ON THE STRUCTURE OF IRRADIANCE IN THE IMAGE PLANE FOR THE LIDAR SENSING OF THE SURFACE WITH THE COMBINED SCATTERING PHASE FUNCTION THROUGH THE ATMOSPHERE

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Received October 19, 1991

The irradiance distribution is studied in the image plane of the lidar receiver for the laser sensing of the surface with a combined scattering phase function through the atmosphere. An expression is derived for the irradiance in the image plane for the sensing of the surface with the scattering phase function comprising the diffuse and quasispecular components through the optically dense aerosol atmosphere. It is shown that the distribution of the irradiance in the image plane can strongly depend on the relation between the diffuse and quasispecular components.

The irradiance distribution in the image plane of the lidar receiver for the sensing of the surface with specular or Lambertian scattering phase function was investigated in a number of papers.[1-4] The distribution of the irradiance in the image plane of the lidar receiver for the sensing of the surface with complicated scattering phase function through the atmosphere is considered below.

Let the surface being sensed be characterized by the brightness $J(\mathbf{R}, \mathbf{m})$ (Ref. 5):

$$J(\mathbf{R}, \mathbf{m}) = \frac{E(\mathbf{R})}{\alpha \dfrac{2\pi}{n+2} + \beta\pi\Delta^2} \left[\alpha \cos^n\theta + \right.$$
$$\left. + \beta \exp\left\{ \frac{(\theta-\theta_0)^2\cos^2\theta_0 + (\varphi-\varphi_0)^2\sin^2\theta_0}{\Delta^2} \right\} \right] , \quad (1)$$

where $E(\mathbf{R}) = AE_s(\mathbf{R})$ is the irradiance of the surface produced by the radiation incident from the source, A is the reflectance, α and β are the coefficients determining the relative contribution from the diffuse and quasispecular reflections, n and Δ are the parameters characterizing the angular width of the diffuse and quasispecular components of reflection, and (θ, θ_0) and (φ, φ_0) are zenith and azimuthal angles of the observation direction and the direction toward the maximum of the reflected radiation (quasispecular component of reflection). The angles θ_0 and φ_0 are related with the corresponding angles θ_s and φ_s, which characterize the direction of incident radiation by the laws of geometric optics.

The brightness of the radiation incident at the receiver[6] and the distribution of the irradiance in the image plane of the lidar receiver[1] can be found from the distribution of brightness $J(\mathbf{R}, \mathbf{m})$ over the scattering surface S. Then using the principle of reciprocity as applied to the scattering medium[6] and performing the calculations analogous to Ref. 1, we derive the following integral equation for the irradiance $E(\mathbf{R}_0)$ in the image plane of the lidar receiver:

$$I(\mathbf{R}_f) = \int_S d\mathbf{R} \int d\Omega(\mathbf{m}) \cos\theta_r \, J(\mathbf{R}, \mathbf{m}) \, J_r(\mathbf{R}, \mathbf{R}_f, \mathbf{m}) , \quad (2)$$

where $J_r(\mathbf{R}, \mathbf{R}_f, \mathbf{m})$ is the brightness of radiation at the point $\mathbf{R}$ of the surface S from the fictitious source (with the parameters of the receiver recording the irradiance[1]). $\mathbf{R}_f$ is the vector in the image plane of the lidar receiver, θ_r is the angle between the normal to the surface S at the point $\mathbf{R}$ and the direction toward the receiver.

For the homogeneous scattering atmosphere with strongly elongated scattering phase function, if the angle at which the receiving aperture can be seen from the points lying on the scattering surface is much smaller than the angular width of the scattering phase function of radiation reflected from the surface and the angular resolution of the receiver, relation (2) for the irradiance $E(\mathbf{R}_f)$ in the image plane of the lidar receiver (assuming that the surface being sensed is flat and lies in the plane XOY while the source, receiver, and their optical axes lie in the plane XOZ and using the results from Refs. 1, 7, and 8) takes the form:

