LIMITING POSSIBILITIES OF ADAPTIVE CORRECTION OF WIND REFRACTION BASED ON THE MODAL CONTROL

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The dependence of the criterion of beam focusing on the number of base modes used for wavefront control is numerically analyzed. The maximum degree of compensation for thermal blooming is found to depend on the product of the path length and the nonlinearity parameter. The relative contribution of the third– and fourth–order modes increases for short path lengths.

The problem of increasing the efficiency of adaptive optical systems covers a wide range of both theoretical and experimental problems. Theoretical analysis based on numerical modeling opens the possibility of optimization of a whole class of the parameters of the adaptive systems. It is of primary interest to find the optimal basis of control of the beam wavefront, as well as to seek for the control algorithms with the highest speed and stability. The present paper is dedicated to investigation of such possibilities.

MATHEMATICAL MODEL AND CONTROL ALGORITHMS OF THE ADAPTIVE SYSTEM

To estimate the limiting possibilities of control, we restrict ourselves to the problem of compensation for stationary wind refraction of a Gaussian beam. This problem is described by the system of dimensionless equations

$$2i\,\frac{\partial E}{\partial z} = \Delta_{\perp}E + RTE\ ,$$

$$\frac{\partial T}{\partial x} = EE^{*}\ ,$$

and the field within the transmitting aperture is given in the form

$$E(x,\ y,\ 0) = \exp\left[\,-\,(x^2 + y^2)/2 + iU(x,\ y)\right]\ .$$

The controllable wavefront $U(x, y)$ is represented as a superposition of the lowest–order optical modes

$$U(x,\ y) = \sum_{j=3}^{7} a_j\, z_j\,(x,\ y)\ ,$$

where $z_3 = 2r^2 - 1$ is defocusing, $z_4 = x^2 - y^2$ is astigmatism, $z_5 = (3r^2 - 2)x$ is coma, $z_6 = (x^2 - 3y^2)x$ is coma, and $z_7 = 6(r^4 - r^2) + 1$ is spherical aberration, where $r^2 = x^2 + y^2$. To reduce the volume of computations, distortion is excluded from the control basis, since it can be easy determined from geometric considerations.

The controllable coordinate $a_j\,(j = 3, 4, 5, 6, 7)$ were chosen from the condition of the minimum energy radius of the beam in the image plane $z = z_0$

$$\sigma = \left\{\frac{1}{P_0}\int\!\!\int\left[(x - x_c)^2 + y^2\right]\,|E|^2_{z=z_0}\ \mathrm{d}x\,\mathrm{d}y\right\}^{1/2}\ ,$$

where P_0 is the total power of the beam and

$$x_c = \frac{1}{P_0}\int\!\!\int x\,|E|^2_{z=z_0}\ \mathrm{d}x\,\mathrm{d}y$$

is the desplacement of the beam energy centroid. Phase was optimized by the method of steepest descent,[1] which was found to reduce the number of measurements of the goal function in the process of numerical modeling by a factor close in value to the number of controllable coordinates. Light field was processed at the point of minimum σ in the image plane to retrieve the limiting energy characteristics of the beam $[\sigma,\ x_c,\ J_f]_{\mathrm{opt}}$, in addition, the criterion of focusing was determined with an account of the windward displacement of the beam

$$J_f = \frac{1}{P_0}\int\!\!\int \rho\,(x - x_c,\ y,\ S_t)\ |E|^2_{z=z_0}\ \mathrm{d}x\,\mathrm{d}y\ .$$

Here ρ is the aperture function in the image plane and S_t is its effective radius.

NUMERICAL RESULTS

Estimate of the optimal number of control channels of the adaptive system sufficient to increase the energy parameters of the beam to the prescribed values in the image plane is of great practical interest. To this end, it is expidient first of all to analyze the relative contribution of modes of different order in the quality of correction as functions of such parameters of propagation as the path length and the power of the beam. To have an averaged description of the field and its structure at the object it is desirable to use the focusing criterion J_f calculated for a number of the radii of receiving aperture S_t, which are scaled by the radius of the diffraction limited focal spot in vacuum a_d. The quality of correction for different dimension of the control basis can be conveniently estimated with the use of the normalized value

$$\eta = J_f/(J_f)_{\max}\ ,$$

where $(J_f)_{\max}$ is the maximum criterion of focusing found for the maximum number of base modes (up to the fourth order, inclusively).

The calculated dependences of $\eta(S_t)$ obtained for the path length $z_0 = 0.5$ and the two values of the nonlinearity parameters $R = -20$ and -40 are shown in Figs. 1 and 2, respectively. Pairwize comparison of curves in these figures show that the contribution of high–order modes increases for extended paths and high–power radiation. It can be seen most clearly from Fig. 3 which shows the dependence of the criterion of focusing J_f on the nonlinearity parameter R within a circle of the doubled diffraction radius at $z_0 = 0.5$. For comparison we show here the same dependence for the case of an ideal corrector with infinite degrees of freedom.[2] While practically complete compensation can be obtained for weak nonlinearity ($|R| = -10$) when we control only by the curvature of the wavefront, in the case of strong nonlinearity ($|R| = -40$) it is necessary to control by the modes up to the fourth order, inclusive by, to obtain an acceptable localization of the field. It is interesting to note that the effects of control by the third– and fourth– order modes are practically equivalent under these conditions.

