DYNAMIC CORRECTION OF NONSTATIONARY WIND–INDUCED REFRACTION BASED ON THE SIMPLEX METHOD

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Possibility of using the simplex method to compensate for thermal blooming of a long light pulse propagating through a regular medium is studied by the method of numerical modeling. The algorithm increasing the efficiency of the beam phase control under nonstationary conditions is proposed. A simplex size is optimized as a function of a nonlinearity parameter, beam control duration, and speed of response of the adaptive system.

The problem of searching after new algorithms for control of a phase of light beams propagating through the natural media is of great importance in connection with the development of atmospheric optics systems. One of the most widely employed principles for accomplishing the control in the optical systems with feedback is currently the method of aperture sounding which provides a reliable search after the extremum of the figure of merit under stationary condition without noise. At the same time, it appears that the gradient procedures of "ascent on a hump," which originally provided the basis for the aperture sounding, are ineffective under conditions of fluctuations in the parameters of the beam and medium, of the transient processes in the "beam–medium" system, etc.

It is therefore of interest to develop the algorithms for control of the light beam phase based on the methods which do not require the calculation of the gradient of the goal function, in particular, this being the simplex search. The preliminary studies[1] showed its efficiency in compensating for the stationary wind–induced refraction. It was found that for attaining maximum in the illumination of the object by the focused radiation the simplex method requires the number of measurements of the control figure of merit 1.5 times smaller than the gradient method does. This paper is aimed at the development of the simplex method for dynamic beam phase control under conditions of nonstationary wind–induced refraction when the position of the maximum in the figure of merit in the space of the control coordinates strongly depends on the trajectory of its search.

MODEL OF THE LIGHT BEAM PROPAGATION THROUGH THE NONLINEAR MEDIUM

The beam propagation through a moving weakly absorbing medium is described by the system of dimensionless equations

$$2i\frac{\partial E}{\partial z} = \Delta_\perp E + RTE,\ \Delta_\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\ , \qquad (1)$$

$$\frac{\partial T}{\partial t} + \frac{\partial T}{\partial x} = EE^*\ . \qquad (2)$$

Here the transverse coordinates x and y are normalized to the initial beam radius α_0, the longitudinal coordinate z is normalized to the diffraction length $z_d = \kappa a_0^2$ (k is the wave number), the time t is normalized to the convective time $\tau_v = \alpha_0/V$, where V is the velocity of the moving medium along the x axis. The nonlinearity parameter R is proportional to the input beam power P_0 and to the time of radiation interaction with the medium.

At the transmitting aperture ($z = 0$) an input field is formed

$$E(x, y, 0, t) = A_0(x, y) \exp (iU(x, y, t)) \qquad (3)$$

with the controllable phase profile of the beam being chosen from the condition of the focusing criterion maximum in the observation plane

$$J_f = \frac{1}{P_0}\int\int \rho(x, y)\ |E(x, y, z_0, t)|^2\ \mathrm{d}x\mathrm{d}y \qquad (4)$$

which is the relative fraction of the light power falling within the given aperture $\rho(x, y)$.

Since the diminishment of the radiation power at the object is primarily related to non–axisymmetric defocusing of the beam and its deviation from rectilinear propagation, the incidence angle θ and two wave front curvatures S_x and S_y determining the beam focusing in perpendicular planes are employed as control coordinates. In accordance with this we have

$$U(x, y, t) = \theta(t)\, x + 0.5\, (S_x(t)\, x^2 + S_y(t)\, y^2)\ . \qquad (5)$$

The numerical modeling was accomplished along the path $z_0 = 0.5$, the amplitude profile upon entering the medium and the aperture function of the object were taken to be Gaussian

$$A_0(x, y) = \exp (- (x^2 + y^2)/2)\ , \qquad (6)$$

$$\rho(x, y) = \exp (- (x^2 + y^2))\ . \qquad (7)$$

ALGORITHM FOR SIMPLEX SEARCH IN THE CASE OF TARGET DRIFT

The beam displacement in the windward direction in the process of evolution of thermal distortion in the space of control engenders the effect which is called a target drift in the theory of optimization.[2] Under these conditions the control can be realized via two processes: ascent on a moving hump and control of this movement. High velocity of the target drift may result in unstable regimes of searching, complicated choice of the step direction, and alternation of searching and ascending. To overcome these difficulties a comprehensive analysis of a searching strategy is needed.

