that the random density and wind speed inhomogeneities must always be taken into account when calculating the statistical characteristics of the sound field in the turbulent atmosphere. Meanwhile, the amplitude and phase fluctuations of the sound wave, propagating through the ocean, are mainly associated with the inhomogeneities in c. Sometimes a significant contribution to the scattered field comes from random density fluctuations and from the velocity of currents.

When constructing the theory of sound propagation in randomly inhomogeneous moving media, we start from the following equation:[2]

$$(\Delta + k^2)p + \left[k^2\varepsilon - \left(\nabla \ln \frac{\rho}{\rho_0}\right)\nabla - \frac{2i}{\omega}\frac{\partial v_i}{\partial x_j}\frac{\partial^2}{\partial x_i \partial x_j} + \frac{2ik}{c_0}\mathbf{V}\nabla\right]p = 0\ , \quad (1)$$

where $p(\mathbf{R})$ is the sound pressure, $\mathbf{R} = (x_1, x_2, x_3) = (x, y, z)$ are the Cartesian coordinates, $k = \omega/c_0$ is the wave number, ω is the frequency, c_0 and ρ_0 are the mean values of c and ρ, $\varepsilon = c_0^2/c^2 - 1$, $\mathbf{v} = (v_1, v_2, v_3)$ is the velocity of the moving medium, and the recurring subscripts denote summation. When deriving Eq. (1) we neglect the terms of the order of $(v/c)^2$ in calculating the moments of the quantity p. We set $c = c_0 + \tilde{c}$ and $\rho = \rho_0 +$ in Eq. (1). The fluctuations in the temperature $\tilde{T}$ and in the concentration $\tilde{C}$ of the component, dissolved in the medium (water vapor in air or salt in ocean water) are primarily responsible for the fluctuations in the sound speed $\tilde{c}$ and in the density . To within the terms of the order of $\tilde{T}$ and $\tilde{C}$ we have

$$\varepsilon = -2\tilde{c}/c_0 = -\beta_c\tilde{T}/T_0 - \eta_c\tilde{C}\ ;$$

$$\ln(\rho/\rho_0) = \tilde{\rho}/\rho_0 = \beta_\rho\tilde{T}/T_0 + \eta_\rho\tilde{C}\ , \quad (2)$$

where T_0 is the mean temperature, while

$$\beta_c = 2\frac{T_0}{c_0}\frac{\partial c}{\partial T} = 1\ (1.73 - 3.46\cdot10^{-2}T°\mathrm{C} + 3.29\cdot10^{-4}(T°\mathrm{C})^2)\ ;$$

$$\eta_c = \frac{2}{c_0}\frac{\partial c}{\partial C} = 0.45\ (1.79 - 1.33 - 10^{-2}(T°\mathrm{C}))\ ;$$

$$\beta_\rho = \frac{T_0}{\rho_0}\frac{\partial r}{\partial T} = -1\ \ (-7.5 - 10^{-2})\ ;$$

$$\eta_q = \frac{1}{\rho_0}\frac{\partial r}{\partial C} = -0.61\ \ (-0.17)\ . \quad (3)$$

In the right sides of these relations we give the values of the coefficients $\beta_{c,\,\rho}$ and $\eta_{c,\,\rho}$ for the atmosphere and for the ocean (in parentheses). Here C is the concentration of the component, dissolved in the given medium (in the atmosphere the partial water–vapor pressure e is related to C in the following way: $e/P = 1.61\ C$, where P is the air pressure).

Assuming that $\langle\tilde{T}v_i\rangle = \langle\tilde{C}v_i\rangle = 0$ and using relation (2), we can derive from Eq. (1) the following relation for the scattering cross section σ of sound wave in a randomly inhomogeneous moving medium:[3,4]

$$\sigma(\mathbf{n} - \mathbf{n}_0) = 2\pi k^4\Big[\beta^2(\theta)\Phi_T(\mathbf{k})/4T_0^2 + \beta(\theta)\,\eta(\theta)\,\Phi_{CT}(\mathbf{k})/2T_0 + \eta^2(\theta)\Phi_C(\boldsymbol{\kappa})/4 + \cos^2\theta\cdot\cos^2\frac{\theta}{2}\cdot F(\mathbf{k})/c_0^2\Big], \quad (4)$$

