LIMITING SENSITIVITY OF AN ABSORPTION SPECTROMETER BASED ON AN OPTO–ELECTRONIC SYSTEM WITH NEGATIVE FEEDBACK

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It is shown that the limiting sensitivity of the absorption spectrometer under consideration is two times better in comparison with a spectrometer in which the Poisson noise of the flux of sounding laser photons (spontaneous intensity fluctuations) is the principal source of noise.

Limiting sensitivity and accuracy of measuring instruments using lasers is determined by a level of the spontaneous intensity fluctuations. The suppression of the photocurrent noise below the shot–noise level was indicated in Refs. 1–3 for a closed opto–electronic system comprising a laser with negative feedback. In this connection it is necessary to consider the possibilities of using the opto–electronic system, e.g., as a basis of an absorption spectrometer.

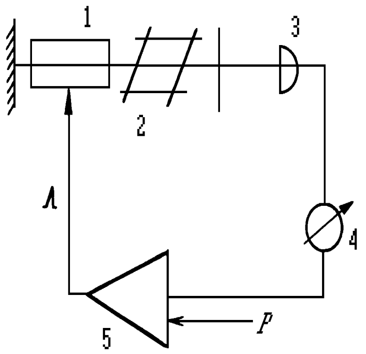

FIG. 1. Block diagram of the spectrometer: 1) tunable laser, 2) absorbing cell, 3) photodetector, 4) recording device, and 5) differential amplifier.

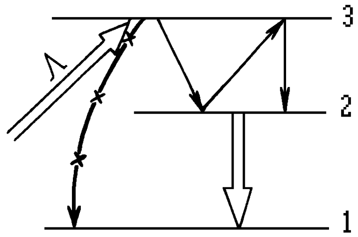

FIG. 2. Scheme of excitation and generation on the working levels of a laser.

Let us determine the limiting sensitivity of the absorption spectrometer based on the opto–electronic system with the negative feedback loop photodetector–pumping (Fig. 1). To this end, let us consider the three–level model of laser operation (Fig. 2). The main peculiarity of the scheme of excitation and generation of an active laser medium is as follows: every pumping photoelectron is changed for a photon of induced radiation, and relaxation from level *3* to level *1* is absent. Let us describe the behavior of photons and photoelectrons by the system of stochastic kinetic equations

$$\dot{n} = - Cn + p - qC_{\rm out}n + F ;$$

$$i = qC_{\rm out}\, n + f , \tag{1}$$

where n is the number density of photons in the laser cavity, i is the photocurrent, C is the rate of photon yield from the laser cavity, $p - C_{\rm out}\, n = \Lambda$ is the resultant pumping of the atoms of the active laser medium, where p is the external pumping, q is the quantum efficiency of the photodetector, $C_{\rm out}$ is the velocity of arrival of photons at the photodetector, F and f are the random Langevin forces. According to corpuscular theory of photodetection, Eqs. (1) are completed by the correlation relation[4] $\langle f_0(0)\, f_0(\tau)\rangle = qC_{\rm out}\bar{n}\ \delta(\tau)$.

If all the generated laser photons reached the photodetector without losses and exchanged for photoelectrons with a 100% efficiency, i.e., $C = C_{\rm out}$ and $q = 1$, as well as there were no losses of photoelectrons in the negative feedback loop, then the balance relation

$$\bar{n} + i = p - Cn \tag{2}$$

would take place. Analogous balance relation appears also in the study of statistics of photons generated by the laser with regularized pumping.[5] Under the aforesaid conditions, the random forces in Eqs. (1) are related by the formula $F_0 = -f_0$. The zero subscripts of F and f stress the validity of this relation only in the above–determined approximations. This circumstance enables us to use the following system of equations for the description of the behavior of photons and photoelectrons

$$\dot{n} = - (C + qC_{\rm out})n + p - f_0 + f_1 ;$$

$$i = qC_{\rm out}n + f_0 + f_2 + f_3 ; \tag{3}$$

$$\langle f_0(0)\, f_0(\tau)\rangle = qC_{\rm out}\, \bar{n}\ \delta(\tau) .$$

Let us note that for $C = C_{\rm out}$, $q = 1$ it follows that $f_1 = f_2 = 0$.