Recall the main idea of the simplex method.[2] In accordance with this approach the movement toward the optimum in the space k of the control coordinates is accomplished by a sequential reflection of the simplex vertex in which a current value of the goal function turns out to be the smallest with respect to the opposite face. A multiple reflection of the "worst" vertices results in a stepwise movement of the simplex to a target along some broken line. As was found in Ref. 1, in the regime of stationary wind–induced refraction a successful simplex search for maximum in the focusing criterion depends on the possible implementation of such procedures as forbidden return to the preceding simplex configuration, decreasing its size while approaching the extremum, recalculation of the goal function at the vertex which has not been replaced for its specular image during the fixed number of steps etc.

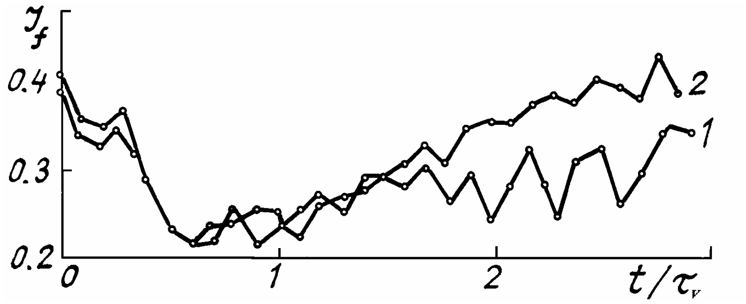

FIG. 1. The focusing criterion J_f as a function of time t in the process of dynamic compensation for nonstationary wind–induced refraction based on the simplex method: 1) algorithm with forbidden return and 2) algorithm with forced reflection of vertices at $z_0 = 0.5$ *for* $R = -20$.

In the regime of nonstationary the wind–induced refraction when the position of the maximum in the goal function strongly depends on the trajectory of its search, the above–described procedures are inefficient. In this case the simplex goes in cycles, i.e., the absence of reflection of one or several vertices during the fixed number of steps ν results in ceasing the progressive movement of the simplex to the target. Therefore a forced reflection of "old" vertices turns out to be a more reasonable strategy. The results of calculations show that for $k \le 2$ it is sufficient to reject only those vertices for which $\nu = k + 1$. When $k \ge 3$, in addition to this procedure we must simultaneously control the vertices for which $\nu = k + 3$, since the simplex goes in cycles about the $(k - 1)$ vertices. As an example, Fig. 1 shows typical time dependences of the focusing criterion for two algorithms of simplex search in a three–dimensional space of control based on Eq. (5): with a forbidden return and forced reflection of vertices. It can be seen that a reasonable implementation of the search allows one to increase the efficiency of dynamic compensation for the effect of nonstationary thermal lens.

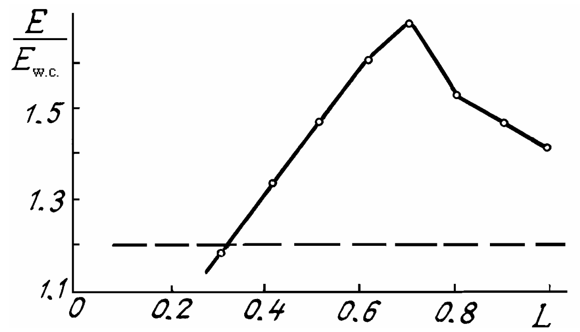

FIG. 2. The total relative light energy E/E_{wc} falling within the receiving aperture during the control time $T = 3\tau_r$ as a function of the simplex edge length L (dashed line is for the gradient method with an optimal step). Parameters of propagation: $z_0 = 0.5$ and $R = -20$. E_{wc} is the energy in the system without control.

SIMPLEX SIZE OPTIMIZATION

As has been found above, the magnitude and position of the extremum in the goal function in space of control coordinates depends on the prehistory of the search which is obviously determined by both the chosen strategy and the simplex size. Since under nonstationary conditions the field parameters in the observation plane vary in time in a complicated manner, it is difficult to choose any regular procedure for changing the simplex edge as approaching the optimum. We must recall that in the process of compensation for the stationary wind–induced refraction[1] it is sufficient to make use of the simplest rules of simplex compression (power–law or exponential) in order to achieve the optimal focusing with any prescribed accuracy. Conversely, in the presence of transient processes along the path accompanying the search for the optimal phase it seems to be reasonable to keep the simplex unchanged.

