SENSITIVITY OF THE VALUES OF MEAN RADIATION FLUXES TO THE CHANGE OF THE FORM OF THE CLOUD SIZE DISTRIBUTION FUNCTION

T.B. Zhuravleva

*Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk*
Received December 3, 1991

The effect of the power–law and exponential functions of horizontal cloud size distribution on the formation of the mean radiative regime under conditions of broken clouds has been evaluated. The mean radiation fluxes have been estimated for the models with the random horizontal size of clouds and with clouds of constant (effective) diameter whose value for the statistically uniform models is determined from the condition that the probabilities for the viewing direction to be occluded are equal. It has been shown that proceeding to the statistically nonuniform models in the presence of a small number of large (up to several kilometres) clouds results in the significant transformation of the mean radiative regime.

At present the methods for study of the radiative properties of cloud fields are most developed for the models of clouds with random geometry (broken clouds). In order to construct the models of broken clouds which adequately describe the statistical properties of real cloud and radiation fields, one should use the available experimental data. Despite the complexity of field measurements, the variety of types, geometric shapes, and size of clouds and their spatio–temporal variability, a large amount of information about the physical parameters of different cloud types and stochastic structure of a cloud field has been accumulated now. In particular, the shape of clouds, the cloud size distribution functions, and the probability for narrow viewing direction to be occluded by clouds were studied in ample detail.[1-6]

The cloud size distribution function $f(D)$ (D is the cloud diameter) is one of the basic characteristics of cloud fields, and its form depends on the observation region, season, cloud type, etc. Thus, the data on the horizontal size of cumulus clouds derived from plotting the air photographs taken over the region of Florida peninsula were discussed in Ref. 5. It was shown that for the cloud fields under consideration the probability density for horizontal cloud size decreases exponentially with increase of D.

The structural characteristics of stratocumulus clouds were derived in Ref. 6 from the data of radiometric measurements made with the help of the instruments used onboard LANDSAT. Twelve types of stratocumulus clouds formed by small cloud cells, elongated cloud bank, and sheet clouds were studied there. The horizontal cloud size distribution function in the majority of cases is most successfully described by the power–law function of the diameter.

The purpose of this study is the use of different horizontal cloud size distribution functions $f(D)$ in constructing the models of broken clouds and in evaluating the effect of the form of the function $f(D)$ on the average radiation balance.

1. Cloud model and solution technique. The optical model of broken clouds is specified in the layer Λ: $0 \le z \le H$ in the form of randomly scalar fields of the extinction coefficient $\sigma\kappa(\mathbf{r})$, of the single scattering albedo $\lambda\kappa(\mathbf{r})$, and of the scattering phase function $g(\boldsymbol{\omega}, \boldsymbol{\omega}')\kappa(\mathbf{r})$, where $\boldsymbol{\omega}$ is the unit direction vector. Here $\kappa(\mathbf{r})$ is the indicator function of the random set of points in the layer Λ in which the cloud material occurs. The mathematical model of the field $\kappa(\mathbf{r})$ is constructed with the help of the Poisson point flux in space.[7,8] It determines the realization of the cloud field as an ensemble of clouds of a fixed configuration randomly distributed in space with their centers lying in one plane. The cloud shape, thickness, probability density of horizontal cloud size $f(D)$, and cloud amount H are the input parameters of the model in addition to σ, λ, and $g(\boldsymbol{\omega}, \boldsymbol{\omega}')$.

The data on the joint distribution function of cloud diameters and heights are limited in number in the literature known to the author, and for this reason we used the relation[6]

$$H/D = \nu(D/D_{\max})^{\beta} ,$$

when simulating the cloud field, where the average values $\nu = 0.955$ and $\beta = 0.031$ were used for ν and β. As a first approximation, one may set $H \sim D$.

If all of the clouds are of the same shape and their projections on the horizontal plane are the discs of diameter D, then the cloud amount N and the two–dimensional Poisson parameter μ (i.e., the mean number of cloud centers per unit area), used when simulating the cloud field, are related by the formula[9]

$$\mu = -\frac{4\ln(1-N)}{\pi\,\overline{D^2}} , \tag{1}$$

where

$$\overline{D^2} = \int\limits_{D_{\min}}^{D_{\max}} D^2 f(D)\mathrm{d}D .$$

We will approximate the clouds by right circular cylinders of the same thickness. The unitary flux of solar radiation is incident on the upper boundary of the layer, $\xi_{\odot}$ and $\varphi_{\odot} = 0$ are the zenith and azimuthal angles of the Sun.

