# ON THE ECHO–SIGNAL POWER RECEIVED FROM THE SURFACE WITH COMPLICATED SCATTERING PHASE FUNCTION IN SOUNDING THROUGH THE ATMOSPHERE

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*The echo signal power in pulsed laser sounding of the surface with complicated scattering phase function through the atmosphere is studied. The expression is derived for the echo–signal power when the surface with scattering phase function comprising diffuse and quasispecular components is sounded through the optically dense aerosol atmosphere. It is shown that the echo–signal shape can depend strongly on the relative contributions of diffusive and quasispecular components.*

Laser sounding of the surface with complicated scattering phase function comprising quasispecular and diffusive components for the case of continuous exposure was considered in Ref. 1.

Below the energy parameters of the echo–signal are studied in pulsed laser sounding of the surface with complicated scattering phase function through the atmosphere.

Let the sounded surface be characterized (for the case of continuous exposure) by the brightness $J_c(\mathbf{R}, \mathbf{m})$ (see Ref. 1)

$$J_c(\mathbf{R}, \mathbf{m}) = \frac{E(\mathbf{R})}{\alpha \dfrac{2\pi}{n+2} + \beta\pi\Delta^2} \times$$

$$\times\left[\alpha \cos^n\theta + \beta \exp\left\{-\frac{(\theta-\theta_0)^2\cos^2\theta_0 + (\varphi-\varphi_0)^2\sin^2\theta_0}{\Delta^2}\right\}\right], \quad (1)$$

where $E(\mathbf{R}) = AE_s(\mathbf{R})$; $E_s(R)$ is the illumination of the surface from the source; $A$ is the reflectance; $\alpha$ and $\beta$ are the coefficients determining the relative contributions of diffusive and quasispecular reflections; $\Delta$ is the parameter characterizing the angular width of the scattering phase function of the quasispecular component of reflection; $n$ is the parameter characterizing the angular width of the scattering phase function of the diffusive component of reflection; $(\theta, \theta_0)$ and $(\varphi, \varphi_0)$ are zenith and azimuth angles of observation and of the reflected radiation maximum (for the quasispecular component of reflection). The angles $\theta_0$ and $\varphi_0$ are related to the corresponding angles $\theta_s$ and $\varphi_s$, which describe the direction of incident radiation by the laws of geometric optics.

Let us assume that the scattering surface is sounded by a pulsed signal through the atmosphere. The brightness $J(\tilde{\mathbf{R}}, \tilde{\mathbf{m}}, t)$ of radiation arriving at the receiver[2] can be determined from the distribution of the brightness $J(\mathbf{R}, \mathbf{m}, t)$ at the scattering surface $S$. Then, using the reciprocity theorem in the scattering medium[2] and the results derived in Ref. 3, we can derive in the small–angle approximation the integral expression for the power recorded by the receiver in pulsed sounding (we assume that shading of surface elements is negligible and $J(\mathbf{R}, \mathbf{m}, t) = J_c(\mathbf{R}, \mathbf{m})\, f\left(t - \dfrac{|\mathbf{r}_s - \mathbf{R}|}{c}\right)$:

$$P(t) = \int_S d\mathbf{R} \int d\Omega(\mathbf{m}) \cos q_r\, J_c(\mathbf{R}, \mathbf{m})\, J_r(\mathbf{R}, \mathbf{m}) \times$$

$$\times f\left(t - \frac{|\mathbf{r}_s - \mathbf{R}| + |\mathbf{r}_r - \mathbf{R}|}{c}\right), \quad (2)$$

where $J_r(\mathbf{R}, \mathbf{m})$ is the brightness of radiation from the "fictitious source" (with the parameters of the receiver) at the point $\mathbf{R}$ of the surface $S$ for the case of continuous exposure[3]; $\theta_r$ is the angle between the normal to the surface $S$ and the direction to the receiver; $\mathbf{r}_s$ and $\mathbf{r}_r$ are the vectors describing the positions of the source and the receiver; $f(t)$ describes the sounding pulse shape.