$$I(\mathbf{R}_f) \simeq \frac{A}{\pi} \frac{1}{\alpha \dfrac{2}{n+2} + \beta\Delta^2} \left[\alpha \cos^n\theta_r \times \right.$$
$$\times \int_S d\mathbf{R} \, E_s(\mathbf{R}') \, E_r(\mathbf{R}'', \mathbf{R}_f) + \beta \int_S d\mathbf{R} \, E_s(\mathbf{R}') \, E_r(\mathbf{R}'', \mathbf{R}_f) \times$$
$$\left. \times \quad \exp\left\{ -\frac{1}{\Delta^2} \left[(\sin\theta_0 - \sin\theta_r + dR_x)^2 + s^2 R_y^2 \right] \right\} \right] , \quad (3)$$

where

$$d = \frac{A_s \cos^2\theta_s}{B_s} + \frac{A_r \cos^2\theta_r}{B_r} \, ; \; A_{s,r} = \frac{1}{2}\sqrt{\alpha_{s,r}^2 + \sigma L_{s,r} \langle\gamma^2\rangle} \, ;$$

$$s = \frac{A_s}{B_s} + \frac{A_r}{B_r} \, ; \; B_{s,r} = \frac{L_{s,r}\left[\dfrac{\alpha_{s,r}^2}{2} + \dfrac{\sigma L_{s,r}\langle\gamma^2\rangle}{4}\right]}{\sqrt{\alpha_{s,r}^2 + \sigma L_{s,r} \langle\gamma^2\rangle}} ;$$

$$\mathbf{R}' = \{R_x \cos\theta_s, R_y\} \, ; \; \mathbf{R}'' = \{R_x \cos\theta_r, R_y\} ,$$

$E_s(\mathbf{R})$ and $E_r(\mathbf{R}, \mathbf{R}_f)$ are the irradiances produced in the atmosphere by the radiation incident at the surface from the real and fictitious (with the parameters of the receiver recording the irradiance) sources, respectively,[1,7] L_s and L_r are the distances from the source and receiver to the surface,

0235-6880/92/04 272-03 $02.00

$2\alpha_s$ and $2\alpha_r$ are the angular divergence of the source and angular resolution of the receiver, σ is the scattering index of the atmosphere, $\langle\gamma^2\rangle$ is the variance of the beam deviation angle during an elementary scattering event.

For $\beta = 0$ and $n = 0$ formula (3) transforms into the relation for the irradiance in the image plane of the receiver for the sensing of the Lambertian surface.[1,2]

By carrying out the calculations and using the results from Refs. 1, 2, and 7 from Eq. (3) we obtain the following analytical formula for the irradiance in the image plane of the lidar receiver for the bistatic sensing of the surface with the combined scattering phase function through the atmosphere:

$$I(\mathbf{R}_f) = c\,[c_1 \exp\{-R^2_{fx}\, b_{1x} - R^2_{fy}\, b_{1y}\} + c^2 \exp\{-(R_{fx} + d)^2 b_{2x} - R^2_{fy}\, b_{2y}\}]\,, \qquad (4)$$

where

$$c = \frac{1}{\alpha \dfrac{2}{n+2} + \beta\Delta^2} \times \frac{AP_0 \cos\theta_s \cos\theta_r\, r_r^2\, \alpha_r^2 \exp\{-(\varepsilon - \sigma)(L_s + L_r)\}}{\pi G_s G_r r_c^2}\,;$$

$$c_1 = \alpha(pq)^{-1/2} \cos^n\theta_r\,;\quad c_2 = \beta(m\bar{n}^{-1/2} \exp(\kappa)\,;$$

$$G_s = 4(D_s + C_s + B_s^2\,;\quad G_r = 4(D_r + C_r + B_r^2)\,;$$

$$m = p + \frac{d^2}{\Delta^2}\,;\quad p = \frac{\cos^2\theta_n}{G_n} + \frac{\cos^2\theta_n}{G_r}\,;\quad q = \frac{1}{G_n} + \frac{1}{G_r}\,;$$

$$\delta = \frac{F\cos\theta_r}{L_r}\,\frac{(\sin\theta_0 - \sin\theta_r)\,d}{\left(\dfrac{\cos^2\theta_s}{G_s} + \dfrac{d^2}{\Delta^2}\right)\Delta^2}\,;\quad \bar{n} = q + \frac{s^2}{D^2}\,;$$

$$k = \frac{\cos^2\theta_r\,(\sin q_0 - \sin\theta_r)^2\, d^2}{\Delta^4\left(p + \dfrac{d^2}{\Delta^2}\right) G_r \left(\dfrac{\cos^2\theta_s}{G_s} + \dfrac{d^2}{\Delta^2}\right)} - \frac{(\sin\theta_0 - \sin\theta_r)^2\, p}{\Delta^2\left(p + \dfrac{d^2}{\Delta^2}\right)}\,;$$