In the calculation of statistical characteristics of solar radiation, the Monte Carlo algorithm was implemented which was developed earlier (see, for example, Refs. 7 and 8). The numerical simulation of the sampling random realizations of the cloud field with the help of a computer, the solution of the radiative transfer equation by the Monte Carlo method

0235-6880/92/03 150-05 $02.00

for the constructed realization, and the subsequent statistical processing are the main points of the algorithm.

Model 1A. In accordance with Ref. 6, the horizontal cloud size distribution function $f(D)$ is represented in the form

$$f_{\mathrm{p}}(D) = \begin{cases} a_1 D^{-\alpha_1}, & D \le D_0, \\ a_2 D^{-\alpha_1}, & D \ge D_0, \end{cases} \quad (2)$$

where $D_{\min} \le D \le D_{\max}$, $D_{\min} = 0.03$ km, $D_{\max} = 5$ km, and $D_0 \simeq 0.5$–0.7 km. The parameters α_i for $i = 1, 2$ range within the limits $1.55 \le \alpha_1 \le 1.86$ and $2.44 \le \alpha_2 \le 2.9$ depending on the cloud type considered in Ref. 6. In the subsequent calculations we will set $D_0 = 0.7$ km, $\alpha_1 = 1.55$, and $\alpha_2 = 2.9$ that correspond to the field of stratocumulus clouds consisting of small cumulus clouds, of large cloud conglomerations, and of clouds of intermediate size. The coefficients a_i for $i = 1, 2$ determined from the conditions of normalization and of continuity of the function $f_{\mathrm{p}}(D)$ at the point $D = D_0$ are $a_1 = 0.092$ and $a_2 = 0.057$. For these values of α_i for $i = 1, 2$ the mean diameter is $\overline{D}_{\mathrm{p}} = \int\limits_{D_{min}}^{D_{max}} D\, f(D)\, \mathrm{d}D = 0.19$ km, $\overline{D^2_{\mathrm{p}}} = 0.112$ km^2.

Model 1B. We consider the cloud model with exponential function of cloud diameter distribution

$$f_{\exp}(D) = a\mathrm{e}^{-bD} . \quad (3)$$

We determine the values of the coefficients a and b in Eq. (3) from the normalization condition for the function $f_{\exp}(D)$ and from the equality $\overline{D^2_{\mathrm{p}}} = \overline{D^2_{\exp}}$ that in accordance with Eq. (1) provides identical values of the Poisson parameter μ for one and the same cloud amount N: $a = 5.16$ and $b = 4.5$ km^{-1}. For these values of a and b and for cloud diameter ranging within the limits $0.03 \le D \le 5$ km we obtain $\overline{D}_{\exp} = 0.25$ km.

When implementing the algorithm for calculation of statistical characteristics of solar radiation, the cloud field $\kappa(\mathbf{r})$ is simulated in a certain sufficiently large (but bounded) region G. Its horizontal size depends strongly on the value of maximum diameter, namely, the larger $D_{\max}$, the greater is the horizontal extension of the region G and therefore, the more laborious is the construction of a cloud realization. The cloud fields described by formulas (2) and (3) consist mainly of the clouds of small size, that is, the probability P of occurence of clouds with $D \lesssim 0.25$ km is about 0.8 while P ($D \gtrsim 0.25$ km) is about 0.004. Therefore in the subsequent calculations it is expedient to set $D_{\max} = 2.5$ km; in so doing, the coefficients in formulas (2) and (3) vary insignificantly. We note that with such a limitation on $D_{\max}$, the procedure of construction of the cloud realization is still remaining too complicated problem because the region G must be represented as a sum of nonoverlapping subregions of smaller area, and one and the same probability density $\mu = -4\ln(1 - N)/(\pi\, \overline{D^2}\,)$ must be used for simulating the number of cloud centers and their horizontal coordinates and diameters in each subregion.