where $\boldsymbol{\kappa} = k(\mathbf{n}_0 - \mathbf{n})$, $\mathbf{n}_0$ and $\mathbf{n}$ are the unit vectors in the direction of incident and scattered waves; θ is the angle between these vectors; Φ_T, Φ_C, and F are the three–dimensional spectral densities of the fields from $\tilde{T}$, $\tilde{C}$, and $\mathbf{v}$; Φ_{CT} is the cross–spectral density of the fields $\tilde{C}$ and $\tilde{T}$; and $\beta = \beta_c + 2\beta_\rho\sin^2\frac{\theta}{2}$, and $\eta = \eta_c + 2\eta_\rho\sin^2\frac{\theta}{2}$. In the inertial interval of the turbulence spectrum the relation for σ takes the form

$$\sigma(\theta) = \frac{55\cdot3^{1/2}\cdot\Gamma(2/3)}{\pi\cdot864\cdot2^{2/3}}\,k^{1/3}\left(\sin\frac{\theta}{2}\right)^{-11/3}\Bigg\{\ \frac{3}{22}\times$$

$$\times\left[\beta^2(\theta)\frac{C_T^2}{T_0^2}+2\eta(\theta)\,\beta(\theta)\frac{C_{CT}}{T_0}+\eta^2C_C^2\right]+\cos^2\theta\cdot\cos^2\frac{\theta}{2}\frac{C_v^2}{c_0^2}\Bigg\}. \quad (5)$$

Here C_T^2, C_C^2, and C_v^2 are the structure constants of the random fields $\tilde{T}$, $\tilde{C}$, and **v**; C_{CT} is the structure constant of the cross–spectral density of the fields $\tilde{C}$ and $\tilde{T}$; and Γ is the γ–function.

For $C_C^2 = C_{CT} = 0$ as well as for the case, in which the sound is scattered in a turbulent atmosphere at $\beta = \cos\theta$, relation (5) transforms into the relation for σ which has been derived by Monin.[6] Note that in Ref. 7, as well as in some other papers, scattering of sound on the humidity fluctuations was examined only qualitatively. This leads to the incorrect coefficients $2\beta^2\eta_c/T_0$ and $\beta^2\eta_c^2$ in front of the structure constants C_{CT} and C_c^2 in Eq. (5).

Since in the atmosphere the coefficients $\beta = \cos\theta$ and $\eta = -0.16 + 0.61\cos\theta$, it follows from Eq. (5) that the sound wave is scattered at the right angle only on the humidity fluctuations. This result is of principal importance. Indeed, using a bistatic configuration of the acoustic sounding system and measuring the scattering cross section $\sigma(\theta = \pi/2) = 1.45\cdot10^{-2}\cdot(\eta_c + \eta_\rho)^2k^{1/3}C_C^2$, one can determine the structure constant C_C^2.

Let θ_1 and θ_2 denote the angles θ which are close to $\pi/2$ for which the term $(3/22)\,\eta^2C_C^2$ in the braces of Eq. (5), is equal to $\cos^2\theta\,\cos^2(\theta/2)C_v^2/c_0^2$. It is obvious that scattering on random humidity fluctuations will make the main contribution to the sound field scattered at the right angle providing the angular half–width of the transmitted beam φ is smaller than or of the order of $|\theta_1 - \theta_2|/2$. Determining the values of the angles θ_1 and θ_2, we obtain

$$\varphi \lesssim \frac{|\theta_1-\theta_2|}{2} = \frac{4.8^\circ b^{1/2}}{1-0.1b}\,,\ \text{where } b = \frac{C_C^2}{C_v^2/c_0^2}\,.$$

In the near–water atmospheric layer the quantity C_C^2 equals $\sim 2.5\cdot10^{-7}$ m$^{-2/3}$ (Ref. 5) and, apparently, may exceed C_T^2/C_0^2. For $b = 1$ it is possible to measure C_C^2, if $\varphi \lesssim 5.3^\circ$.

