PROBABILITY DENSITY OF FLUCTUATIONS OF THE DIFFERENCE IN THE OPTICAL BEAM INTENSITIES IN THE TURBULENT ATMOSPHERE

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On the assumption that optical wave intensities follow the two–dimensional lognormal distribution in the turbulent atmosphere, the probability density of the difference in the intensity fluctuations is considered. It is shown that the results obtained employing the model agree fairly well with the experimental data for weak fluctuations of the intensity.

When measuring the angular coordinates of the objects with the use of the opto–electronic single–pulse technique[1] in the turbulent atmosphere, the question of the determining the measurement errors arises. The atmosphere–induced component of measurement errors was studied in the equisignal direction[2,3] for the direction–finding characteristic u_1 of the form

$$u_1 = \frac{I_1 - I_2}{\langle I_1\rangle + \langle I_2\rangle} = c(I_2 - I_1)\ , \quad I_1,\ I_2 \ge 0\ ,$$

where I_1 and I_2 are the instantaneous values of the signals and $\langle I_1\rangle$ and $\langle I_2\rangle$ are their average values.

The probability density of fluctuations of the direction–finding characteristic $P(u_1)$ is the generalized characteristic of the error. The sought–after density can be obtained if the two–dimensional probability density of intensity fluctuations $P(I_1, I_2)$ is known. Starting from the available theoretical and experimental[4] results, let us prescribe the probability density of intensity fluctuations $P(I_1, I_2)$ in the form of the two–dimensional lognormal distribution:

$$P(I_1, I_2) = \frac{I_1^{-1}\cdot I_2^{-1}}{\sqrt{1-R^2}\,2\pi\sigma_1\sigma_2}\exp\left\{-\frac{1}{2(1-R^2)}\left[\frac{(\ln I_1 - \mu_1)^2}{\sigma_1^2} - 2R\frac{\ln I_1 - \mu_1}{\sigma_1}\frac{\ln I_2 - \mu_2}{\sigma_2} + \frac{(\ln I_2 - \mu_2)^2}{\sigma_2^2}\right]\right\},\quad \mu_{1,2} = \langle \ln I_{1,2}\rangle, \quad (1)$$

$$\sigma_{1,2}^2 = \langle \ln^2 I_{1,2}\rangle - \mu_{1,2}^2\ ,\quad R = \langle(\ln I_1 - \mu_1)(\ln I_2 - \mu_2)\rangle/\sigma_1\sigma_2\ ,$$

where $\mu_{1,2}$, $\sigma_{1,2}$, and R are the parameters of the corresponding two–dimensional normal distribution.

Based on Eq. (1), let us represent the probability density of the intensity difference $u = I_1 - I_2$ in the form

$$P(u) = \frac{1}{\sqrt{1-R^2}\,2\pi\sigma_1\sigma_2}\int\limits_{-u}^{\infty}\frac{dI_2}{(I_2+u)I_2}\exp\left\{-\frac{1}{2(1-R^2)}\times\left[\frac{(\ln(I_2+u) - \mu_1)^2}{\sigma_1^2} - 2R\frac{(\ln I_2 + u) - \mu_1}{\sigma_1}\frac{(\ln I_2 - \mu_2)^2}{\sigma_2} + \frac{(\ln I_2 - \mu_2)^2}{\sigma_2^2}\right]\right\}\ ,\quad I_2 + u \ge 0\ . \quad (2)$$

It can be shown that in the equisignal direction, when $\mu_1 = \mu_2$ and $\sigma_1 = \sigma_2$, $p(u)$ becomes an even function.