It is clear that in the case in which the cloud diameter is a random variable, the complexity of the algorithm increases since additional averaging over this random variable must be carried out. Therefore, the question arises pertaining to the choice of such a constant diameter D for which the mean radiation fluxes for model 1 with the randomly horizontal cloud size and for the model with the constant cloud diameter (further it is referred to as model 2) become close in value with the rest of the optical–geometric parameters and the illumination conditions being identical. If we find such a diameter then a simpler model of cloud field consisting of the clouds of the constant diameter may be used for calculation of the mean fluxes.

As is well known (see, for instance, Ref. 3) the mean flux of unscattered radiation $\overline{S}$ is determined to a considerable degree by the probability N_θ for the viewing direction to be occluded, where $\theta = \xi_\odot$. In accordance with Ref. 2, for model 1 we have

$$N^{(1)}_\theta = 1 - \exp[-\mu(\pi\, \overline{D^2}\ + \overline{D}\, H \tan\theta)] , \quad (4)$$

while for model 2

$$N^{(2)}_\theta = 1 - \exp[-\mu_1(\pi D^2 + D\, H \tan\theta)] , \quad (4')$$

where $\mu_1 = -4\ln(1 - N)/(\pi D^2)$.

Let us choose such a value of the diameter D for model 2 for which the equality $N^{(1)}_\theta = N^{(2)}_\theta$ holds (in what follows it is referred to as effective diameter)

$$D^{\mathrm{eff}} = \overline{D^2/D} . \quad (5)$$

In accordance with Eq. (6), the $D^{\mathrm{eff}}_{\mathrm{p}} = 0.6$ km and $D^{\mathrm{eff}}_{\exp} = 0.45$ km.

In the case in which the probabilities for the viewing direction to be occluded coincide, i.e., $N^{(1)}_\theta = N^{(2)}_\theta$, one may expect that at large optical thickness, the mean fluxes of unscattered and, possibly, of scattered radiation become close in value. To check this assumption, we consider the calculated results.

2. Calculated results. We will neglect the molecular and aerosol scattering within the layer Λ and will set the albedo of the underlying surface equal to zero. The scattering phase function is taken for the cloud model C_1 (see Ref. 10) at a wavelength of 0.69 μm.

We denote by $\overline{S}$, $\overline{Q}_s$, and $\overline{A}$ the unscattered, scattered transmitted, and reflected radiation, respectively, for model 2, while by $\overline{S}_l$, $\overline{Q}_{sl}$, and $\overline{A}_l$ the corresponding fluxes for model 1, where l = p or l = exp depending on the form of the function $f(D)$.

The calculations were performed for the optical–geometric parameters of clouds ranging within the limits $0.1 \le N \le 0.7$, $0 \le \xi_\odot \le 60°$, and 10 km$^{-1} \le \sigma \le 60$ km^{-1} and for $H = 0.5$ km. The relative calculational error Δ did not exceed 5%. First we will discuss the effect of the form of the function $f(D)$ (models 1A and 1B) on the mean fluxes of solar radiation and then we will compare the calculated results obtained for models 1 and 2.

We will start from the analysis of formula (4) in more detail. In accordance with Eq. (1)

$$1 - N_\theta = \exp\left(\ln(1 - N)\left(1 + \frac{4\overline{D}H\tan\theta}{\pi\,\overline{D^2}}\right)\right).$$

Since in a first approximation the assumption can be made that $\overline{S} \simeq 1 - N_\theta$ (see, for instance, Ref. 3), then

$$g_s = \frac{\overline{S}_{\rm exp}}{\overline{S}_{\rm p}} \approx \exp\big(\ln(1 - N)\tan\theta\,\gamma\big)\,, \qquad (6)$$

where

$$\gamma = 4H(\overline{D}_{\rm exp} - \overline{D}_{\rm p}))/(\pi\,\overline{D^2}\,) > 0\,.$$

Due to the negative exponent in Eq. (6), the inequality $\overline{S}_{\rm exp} \le \overline{S}_{\rm p}$ is seemed to be valid. For larger cloud amounts, the value $|\ln(1 - N)|$ increases; therefore, the increase in the difference between $\overline{S}_{\rm p}$ and $\overline{S}_{\rm exp}$ is possible. As follows from Eq. (7), for fixed cloud amount N at larger solar zenith angle $\xi_\odot$ the decrease of g_s is possible because $\tan\theta$ is an increasing function.