In the ocean under conditions of small–scale inhomogeneities, whose size does not exceed several meters, in accordance with Ref. 8, $C_T^2\,/\,T_0^2$ vary within the limits $2\cdot10^{-10} - 2\cdot10^{-6}$ m$^{-2/3}$. Accounting for the fact that in the ocean the turbulent energy dissipation rate $\varepsilon = 10^{-7}$–10^{-4} m^2/s^3 (see Ref. 9) we determine the range of variation of the ratio

$$C_v^2/c_0^3 = \frac{1.9\varepsilon^{2/3}}{c_0^2}\ :\ 2\cdot10^{-11} - 2\cdot10^{-9}\text{m}^{-2/3}\,.$$

Thus, the ranges of variation of C_T^2/T_0^2 and C_v^2/c_0^2 overlap. Moreover, the numerical coefficient in front of C_T^2/T_0^2 in Eq. (5) equals 0.1 – 0.3. For this reason, in the ocean sound scattering on the fluctuations of the velocity **v** of the moving medium may be significant in some cases. As regards sound–wave scattering on the density fluctuations, it follows from Eqs. (3) and (5) that it intensifies as the angle θ increases and may be about ~10% of the scattering on the sound speed fluctuations.

The inner scale of turbulence l (the size of the smallest inhomogeneities) in the atmosphere is of the order of 1 mm and is smaller than or near 1 cm in the ocean. For this reason, in acoustics the sound wavelength $\lambda \gg l$ virtually for all frequencies employed. Let $\lambda \ll L$, where L is the outer scale of turbulence (the size of the largest inhomogeneities). Then, since the main portion of the turbulent energy is concentrated within the region of large scales of turbulence the amplitude and phase fluctuations of the sound wave in the direction of propagation of the undisturbed wave are mostly caused by the large–scale inhomogeneities while the small–scale inhomogeneities result in relatively weak sound scattering in all directions. We will more rigourously justify this statement with the help of the multiple scattering theory. Thus, if $l \ll \lambda \ll L$ then in calculating the statistical characteristics of the field in the direction of propagation of undisturbed wave, the random inhomogeneities may be regarded as the large–scale ones in comparison with λ. In this case it is reasonable to transfer from Eq. (1) to the parabolic equation

$$2ik\frac{\partial A}{\partial x} + \Delta_\perp A + k^2\varepsilon_{\text{ef}}A = 0\,. \quad (6)$$

Here A is the complex amplitude, which is related to p by the formula $p(\mathbf{R}) = A(\mathbf{R})e^{ikx}$; the x axis coincides with the direction of wave propagation, and the function $\varepsilon_{\text{ef}} = \varepsilon - 2vx/c_0$. Relation (6) coincides with the parabolic equation for sound and electromagnetic waves in a stationary medium, in which the function ε plays the role of the function ε_{ef}. For this reason, in the parabolic–equation approximation (and, consequently, in the geometric–acoustics approximation and in the method of smooth perturbations) the relations for the statistical moments of the quantity p in the moving medium agree with the analogous well–known relations for the moments of p in the stationary medium, if only the structure function $D_\varepsilon(\mathbf{R})$ of the random field ε or its three–dimensional spectral density $\Phi_\varepsilon(\boldsymbol{\kappa})$ is replaced by the structure function $D_{\text{ef}}(\mathbf{R})$ of the random field ε_{ef} or by its three–dimensional spectral density $\Phi_{\text{ef}}(\boldsymbol{\kappa})$.

This result is valid for the arbitrary randomly inhomogeneous moving medium. For locally homogeneous and locally isotropic random fields ε and **v** as well as for the inertial interval of turbulence the functions $D_{\text{ef}}(\mathbf{R})$ and $\Phi_{\text{ef}}(\boldsymbol{\kappa}) = \Phi_{\text{ef}}(\kappa_x, \boldsymbol{\kappa}_\perp)$ can be specified[10]

$$D_{\text{ef}}(\mathbf{R}) = D_\varepsilon(R) + \frac{4}{c_0^2}\left[\sin^2\alpha\cdot D_{tt}(R) + \cos^2\alpha\cdot D_{RR}(R)\right] = C_N^2R^{2/3};$$

$$\Phi_{\text{ef}}(0, \boldsymbol{\kappa}_\perp) = \Phi_\varepsilon(0, \boldsymbol{\kappa}_\perp) + \frac{4F(\mathbf{k}_\perp)}{c_0^2} = \frac{5\sqrt{3}\,\Gamma(2/3)}{36\pi^2}\,C_{\text{ef}}^2\,\mathbf{k}_\perp^{-11/3}\,, \quad (7)$$

where

$$C_N^2 = C_\varepsilon^2 + 4\left(1 + \sin^2\frac{\alpha}{3}\right)\frac{C_v^2}{c_0^2}\,,\quad C_{\text{ef}}^2 = C_\varepsilon^2 + \frac{22}{3}\frac{C_v^2}{c_0^2}\,,$$

$$C_\varepsilon^2 = \beta_c^2\frac{C_T^2}{T_0^2} + 2\eta_c\,\beta_c\frac{C_{CT}}{T_0} + \eta_c^2\,C_c^2\,. \quad (8)$$