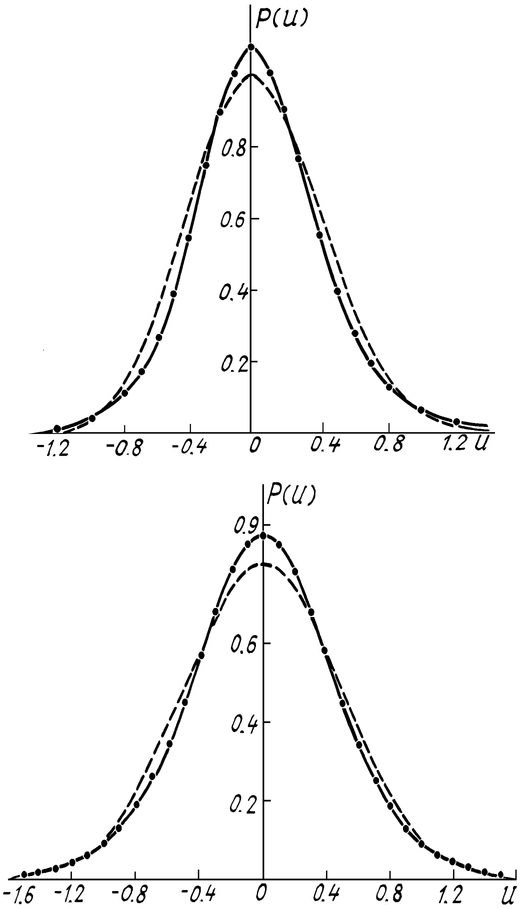

FIG. 1.

To compare derived distribution (2) with experimental data, we used the results of measurements of the statistical characteristics of the intensity fluctuations I_1 and I_2 of the two beams forming the equisignal direction in the turbulent atmosphere obtained in Ref. 5. The relative variances of the intensity fluctuations

$$\beta_{1,2}^2 = \frac{\langle I_{1,2}^2\rangle - \langle I_{1,2}\rangle^2}{\langle I_{1,2}\rangle^2} = \frac{\sigma_{1,2}^2}{\langle I_{1,2}\rangle^2}\ ,$$

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and the cross correlation coefficient

$$r = [\langle I_1, I_2\rangle - \langle I_1\rangle\langle I_2\rangle]/\sigma_1\sigma_2$$

were measured, and the histogram was obtained in this experiment.

Figures 1*a* and *b* show the results of comparison of the experimental data denoted by points with derived distribution (2) indicated by solid curve for the narrow collimated beam with positive and significant negative correlation of the intensity fluctuations.[5]

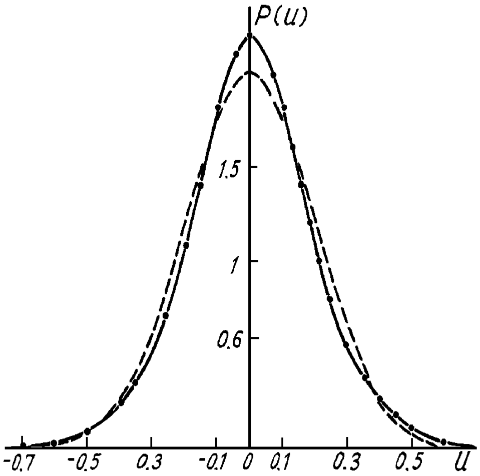

FIG. 2.

By comparing the results, we assumed that the relations peculiar to the two–dimensional lognormal distribution exist between the measured characteristics $\beta_{1,2}$ and r and the parameters of the two–dimensional normal distribution $\sigma_{1,2}$ and R in the equisignal direction

$$\beta_{1,2}^2 = \exp(\sigma_{1,2}^2) - 1\ ,\quad r = [\exp(R\sigma_{1,2}^2) - 1]/[\exp(\sigma_{1,2}^2) - 1].$$

For the directional–spherical waves forming the equisignal direction in the turbulent atmosphere, the large values of the cross correlation coefficient shown in Fig. 2 are typical. As can be seen from these results, the agreement between model (2) and experimental data is observed in both cases. It should be noted also that distribution (2) has much larger modal value than the normal distribution indicated in these figures by dashed lines for the same variances σ_n^2.

Thus, the experimental data on the probability density of intensity difference are adequately described by the model of the two–dimensional logarithmic intensity distribution for weak fluctuations.

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