The above–formulated assumptions are confirmed by the results of calculation of the mean fluxes shown in Fig. 1. As N increases from 0.1 to 0.7 at $\xi_\odot = 30°$ (Fig. 1*a*), g_s decreases from 1 to 0.7 while the ratio $\overline{Q}_{s,\rm exp}/\overline{Q}_{s,\rm p}$ increases by a factor of 1.1. With growth of the solar zenith angle from 0 to 60° for $N = 0.5$ (Fig. 1*b*), g_s decreases from 1 to ~0.6 while $\overline{Q}_{s,\rm exp}/\overline{Q}_{s,\rm p} \approx 1.1$ at the angles under consideration. The mean albedo depends weakly on the type of the function of the horizontal cloud size distribution attendant to changes in the cloud amount and in the solar zenith angle $0.1 \le N \le 0.7$ and $0 \le \xi_\odot \le 60°$, respectively.

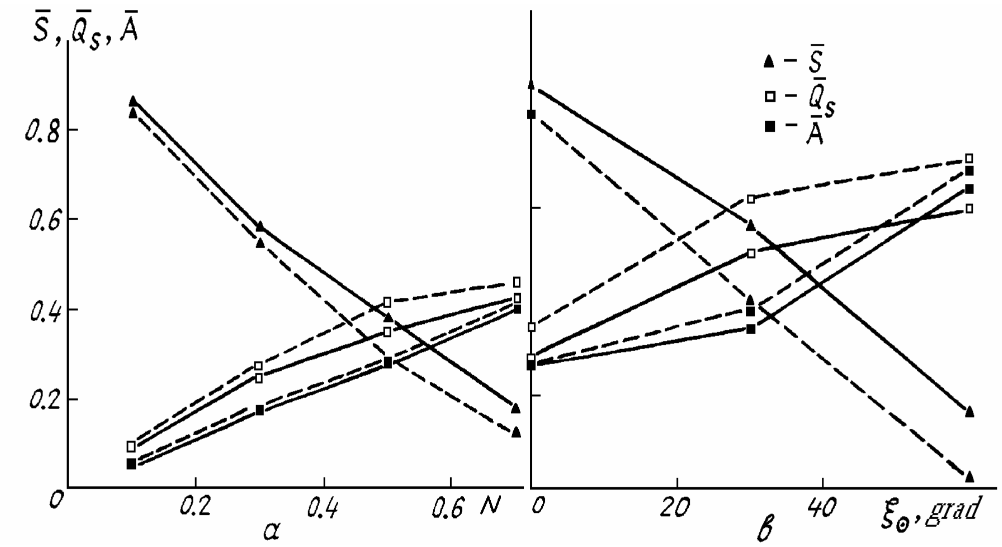

FIG. 1. The effect of the form of function f(D) of horizontal cloud size distribution on the mean radiation fluxes $\overline{S}$, $\overline{Q}_s$, and $\overline{A}$ for different cloud amounts ($\xi_\odot = 30°$) (a) at different solar zenith angles ($N = 0.5$) (b) for $H = 0.5$ km. Solid curves refer to $f(D) = f_{\rm p}(D)$ and dashed curves refer to $f(D) = f_{\rm exp}(D)$.

For intermediate cloud amount ($N = 0.5$) at a large solar zenith angle ($\xi_\odot = 60°$), a variation in the extinction coefficient within the limits $10\ {\rm km}^{-1} \le \sigma \le 60\ {\rm km}^{-1}$ results in the insignificant variations in g_s ($g_s \simeq 0.6$) and $\overline{Q}_{s,\rm exp}/\overline{Q}_{s,\rm p}$ decreases from 1.1 to 1 while $\overline{A}_{\rm exp}/\overline{A}_{\rm p} \lesssim 1.1$.

For comparison of the mean fluxes for models 1 and 2, we will use as a criterion for proximity

$$\delta F = \frac{\overline{F}_l - \overline{F}}{\overline{F}_l} \times 100\%\,,$$

where $F = S, Q_s, A$.