$$C_{s,r} = \frac{\alpha^2_{s,r}\,\sigma L^3_{s,r}\,\langle\gamma^2\rangle}{16(\alpha_s^2 + \sigma L_{s,r}\,\langle\gamma^2\rangle)}\,;\quad D_{s,r} = \frac{r^2_{s,r}}{4} + \frac{\sigma L^3_{s,r}\,\langle\gamma^2\rangle}{48}\,;$$

$$b_{1x} = \left(\frac{L_n}{F\cos\theta_r}\right)^2 \left(\frac{G_r}{\cos^2\theta_r} + \frac{G_s}{\cos^2\theta_s}\right)^{-1}\,;$$

$$b_{1y} = \left(\frac{L_r}{F}\right)^2 (G_r + G_s)^{-1}\,;$$

$$b_{2x} = \left(\frac{L_r}{F}\right)^2 \left(\frac{\cos^2\theta_s}{G_s} + \frac{d^2}{\Delta^2}\right)\left(p + \frac{d^2}{\Delta^2}\right)^{-1} \frac{1}{G_r}\,;$$

$$b_{2y} = \left(\frac{L_r}{F}\right)^2 \left(1 + \frac{s^2}{\Delta^2}\,G_s\right)\left(G_s + G_r + G_s G_r \frac{s^2}{\Delta^2}\right)^{-1}\,;$$

P_0 is the power of radiation emitted by the source; r_s and r_r are the effective diameters of the apertures of the source and receiver, $\boldsymbol{r}_c$ is the effective diameter of the least circle of aberration of the receiving optical system, and ε is the extinction index of the atmosphere.

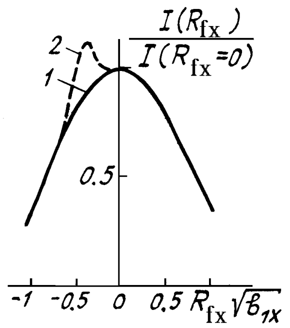

FIG. 1. Spatial distribution of irradiance for the transparent atmosphere.

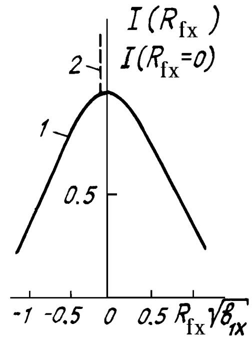

FIG. 2. Spatial distribution of irradiance for the optically dense atmosphere.

For $\beta = 0$, $n = 0$, and $\sigma = 0$ Eq. (4) transforms into the relation for irradiance produced by a flat Lambertian surface in the transparent aerosol atmosphere.[1,2]

The results of calculating the spatial distribution of irradiance in the image plane of the receiver for different values of the parameter $\gamma = \beta/\alpha$ (for different contributions from the diffuse and quasispecular components of the phase function of reflection of the surface being sensed) are shown in Figs. 1 and 2.

Calculations of the value $I(R_{fx}, R_{fy})/I(R_{fx} = 0, R_{fy})$ were carried out from formula (4) for the following values of the parameters: $n = 1$, $\theta_s = -60°$, $\theta_r = 55°$, $L_s = 10^4$ m, $L_r = 10^2$ m, $\alpha_s = 10^{-2}$, $\alpha_r = 10^{-2}$, $\Delta = 0.3$, $\theta_0 = \theta_s$, $\sigma\langle\gamma^2\rangle = 0$ (Fig. 1) and $\sigma\langle\gamma^2\rangle = 10^{-6}$ (Fig. 2). Curve *1* corresponds to $\gamma = 0$ and curve *2* – $\gamma = 0.3$.

It can be seen from the figures that the relation between the quasispecular and diffuse components of the scattering phase function of the surface being sensed substantially affects the irradiance distribution in the image plane at the receiving angles which are close to the angles of specular reflection.

Atmospheric turbidity results in increasing the size of the image (because of increasing the size of the illuminated spot on the scattering surface) and relative (in comparison with the size of the entire image) decrease of the element of the image in which the contribution of the quasispecular component of the scattering phase function of the surface to lidar return signal is significant.

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