In the case of the homogeneous scattering atmosphere with strong elongation of the scattering phase function, when the angle, at which the receiving aperture is observed from the points of the scattering surface, is much smaller than the angular width of the scattering phase function of radiation reflected from the surface and the field–of–view angle of the receiver, Eq. (2) for the power recorded by the receiver takes the form (assuming that the sounded surface is flat and coincides with the XOY plane while the source, the receiver, and their optical axes lie in the XOZ plane and using the results obtained in Refs. 3–5)

$$P(t) = \frac{A}{\pi} \frac{1}{\alpha \dfrac{2}{n+2} + \beta\Delta^2}\left[\alpha \cos^n \theta_r \int_S d^2R E_s(\mathbf{R}') E_r(\mathbf{R}'') \times\right.$$

$$\times f\left(t - \frac{L_s + L_r}{c} + \frac{R_x(\sin\theta_s + \sin\theta_r)}{c}\right) + \beta \int_S d^2R E_s(\mathbf{R}') E_r(\mathbf{R}'') \times$$

$$\times \exp\left\{-\frac{1}{\Delta^2}\left[(\sin\theta_0 - \sin\theta_r + R_x d)^2 + R_y^2 s^2\right]\right\} \times$$

$$\left.\times f\left(t - \frac{L_s + L_r}{c} + \frac{R_x(\sin\theta_s + \sin\theta_r)}{c}\right)\right], \quad (3)$$

where

$$s = \frac{A_s}{B_s} + \frac{A_r}{B_r}; \quad d = \frac{A_s \cos^2\theta_s}{B_s} + \frac{A_r \cos^2\theta_r}{B_r};$$

$$A_{sr} = \left[\frac{\alpha_{sr}^2}{4} + \frac{\sigma L_{sr}\langle\gamma^2\rangle}{4}\right]^{1/2};$$

$$B_{sr} = L_{sr}\left[\frac{\alpha_{sr}^2}{2} + \frac{\sigma L_{sr}\langle\gamma^2\rangle}{4}\right]\left[\alpha_{sr}^2 + \sigma L_{sr}\langle\gamma^2\rangle\right]^{-1/2};$$

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$\mathbf{R}' = \{R_x \cos\theta_s, R_y\}$ , $\mathbf{R}'' = \{R_x \cos\theta_r, R_y\}$ ;

where $E_s(\mathbf{R})$ and $E_r(\mathbf{R})$ are the illuminations from the real and "fictitious" (with the parameters of the receiver) sources, respectively;[3,4] $L_s$ and $L_r$ are the distances from the source and the receiver to the surface; $2\alpha_s$ and $2\alpha_r$ are the angular divergence of the source and the field–of–view angle of the receiver; $\sigma$ is the scattering coefficient of the atmosphere; $\langle\gamma^2\rangle$ is the variance of the angle of deflection due to the elementary scattering event.

For $\beta = 0$ and $n = 0$ formula (3) transforms into the expression for the power received from the Lambertian surface.[3] For $\alpha = 0$ as $\Delta \to 0$ formula (3) transforms into the expression for the power received from the specular surface.

Calculating the integrals appearing in Eq. (3), we derive (assuming that the sounding pulse shape is Gaussian, i.e., $f(t) = \dfrac{2}{\sqrt{\pi}} \exp\left(-\dfrac{4t^2}{\tau_p^2}\right)$ and using the results obtained in Ref. 3)

$$P(t) = C_1 \left[ C_2 \exp\left\{-\frac{(t')^2 4}{\tau_p^2} \frac{q}{q+m}\right\} + C_3 \exp\left\{ - b \frac{q+m}{q+m+\dfrac{d^2}{\Delta^2}} - \right.\right.$$

$$\left.\left. - \frac{4\,(t')^2}{\tau_p^2} \frac{q + \dfrac{d^2}{\Delta^2}}{q + \dfrac{d^2}{\Delta^2} + m} + \frac{t'\tilde{b}\, d}{q + m + \dfrac{d^2}{\Delta^2}} \frac{8}{\tau_p^2 c} \right\}\right], \qquad (4)$$