Finally we are interested in the sensitivity of the absorption spectrometer. For this purpose using Eq. (3) after linearization, we derive for the power spectrum of the photocurrent

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$$< (\delta i)^2 >_\Omega = qC_{\rm out}\bar{n}\left\{1 + \frac{C_{\rm out}^2 + \alpha}{\Omega^2 + (C + qC_{\rm out})^2} - \right.$$

$$\left. -2C_{\rm out}\frac{C + qC_{\rm out}}{\Omega^2 + (C + qC_{\rm out})^2} + (1 - q) + < f_3^2 > / qC_{\rm out}\bar{n}\right\}\Delta f, \quad (4)$$

where

$$\alpha \equiv (Im\kappa cth\,\hbar\,\omega / 2k_{\rm B}T) / qC_{\rm out}n \ .$$

The correlator $\langle f_1(0)f_1(\tau)\rangle_\Omega$ is determined from the fluctuation–dissipation theorem,[6] which relates this correlator with the absorption coefficient $\Delta C \equiv \mathrm{Im}\ \kappa$ and with the average number of thermal photons $\mathrm{cth}\omega/2k_{\rm B}T$ by the formula

$$<f_1(0)\ f_1(\tau)>_\Omega = \mathrm{Im\ k\ cth}\omega/2k_{\rm B}T \ .$$

The random force f_2 takes into account the source of noise caused by nonideal photodetector with $q < 1$. The correlation properties of f_2 are given by the formula $<f_2(0)f_2(\tau)> = q(1 - q)C_{\rm out}\ \bar{n}\delta(\tau)$.[7]

The last term in Eq. (4) represents the thermal noise of the photocurrent.

The photocurrent response on a stationary attenuation of the flux of photons passed through an absorbing cell has the form

$$<(\delta i)^2>^2_{\Omega=0} = \eta\,\frac{q^2C_{\rm out}^{\ 2}\bar{n}^2}{1 + q\beta}\ , \quad (5)$$

where $\eta \equiv (\Delta C/C)^2$ and $\beta \equiv C_{\rm out}/C$. On the basis of Eqs. (4) and (5) let us find the minimum detectable value $\eta_{\rm min}$ from the condition signal/noise = 1

$$\eta_{\rm min} = \left\{\left[\left(\frac{[(1-\beta)+q\beta]^2}{1+q\beta} + \frac{\alpha/C^2}{1+q\beta}\right)\Big/ (qC_{\rm out}\,\bar{n})\right] + [(1 - q) + \right.$$

$$\left. + <f_3^2>/qC_{\rm out}\ \bar{n}]\ (1 + q\beta)/qC_{\rm out}\ \bar{n}\right\}\ \Delta f\ . \quad (6)$$

From Eq. (6) it follows that the maximum sensitivity of the spectrometer is reached for the following parameters of the opto–electronic scheme: $q = 1$ and $\beta = 1$ and under the conditions $<f_3^2>/qC_{\rm out}\ \bar{n} \ll 1$ and $2\alpha/C^{\,2} \ll 1$.

$$\eta_{\rm min} = 0.5\ (C_{\rm out}\bar{n})^{-1}\ . \quad (7)$$

Thus, the limiting sensitivity of the analysed spectrometer is two times better in comparison with the spectrometer in which the noise of an ideal laser is the principal source of noise (the Poisson noise of photons).

In conclusion I note that the problem of finding the limiting detectable value of absorption for the absorption spectrometer based on a laser with negative feedback was discussed in Ref. 8 on the basis of other assumptions for the starting relations.

REFERENCES

1. S. Machida and Y. Yamamoto, Opt. Commun. **57**, 270 (1986).
2. Y. Yamamoto, N. Imoto, and S. Machida, Phys. Rev. **33**, 3243 (1986).
3. Ya.A. Fofanov, Radio–Elektron. **33**, 177 (1988); Kvant. Elektron. **12**, 2593 (1989).
4. E.B. Aleksandrov, Yu.M. Golubev, A.V. Lomakin, and V.A. Noskin, Usp. Fiz. Nauk **104**, 547 (1983).
5. Yu.N. Golubev and I.V. Sokolov, Zh. Eksp. Teor. Fiz. **87**, 408 (1984).
6. H.B. Callen and T.A. Welton, Phys. Rev. **83**, 34 (1951).
7. A.S. Troshin, Opt. Spektrosk. **70**, No. 3, 662 (1991).
8. J.H. Churnside and E.P. Gordov, Atm. Opt. **4**, No. 2, 111–114 (1991).