In relations (7) – (8) α is the angle between the vector **R** and the x axis, D_{tt} and D_{RR} are the transverse and longitudinal structure functions of the random field $\mathbf{v}$, C_{ef}^2 is the effective structure constant, and C_N^2 is the structure constant of the random field ε_{ef}.

For the case of sound propagation through the turbulent atmosphere it is well known[11,12] that it is possible to introduce of the effective functions D_{ef} and C_N^2 in the geometric–acoustics approximation and smooth perturbation method. . However, it is conventionally assumed in literature that the angle $\alpha = 0$ in formulas (7)–(8). This assumption results in incorrect values of the functions D_{ef} and C_N^2 as well as in the fact that the coefficient in front of C_v^2 in the relation for C_{ef}^2 turns out to be equal to 4, rather than to 22/3. Note that we introduce the effective functions D_{ef}, C_N^2, and so on, not only in the geometric–acoustics approximation and in the smooth perturbation method, but also in a more general parabolic–equation method. Moreover, these functions are introduced in our examination of the sound propagation through an arbitrary randomly inhomogeneous medium.

The effect of regular refraction on the statistical moments of the sound field in the randomly inhomogeneous moving medium was examined in Ref. 3 in the Markovian approximation of the parabolic–equation method.

Let us consider a single–point sound source, which is located at the point $\mathbf{R}_0$. The field of this source $G(\mathbf{R}, \mathbf{R}_0)$ obeys Eq. (1) with the function $(1 + i\mathbf{v}(\mathbf{R}_0)\nabla/\omega)\delta(\mathbf{R} - \mathbf{R}_0)$. For $|\mathbf{R} - \mathbf{R}_0| \gg L$ the effect of the random vector $\mathbf{v}(\mathbf{R}_0)$ on the moments of G is negligible. In this case, from Eq. (1) we proceed to the equivalent integral equation

$$G(\mathbf{R}, \mathbf{R}_0) = G_0(\mathbf{R} - \mathbf{R}_0) - \int d^3R' G_0(\mathbf{R} - \mathbf{R}') \times$$

$$\times \left[k^2\varepsilon(\mathbf{R}') - \left(\nabla' \ln \frac{\rho(\mathbf{R}')}{\rho_0}\right) \nabla' - \frac{2i}{\omega} \frac{\partial v_i(\mathbf{R}')}{\partial x_j'} \frac{\partial^2}{\partial x_i' \partial x_j'} + \right.$$

$$\left. + \frac{2ik}{c_0} \mathbf{v}(\mathbf{R}')\nabla' \right] G(\mathbf{R}', \mathbf{R}_0) , \tag{9}$$

where $G_0(\mathbf{R}) = -\exp(ikR)/4\pi R$ and $\nabla' = \dfrac{\delta}{\delta \mathbf{R}'}$. When constructing our theory of multiple scattering, we start from Eq. (9) with the functions ε and $\ln(\rho/\rho_0)$ given by Eq. (2). Let us go over to a spectral representation of all functions entering into Eq. (9). We will write the solution of this equation in the form of a series of the perturbation theories and make use of the diagram technique. As a result, for the spectral density of the mean Green's function G we succeed in obtaining the Dyson equation. Solving it and then calculating the function G, we find

$$G(|\mathbf{R} - \mathbf{R}_0|) = -\exp(ikN_{ef}|\mathbf{R} - \mathbf{R}_0|)/4\pi|\mathbf{R} - \mathbf{R}_0| . \tag{10}$$