where

$$C_1 = \frac{1}{\alpha \dfrac{2}{n+2} + \beta\Delta^2} \frac{A P_0 \cos\theta_r \cos\theta_r r_r^2 \alpha_r^2}{\sqrt{\pi}\, 8 B_s^2 B_r^2} \times$$

$$\times \exp\left[-(\varepsilon - \sigma)(L_s + L_r)\right] ;$$

$$C_2 = \alpha \cos^n\theta_r\, p^{-1/2} (q+m)^{-1/2} ;$$

$$C_3 = \beta \left(p + \frac{s^2}{\Delta^2}\right)^{-1/2} \left(q + m + \frac{d^2}{\Delta^2}\right)^{-1/2} ;$$

$$d = \frac{1}{4B_s^2} + \frac{1}{4B_r^2} ; \ q = \frac{\cos^2\theta_s}{4B_s^2} + \frac{\cos^2\theta_r}{4B_r^2} ;$$

$$\tilde{b} = \frac{(\sin\theta_0 - \sin\theta_r)(\sin\theta_s + \sin\theta_r)}{\Delta^2} ;$$

$$m = \frac{4(\sin\theta_s + \sin\theta_r)^2}{\tau_p^2 c^2} ; \ b = \frac{(\sin\theta_s + \sin\theta_r)^2}{\Delta^2} ; \ t' = t - \frac{L_s + L_r}{c} ;$$

where $P_0$ is the emitted power, $r_r$ is the effective radius of the receiving aperture, $\varepsilon$ is the extinction coefficient of the atmosphere, and $\tau_p$ is the sounding pulse width.

For $\beta = 0$, $n = 0$, and $\sigma = 0$ formula (4) transforms into the expression for $P(t)$ received from the flat Lambertian surface through the transparent aerosol atmosphere.[3]

Figures 1 and 2 show the results of calculations of the echo–signal shape received from the surface with complicated scattering phase function for different values of the parameter $\beta/\alpha$ (for different relative contribution of diffusive and quasispecular components). The quantity $\dfrac{P(t')}{P(t'=0)}$ was calculated using formula (4) for the following values of the parameters: $\beta/\alpha = 0.1$ (curve *1*), $\beta/\alpha = 0.9$ (curve *2*), $n = 1$, $\theta_s = 70^\circ$, $\theta_r = -65^\circ$, $L_s = 10^4$ m, $L_r = 10^3$ m, $\alpha_s = 10^{-2}$, $\alpha_r = 10^{-1}$, $\tau_p = 10^{-9}$ s, $\Delta = 10^{-2}$, $\theta_0 = -\theta_s$, $\sigma\langle\gamma^2\rangle = 0$ (Fig. 1), $\sigma\langle\gamma^2\rangle = 10^{-4}$ m$^{-2}$ (Fig. 2).

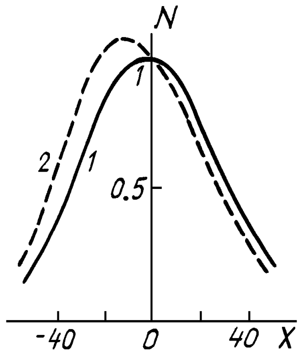


*FIG. 1. The echo–signal shape in the transparent atmosphere.*

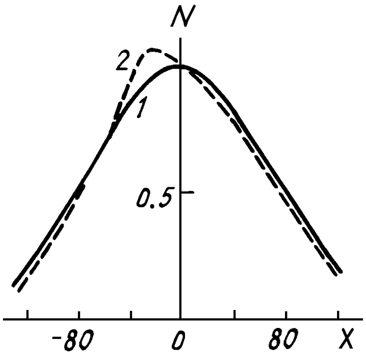


*FIG. 2. The echo–signal shape in the optically dense atmosphere.*

It can be seen from the figures that the shape of the received echo–signal depends strongly on the relative contribution of the quasispecular and diffusive components of the scattering phase function. For optically denser atmosphere this dependence is weak. Physically this is explained by the increase in the effective angular width of the quasispecular component of the scattering phase function of the surface in the atmosphere.

The relations derived in this work can be used with the purposes of developing laser remote sounding systems and analyzing their operation.

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