Let us study the dependence of δS, δQ_s, and δA on the cloud amount for model 1A with $f(D) = f_{\rm p}(D)$ and for model 2 with $D = D^{\rm eff}$ given by formula (5). The values of $|\delta S_{\rm p}|$ for small cloud amounts $N \lesssim 0.3$ are within the limits of calculational error, while for intermediate cloud amounts $N \simeq 0.5$ they are about 10% and in addition, $\overline{S} < S_{\rm p}$. The latter is possibly due to the fact that the volume of cloud material for model 2 is larger than for model 1A, i.e., $((D^{\rm eff})^2/\overline{D^2} \simeq 3)$, and the inequality $1 - \overline{S} > 1 - \overline{S}_{\rm p}$ holds for the scattered fraction of energy. The further increase in the cloud amount value up to $N \simeq 0.7$ results in the fact that the radiative interaction of clouds starts to play an important role. A great number of small clouds for model 1A results in the increase in the fraction of scattered solar radiation compared to model 2, and for this reason the unequality $\overline{S}_{\rm p} < \overline{S}$ becomes valid. For $N \le 0.7$, $|\delta Q_{s,\rm p}| \le \Delta$ while $|\delta A_{\rm p}|$ decreases from ~20% for $N = 0.1$ to 6–7% for $N \gtrsim 0.5$.

For a fixed cloud amount as the solar zenith angle increases, the conditions of cloud illumination change and therefore the fraction of scattered radiation increases due to the illumination of cloud sides. The quantity $(\overline{S}_{\rm p} - \overline{S})$

varies insignificantly at $0 \le \xi_{\odot} \le 60°$, but due to decrease of $\bar{S}_{\rm p}$ the value of $|\delta S_{\rm p}|$ increases up to 15% (Fig. 3). At the above–indicated angles, $\delta A_{\rm p}$ and $\delta Q_{s\rm p}$ are negative and vary in the following way (for the cloud parameters indicated in caption to Fig. 3): $|\delta A_{\rm p}|$ increases from 5 to 10%, and $|Q_{s,\rm p}|$ decreases from 8 to 4%.

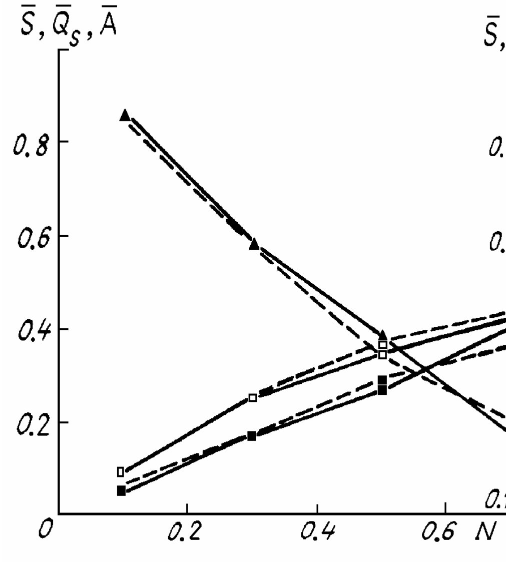

FIG. 2. The dependence of mean radiation fluxes $\bar{S}$, $\bar{Q}_s$, and $\bar{A}$ on the cloud amount for $\sigma = 30$ km^{-1} and $H = 0.5$ km at $\xi_{\odot} = 30°$. Here and in other figures the solid curves refer to $f(D) = f_{\rm p}(D)$, and dashed curves refer to $D^{\rm eff} = 0.6$ km.

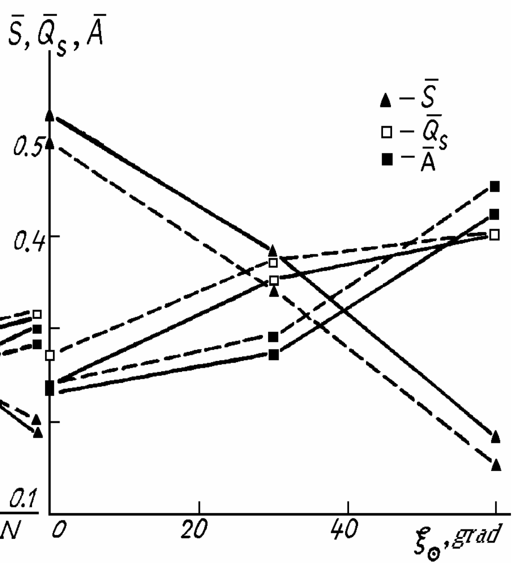

FIG. 3. The mean radiation fluxes $\bar{S}$, $\bar{Q}_s$, and $\bar{A}$ vs the solar zenith angle $\xi_{\odot}$ for $\sigma = 30$ km^{-1}, $N = 0.5$, and $H = 0.5$ km.