Under nonstationary conditions it is worth to perform the beam control within the finite time T starting from the moment of switching on the laser radiation source. Since the characteristic time of establishing the thermal lens on the path is $t_{est} \cong 3\tau_v$, it is of primary importance to consider the beam control at such times. Figure 2 shows the calculated dependence of the total energy falling within the given aperture during $T = 3\tau_v$ on the simplex edge length L. The value obtained when using a gradient procedure with an optimal step[3] is shown for comparison here too. It can be seen that the efficiencies of both these approaches are of the same order of magnitude for simplex edge length $L = 0.3$. As L increases up to $L = 0.7$, the efficiency of the simplex search also increases and then it decreases as a result of early oscillations of the figure of merit due to excessive high–amplitude changes of the beam phase (Fig. 3).

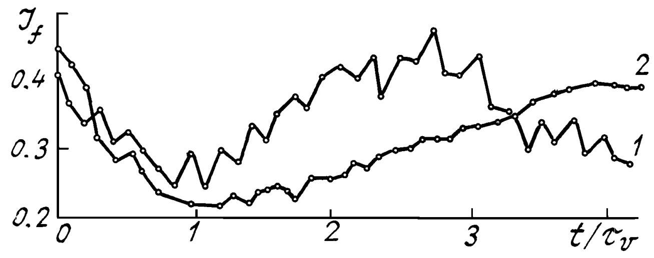

FIG. 3. Dynamic control of the beam phase based on the simplex method for two values of the simplex edge length: $L = 0.6$ *(curve 1) and* 0.3 *(curve 2). Parameters of propagation:* $z_0 = 0.5$ *and* $R = -20$.

More detailed calculations reveal that the optimal size of simplex $L_{\rm opt}$ is primarily determined by the nonlinearity parameter and is less dependent on the control time T. If T does not exceed $3\tau_v$, $L_{\rm opt}$ can be evaluated by the empirical formula $L \ge k\nu$, where k is the dimensionality of the control space and ν is the mean normalized rate of displacement of the energy centroid of the uncontrollable beam. For more prolonged beam control ($T > 3\tau_v$) it is more efficient to employ a simplex of smaller size, i.e., $L_{\rm opt} \simeq \nu$.

SPEED OF RESPONSE AND CONTROL EFFICIENCY

The model of the adaptive system with finite speed of response determined by the time between sequential wave front corrections τ_c (in the calculations we assumed $\tau_c = 0.1\,\tau_v$) has been considered above. It is obvious that this model is incapable of controlling the thermal spreading and drift of the beam at the initial stage of heating up the medium, i.e., at times of control $T \cong \tau_v$. This is due to the fact that within the time $(\kappa + 1)\tau_c$ needed for determining the starting simplex configuration the thermal lens is strongly deformed and first operating steps result only in deterioration of the focusing criterion. Within the framework of the model under study the speed of response of the system can easily be varied by changing τ_c at a fixed relaxation time of the medium τ_r. In particular, infinite decrease in τ_c makes it possible to transfer over to a model of an idealized adaptive system with infinitely high speed of response. The search for an optimal phase and transient processes occurring simultaneously in the real system are approximately discriminated in time. The phase optimization for each instantaneous state of the medium can be carried out in a "frozen" temperature field with any prescribed accuracy. The succeeding relaxation of the medium during the time τ_r allows one to match the obtained phase and the nonlinear thermal lens.

The calculations show that the control within $T \cong \tau_r$ is efficient, if $\tau_c \le \frac{1}{2(\kappa + 1)}\tau_r$. The simplex size optimization also provides the possibilities for increasing the field concentration at the target (Fig. 4).

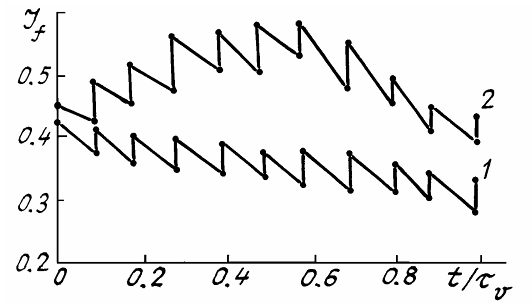

FIG. 4. Beam phase control for "frozen" instantaneous states of the medium ($\tau_c = 0.01\tau_v$). *1)* $L = 0.1$ and 2) $L = 0.2$. *Parameters of propagation:* $z_0 = 0.5$ *and* $R = -20$.

The analysis of the model problems carried out in this paper allows us to conclude that the simplex search for an optimal phase is applicable to the dynamic compensation for nonstationary wind–induced refraction. A simplex of different size depending on the duration T and speed of response τ_c should be employed due to the fact that for each concrete T and τ_c there exists an optimal amplitude of beam phase changes directly related with the simplex edge length. The *a priori* estimates of $L_{\rm opt}$ can be made for typical situations along the path by means of the numerical experiment.

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