The quantity N_{ef}, which enters into this relation, may be treated as the effective refractive index of the randomly inhomogeneous moving medium. The value of N_{ef} has been calculated in the Burre approximation. The imaginary part of N_{ef} defines the extinction coefficient γ of the mean field in the randomly inhomogeneous moving medium

$$\gamma = k \operatorname{Im} N_{ef} = \frac{\pi^2 k^2}{2} \int_0^{2k} d\mathbf{k}\mathbf{k} \left[\left(\beta_c + \frac{\mathbf{k}^2\beta_\rho}{2k^2}\right)^2 \frac{\Phi_T(\mathbf{k})}{T_0^2} + \right.$$

$$+ 2\left(\beta_c + \frac{\mathbf{k}^2\beta_\rho}{2k^2}\right)\left(\eta_c + \frac{\mathbf{k}^2\eta_\rho}{2k^2}\right)\frac{F_{CT}(\mathbf{k})}{T_0} +$$

$$\left. + \left(\eta_c^2 + \frac{\mathbf{k}^2\eta_\rho}{2k^2}\right)^2 \Phi_c(\mathbf{k}) + 4\left(1 - \frac{\mathbf{k}^2}{2k^2}\right)^2\left(1 - \frac{\mathbf{k}^2}{4k^2}\right) F(\mathbf{k})/c_0^2 \right] .$$

For $\beta_c = -\beta_\rho = 1$ (propagation of sound through the atmosphere) and for $\eta_c = \eta_\rho = 0$ formula (10) has been also derived by another method in Ref. 13.

Let the size of the largest inhomogineities in the medium $L \gg \lambda$. We also assume that the spectral densities $\Phi_T(\kappa)$, $F(\kappa)$, etc., decrease not too slowly with increase of κ (such spectral densities are, e.g., the Gaussian spectral densities as well as the power–law spectral densities, proportional to $(\kappa^2 + 4\pi^2/L^2)^{-\nu}$ for $\nu > 1$). In this case, the main contribution into γ comes from integrating over the range of small κ, corresponding to the inhomogeneities with $d \gg \lambda$, while the effect of small–scale inhomogeneities with $d \ll \lambda$ is negligible.

For the spectral density of the coherence function $\langle G(\mathbf{R}, \mathbf{R}_0)G^*(\mathbf{R}', \mathbf{R}_0)\rangle$ of the sound field from two single–point sources the Bethe–Salpeter equation has been derived. Starting from it, we derive the equation for the spectral density $b(\kappa, \kappa')$ of the correlation function

$$B(\mathbf{R}, \mathbf{R}') = \left\langle \left[p(\mathbf{R}) - \bar{p}(\mathbf{R})\right]\left[p^*(\mathbf{R}') - \bar{p}^*(\mathbf{R}')\right] \right\rangle =$$

$$= \int\int d^3\mathbf{k}\, d^3\mathbf{k}' \exp\left(i\mathbf{k}\mathbf{R} - i\mathbf{k}'\mathbf{R}'\right) b(\mathbf{k}, \mathbf{k}')$$

of the arbitrary sound field $p(\mathbf{R})$. An equation for the function b is a rigourous consequence of Eq. (9) and has the form

$$\left[a(\kappa) - a^*(\kappa')\right] b(\kappa, \kappa') = \left(1/a^*(\kappa') - 1/a(\kappa')\right) \times$$

$$\times \int\int d^3\kappa_1 d^3\kappa_2 \Lambda(\kappa, \kappa', \kappa_1, \kappa_2) \left[b(\kappa_1, \kappa_2) + \bar{\Pi}(\kappa_1)\bar{\Pi}^*(\kappa_2) \right] . \tag{11}$$

Here $a = k^2 - \kappa^2 - D(\kappa)$. The functions , D, and Λ are the spectral densities of the mean field $\bar{p}(\mathbf{R})$ and of the mass–operator and the intensity–operator kernels, respectively. The functions D and Λ can be represented by infinite series in terms of all strongly coupled diagrams. Retaining only the first terms in these series, i.e.,using the Burre approximation and the ladder approximation, we find

$$D(\mathbf{k}) = \int d^3\mathbf{k}_1 U(\mathbf{k}, \mathbf{k}_1, \mathbf{k}')/\left(k^2 - \mathbf{k}_1^2\right) ; \tag{12}$$

$$\Lambda(\kappa, \kappa', \kappa_1, \kappa_2) = \delta(\kappa - \kappa' - \kappa_1 + \kappa_2)\, W(\kappa, \kappa_1, \kappa') . \tag{13}$$