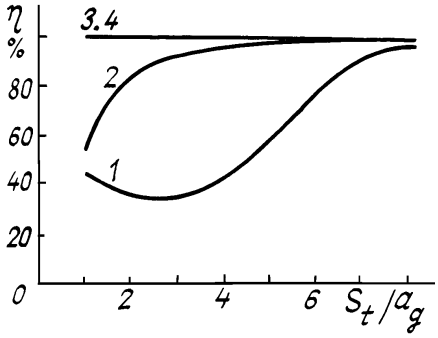

FIG. 1. The normalized criterion of correction quality η as a function of the radius S_t of the receiving aperture for different number of the controllable modes. 1) without correction, 2) second–order modes, and 3, 4) third– and fourth–order modes, respectively. Path length $z_0 = 0.5$ and the nonlinearity parameter $R = -20$.

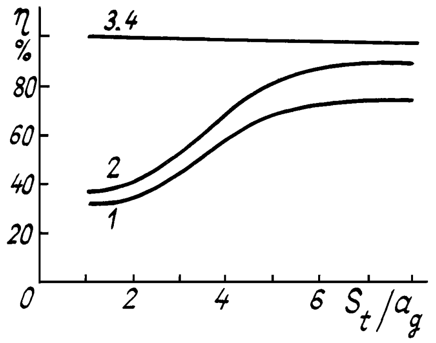

FIG. 2. The normalized criterion of correction quality η as a function of the radius of the receiving aperture S_t for different number of the modes. 1) without correction, 2) second–order modes, and 3, 4) third– and fourth–order modes, respectively. Path length $z_0 = 0.5$ and the nonlinearity parameter $R = -40$.

When strongly focused beam propagates along short paths, the induced thermal lens has a more complicated spatial structure than that in the case considered above. We could expect that the relative contribution of the fourth–order mode will increase in the case of sharp focusing.

Calculated data on compensation for wind refraction along the path $z_0 = 0.2$ for the nonlinearity parameter are shown in Fig. 4. (See notation in Figs. 1 and 2.) It can be seen that the control by the second– and third–order modes in this case is equally inefficient. The main contribution to the field concentration comes from the control by the spherical aberration alone.

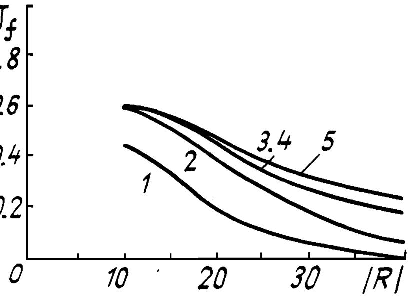

FIG. 3. The criterion of focusing J_f within the circle of radius $S_t = 2a_d$ as a function of. the nonlinearity parameter R. 1) without correction, 2) second– order mode, 3, 4) third– and fourth–order modes, respectively, and 5) ideal corrector. Path length $z_0 = 0.5$.

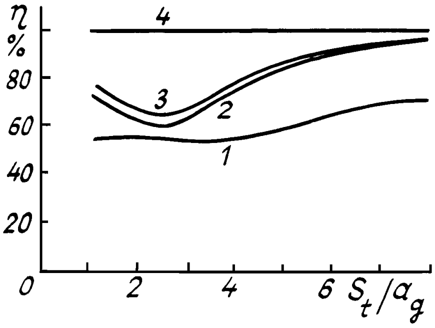

FIG. 4. The normalized criterion of correction quality η as a function of the radius of receiving aperture S_t for different number of the controllable modes. 1) without correction, 2) second–order modes, and 3, 4) third– and fourth–order modes, respectively. Path length $z_0 = 0.2$ and the nonlinearity parameter $R = -100$.

In addition to the estimate of the relative contribution of different modes to localization of the field at the object, it is also of interest to estimate the maximum degree of compensation for thermal blooming by the ideal corrector depending on the path length and on the nonlinearity parameter. Calculations carried out for the wide range of variations of the parameters z_0 and R demonstrate that the main parameter which determines the maximum degree of compensation, is the product $\zeta = |R| z_0$. For example, the relative fraction of light power which can be delivered at the aperture of radius $S_t = 2a_d$ does not exceed 75% for $\zeta = 10$–50 and is about 25% for z = 20, moreover, it remains practically independent of the path length z_0 for $\zeta = 20$. At the same time, the smaller is z_0, the higher order modes of the beam must be controlled to obtain the above–indicated degree of compensation.

Thus the studying of the dependence of the maximum criterion of focusing on the number of base modes demonstrate that in contrast to the case of long paths and of moderate nonlinearity of the medium, when it is sufficient to control by tilt, curvature, and astigmatism of the wavefront, the relative contribution of the third– and fourth–order modes increases for short paths. Moreover, the increase of the dimension of the control basis from 3 to 8 modes modes result in the increase of concentration of the field at the object by 20–30%. The maximum degree of compensation for thermal blooming is determined by a single parameter $\zeta = |R| z_0$.

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2. M.A. Vorontsov and S.S. Chesnokov, Izv. Vyssh. Uchebn. Zaved. SSSR, Radiofiz. 22, No. 11, 1318 (1979).