As is well known,[11] the derivatives of the mean radiative fluxes with respect to the extinction coefficient are by 2–3 orders of magnitude less than the derivatives with respect to N and D depending on the parameters of the problem. For this reason we may expect that the mean radiative regime of cumulus clouds for both models changes not very significantly when σ varies within the limits $10 \le \sigma \le 60$ km^{-1}. It is obvious from the results of calculations that for $N = 0.5$ at $\xi_{\odot} = 60°$ the value of δS_p remains unchanged ($\delta S_p \sim 15\%$), $|\delta Q_{s,\rm p}| < \Delta$, while the value of δA_p is negative, and $|\delta A_p|$ decreases from 12 to 4%.

Mean radiation fluxes for the model with the exponential function of horizontal cloud size distribution and for model 2 with $D^{\rm eff} = 0.45$ km differ insignificantly, namely, $|\delta S_{\rm exp}|$, $|\delta Q_{s,\rm exp}|$, and $|\delta A_{\rm exp}|$ are less than Δ in the considered range of variation of the cloud parameters. The relation between the content of cloud material, for the above–indicated models, being equal to $(D^{\rm eff})^2/\overline{D^2}$, on the average, is about 1.8. It is less than in the case in which $f(D) = f_{\rm p}(D)$ and is possibly one of the reasons for better agreement between the mean radiation fluxes.

3. The above–described results have been obtained for statistically uniform models of cloud field. Let us consider a statistically nonuniform cloud model in which the horizontal cloud size is described by functions (2) and (3) while the cloud height is equal to the diameter of the cloud base (model 3). Thus the cloud field consists of cylinders whose vertical and horizontal size ranging from several tens of meters ($D_{\rm min} = 0.03$ km) to several kilometers ($D_{\rm max} = 2.5$ km). The probability N_θ for the viewing direction to be occluded is the function of the observation height z.

The purpose of the calculations presented below is to find such an analogue of the statistically uniform model with clouds of constant diameter D for statistically nonuniform cloud model with random horizontal cloud size that their mean radiative regimes differ insignificantly.

Let us consider the cloud field model consisting of the cylinders of constant diameter D whose thickness $H = D$. Let us choose the value of D from the condition of equality of the Poisson parameters for the fixed cloud amount N: $D = \sqrt{\overline{D^2}}$.

The values of mean fluxes are presented in Table I.

TABLE I. $\sigma = 30$ km^{-1}, $N = 0.5$, *and* $\xi_{\odot} = 0°$.

	$f_{ct}(D)$	$f_{\rm exp}(D)$	$D = \sqrt{\overline{D^2}} = 0.33$ km
$\bar{S}$	0.526	0.505	0.501
$\bar{Q}_s$	0.185	0.239	0.318
$\bar{A}$	0.289	0.256	0.181
$\overline{D^3}$	0.1226	0.075	0.037

In the case under consideration $\xi_{\odot} = 0°$, the cloud amount N equals to the probability N_θ for the viewing direction to be occluded, therefore the value of $\bar{S}$ is practically independent of the type of the function of horizontal cloud size distribution. Substantial differences between the mean fluxes of scattered radiation are caused to a considerable degree by different content of a cloud material within the layer Λ. Thus, for $f(D) = f_p(D)$ the relation $\overline{D^3}/D^3 \simeq 3$ holds primarily due to the presence of even small number of clouds of large diameter. These are the clouds with large optical thickness that determine the ratio of transmitted to reflected radiation, in particular, for model 3 with power–low function of cloud diameter distribution: $\bar{Q}_s/\bar{A} \simeq 0.6$, while for model 2 this ratio is about 1.7. For the above–indicated cloud parameters the mean albedo for model 3 is greater by about 30% than for model 2 while $|\delta Q_{s,\mathrm{p}}|$ reaches 60%.