Here the function W is given by the relation

$$W(\kappa, \kappa_1, \kappa') = \left[k^2\beta_c - \kappa_0\kappa_1\beta_\rho\right]\left[k^2\beta_c - \kappa_0(\kappa' - \kappa_0)\beta_\rho\right] \times$$

$$\times \Phi_T(\mathbf{k}_0)/T_0^2 + 4\left[k^2 + \mathbf{k}_0\mathbf{k}_1\right]\left[k^2 + \mathbf{k}_0(\mathbf{k}' - \mathbf{k}_0)\right] \times$$

$$\times \left[\kappa_1 \kappa' \kappa_0^2 - (\kappa_0 \kappa_1) \right] (\kappa_0 \kappa') F(\kappa_0) / \omega^2 \kappa_0^2 , \qquad (14)$$

where $\kappa_0 = \kappa - \kappa_1$ and for the simplicity we assume that $\Phi_C = \Phi_{CT} = 0$. The value of the function W for $\Phi_C \neq 0$ and $\Phi_{CT} \neq 0$ has been given in Ref. 1. The function $U(\kappa, \kappa_1, \kappa')$ coincides with W, if only $-\kappa'$ and $-F$ are substituted for κ' and F in the right side of Eq. (14). Note that one fails to regourously solve Eq. (11).

Let us introduce the summary $\mathbf{R}_+ = (\mathbf{R} + \mathbf{R}')/2$ and difference $\mathbf{R}_- = \mathbf{R} - \mathbf{R}'$ coordinates of the observation points $\mathbf{R}$ and $\mathbf{R}'$ as well as denote the correlation function by $B_p(\mathbf{R}_+, \mathbf{R}_-) = B(\mathbf{R}_+ + \mathbf{R}_-/2, R_+ - \mathbf{R}_-/2)$. The last function B_p may be represented in the form

$$B_p(\mathbf{R}_+, \mathbf{R}_-) = \oint \mathrm{d}\Omega(\mathbf{n})\, \mathrm{e}^{ik\mathbf{n}\mathbf{R}_-} J(\mathbf{R}_+, \mathbf{n}) ,$$

where $\Omega(\mathbf{n})$ is the solid angle in the direction of the unit vector $\mathbf{n}$, and the function $J(\mathbf{R}_+, \mathbf{n})$ may be interpreted as a ray intensity of the sound field at the point $\mathbf{R}_+$ in the direction of unit vector $\mathbf{n}$. Let us now assume that the variation scale of the function $B_p(\mathbf{R}_+, \mathbf{R}_-)$ along the $\mathbf{R}_+$ coordinate exceeds that along the $\mathbf{R}_-$ coordinate. We also employ Burre approximation (12) and ladder approximation (13). As a result, equation (11) reduces to the radiative transfer equation

$$\left(\mathbf{n} \frac{\partial}{\partial \mathbf{R}_+} + 2\gamma \right) J(\mathbf{R}_+, \mathbf{n}) = \oint \mathrm{d}\Omega(\mathbf{n}_0)\, J(\mathbf{R}_+, \mathbf{n}_0)\, \sigma(\mathbf{n} - \mathbf{n}_0) +$$

$$+ \pi/2 \int \mathrm{d}^3\mathbf{k}_0 W(k\mathbf{n}, \mathbf{k}_0, k\mathbf{n}) \times$$

$$\times \int \mathrm{d}^3K\, \mathrm{e}^{i\mathbf{K}\mathbf{R}_+} \overline{\Pi}(\mathbf{k}_0 + \mathbf{K}/2) \cdot \overline{\Pi}^*(\mathbf{k}_0 - \mathbf{K}/2) , \qquad (15)$$

which describes the radiant intensity J.

In Eq. (15) $\sigma(\mathbf{n} - \mathbf{n}_0) = \frac{\pi}{2} W(k\mathbf{n}, k\mathbf{n}_0, k\mathbf{n})$ is the scattering cross section of the sound wave, given by relation (4). Although one fails to regourously solve the radiative transfer equation (15), there are different well–known approximate and numerical methods for solving it. For example, if the value of the scattering cross section $\sigma(\mathbf{n} - \mathbf{n}_0)$ rapidly decreases as the angle θ between the vectors $\mathbf{n}$ and $\mathbf{n}_0$ increases, then equation (15) can be solved in the small–angle approximation. Note that for Kholmogorov's turbulence spectrum as well as when the inequality $\lambda \ll L$ is valid, the quantity σ rapidly decreases as θ increases, no matter if the sound wavelength is shorter or longer than the internal scale of turbulence.

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