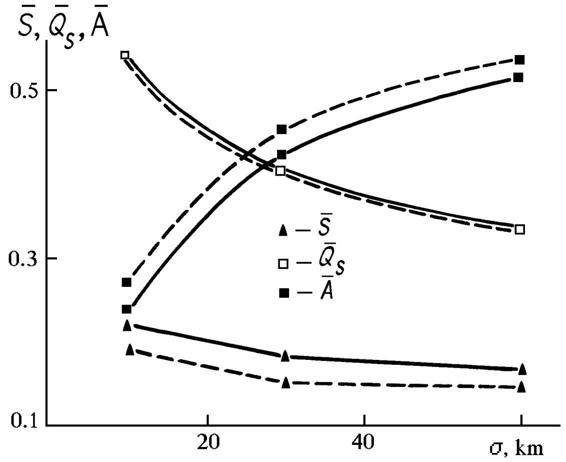

FIG. 4. The effect of the extinction coefficient σ on the mean fluxes of solar radiation for $N = 0.5$ and $H = 0.5$ km at $\xi_\odot = 60°$.

For the model with $f(D) = f_{\exp}(D)$, the ratio $\overline{D^3}/D^3 \simeq 2$ and the differences $|\delta Q_{s,\exp}|$ and $|\delta A_{\exp}|$ are about 30%.

Analysis of the results indicates that the error in estimating the fraction of transmitted radiation $\bar{S}_p + \bar{Q}_{s,p}$ for the statistically uniform model consisting of clouds of constant diameter is no greater than Δ while $|\delta S_p|$ can reach ~ 15° at $\xi_\odot > 30°$. In the estimation of the mean albedo we must take into account that for small cloud amounts $N \lesssim 0.3$ at $\tau \simeq 15$ and at $\tau \simeq 5$ even for intermediate N, $|\delta A_p|$ is about 10–20%. The error in determining $\bar{A}_p$ decreases for $N \sim 0.5$ with increase of τ: for $\tau = 15$ at $\xi_\odot < 30°$ $|\delta A_p| \le \Delta$ while at $\xi_\odot \ge 30°$ $|\delta A_p|$ becomes comparable to the relative calculational error and is about 6–7%.

Comparison of the mean radiative regimes for the statistically nonuniform cloud model with random horizontal cloud size with that for the statistically uniform model with D = const shows that mean fluxes of scattered solar radiation differ substantially. Even small number of clouds of large diameter (and therefore, of large optical thickness) results in the discrepancy in the values of $\bar{A}$ and Q_s, which may be as large as 60%.

The author would like to acknowledge Dr. G.A. Titov for the formulation of the problem and for the discussions of the results and E.V. Trusova for assistance in performing the calculations.

REFERENCES

1. Yu.–A.R. Mullamaa, ed., *Stochastic Structure of Cloud and Radiation Fields* (Tartu, 1972), 281 pp.
2. E.M. Feigel'son and L.R. Tsvang, eds., *Heat Transfer in the Atmosphere* (Nauka, Moscow, 1972), 146 pp.
3. Yu.K. Ross, ed., *Clouds and Radiation* (Tartu, 1975), 251 pp.
4. Yu.K. Ross, et al., ed., *Variability of Cloudiness and of Radiation Fields* (Tartu, 1978), 142 pp.
5. V.G. Plank, J. Appl. Meteor. **8**, No. 1, 46–67 (1967).
6. R.M. Welch, K.S. Kuo, B.A. Wielcki, et al., J. Appl. Meteor. **27**, 341–361 (1988).
7. G.A. Titov, Izv. Akad. Nauk SSSR, Ser. FAO **15**, No. 6, 633–638 (1979).
8. O.A. Avaste, G.M. Vainikko, et al., *in: Monte Carlo Methods in Computing Mathematics and Mathematical Physics* (Computing Center of the Siberian Branch of the Academy of Sciences of the USSR, Novosibirsk, 1974), pp. 232–239.
9. G.N. Glazov and G.A. Titov, Izv. Vyssh. Uchebn. Zaved. SSSR, Ser. Fizika, No. 9, 151–154 (1975).
10. D. Deirmedjian, *Electromagnetic Scattering on Spherical Polydispersions,* (American Elsevier, New York, 1969).
11. G.A. Titov, Izv. Akad. Nauk SSSR, Ser. FAO **23**, No. 8, 851–858 (1987).