DISTRIBUTION OF THE RADIATION INTENSITY IN THE IMAGE PLANE OF A POINT–SIZE REFLECTOR WITH ARBITRARY ANGULAR POSITION ON A RANGING TURBULENT PATH

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Received June 27, 1991

It is shown that correlation of oncoming waves in the case of double passage of radiation through the same inhomogeneities leads to the biased estimate of the angular position of the reflector. The approximate analytic relation for this bias is derived. It is shown that an enhancement of the backscatter is accompanied with growth of the bias of the estimate.

It is well known[1-3] that correlation of oncoming waves after double passage of radiation through the same inhomogeneities located on ranging turbulent path leads to the backscattering intensification effects and doubling of the variance of the phase fluctuations of the reflected wave. In this paper we show that after displacement of the reflector from the axis of polar diagram of the source the correlation of the oncoming waves causes the bias of the estimate of the angular position of the reflector.

To calculate the complex amplitude of the field we make use of the phase approximation of the generalized Huygens––Kirchhoff method in which the Green's function describing the field of the point–size source is prescribed in the form $G = G_0\exp(i\psi)$, where ψ is the random run–on of the phase having a Gaussian distribution with zero mean and G is the unperturbed Green's function determined by the Helmholtz equation

$$\Delta G_0 = k^2 G_0 = \delta(\mathbf{r} - \rho) , \tag{1}$$

where Δ is the Laplacian operator and k is the wave number.

Let us first consider the case in which the radiating and receiving apertures are collocated. Let us introduce the angular coordinate $\theta = \rho/z$, where ρ is the radius vector in the image plane and z is the distance from the plane of the receiving aperture to the image plane. Let us express the complex amplitude of the field at the radiating aperture in the form $U(\mathbf{r}) = U\exp[i\varphi(\mathbf{r})]$' where U is the field amplitude, $\varphi(\mathbf{r})$ is the random phase distribution with the partial spatial coherence of the radiated wave taken into account. It is assumed that $\varphi(\mathbf{r})$ has the Gaussian distribution of probability with zero mean. In what follows, in the approximation of the Fresnel diffraction by virtue of the reciprocity theorem for the Green's function,[4] the distribution of the average intensity in the image plane has the form

$$<I(\theta)> = \left(\frac{k}{2\pi}\right)^6 \frac{K_0U^2}{L^4z^2} \int\int\int\int W_r\left(\mathbf{R} + \frac{\mathbf{r}}{2}\right) \times$$

$$\times W_r^*\left(\mathbf{R} - \frac{\mathbf{r}}{2}\right) W\left(\mathbf{R}' + \frac{\mathbf{r}'}{2}\right) W^*\left(\mathbf{R}' - \frac{\mathbf{r}'}{2}\right) \times$$

$$\times \exp\left\{ik\left[\mathbf{r}\left(\frac{\mathbf{R}}{L} - \theta_0\right) - \mathbf{r}'(\theta + \theta_0)\right] - \right.$$

$$- \frac{1}{2}\left[D_\varphi(\mathbf{r}) + D_\psi(\mathbf{r}) + D_\psi(\mathbf{r}') + \sum_{i,\, j=1}^{2} (-1)^{i+j} D_\psi \times \right.$$

$$\left.\left. \times \left(\mathbf{R} - \mathbf{R}' + \frac{(-1)^i}{2}\mathbf{r} + \frac{(-1)^j}{2}\mathbf{r}'\right)\right]\right\} d\mathbf{r}\, d\mathbf{R}\, d\mathbf{r}'\, d\mathbf{R}' , \tag{2}$$

where K_0 is the coefficient with the properties of the reflecting surface taken into account; L is the path length; $W_r(\cdot)$ and $W(\cdot)$ are the transmission functions of the radiating and receiving aperture, respectively; $D_\varphi(\cdot)$ and $D_\psi(\cdot)$ are the structure functions of the phase distributions $\varphi(\mathbf{r})$ and $\psi(\mathbf{r})$; $\theta_0 = \rho_0/L$ is the vector of the angular coordinates of the reflector; and, ρ_0 is the vector of the transverse shift of the reflector off the axis of the directional pattern of the source.

If within the apertures not more than one spot of the coherence is laid, we can use the quadratic dependences $D_\varphi(\mathbf{r}) = r^2/\rho_\varphi^2$ and $D_\psi(\mathbf{r}) = r^2/\rho_\psi^2$, where ρ_φ and ρ_ψ are the coherence lengths of the field at the radiating aperture and of the field propagating through the turbulent atmosphere, respectively.

Integral (2) can be calculated analytically for the Gaussian approximation of the transmission functions $W_r(\mathbf{r}) = \exp(-\pi r^2/S_r)$ and $W(\mathbf{r}) = \exp(-\pi r^2/S)$, where S_r and S are the areas of the radiating and receiving apertures. In these approximations the relation for the average intensity with collocated apertures has the form

$$<I(\theta)> = \left(\frac{k}{\pi z}\right)^2 \frac{K_0U^2\,\Omega_r^2\,\Omega^2 Q}{[1 + 2(l + \Omega_r^2)]\,(1 + 2n) + 2m} \times$$

$$\times \exp\left\{-\frac{5\mu\,[1 + 2(l + m + \Omega_r^2)]\,[\theta + \theta_0(1 - q)]^2}{\theta_0^2\{[1 + 2(l + \Omega_r^2)]\,(1 + 2n) + 2m\}}\right\} , \tag{3}$$

where $\Omega_r = kS_r/2\pi L$ and $\Omega = kS/2\pi L$ are the Fresnel parameters for the radiating and receiving apertures; $l = S_r/\pi$ is the number of the coherence spots in the distribution of the intensity of the radiated wave; $m = S_r/\pi\rho_\psi^2$ and $n = S/\pi\rho_\psi^2$ are the number of the coherence spots on the radiating and receiving apertures for the wave having passed through the turbulent

0235-6880/92/04 249-02 $02.00

medium; $\mu_r = k^2 S_r \theta_0^2/4\pi$ and $\mu = k^2 S \theta_0^2/4\pi$ are the ratios of the solid angle corresponding to the transverse shift of the reflector to the solid angle of the diffraction divergence for the radiating and receiving apertures, respectively; $Q = \exp\left[-\frac{4\mu_r}{(1 + 2(l + m + \Omega_r^2))}\right]$ is the multiplier characterizing the diminishment of the average intensity of the image after the reflector displaced off the axis of the directional pattern of the source; and, $q = \frac{2m}{(1 + 2(l + m + \Omega_r^2))}$ is the bias of the estimate of the angular position of the reflector.

When the centers of the radiating and receiving apertures are separated at the distance $d > l_0$, where l_0 is the inner scale of the turbulence, the last term in the exponent involved in Eq. (2) comparising the double sum and describing the correlation of the oncoming waves vanishes and distribution of the average intensity has the form

$$<I'(\theta)> = \left(\frac{\kappa}{\pi z}\right)^2 \frac{K_0 U^2 \Omega_r^2 \Omega^2 Q}{[1 + 2(l + m + \Omega_r^2)](1 + 2n)} \times$$

$$\times \exp\left[-\frac{4\mu\,(\boldsymbol{\theta} + \boldsymbol{\theta}_0')^2}{\theta_0^2\,(1 + 2n)}\right], \quad (4)$$

where $\boldsymbol{\theta}_0' = \boldsymbol{\theta}_0 + \mathbf{d}/L$, i.e., in the process of propagating of the radiation through different inhomogeneities, the estimate of the angular position of the reflector turns out to be unbiased.

The one–dimensional distributions $<I(\theta/\theta_0)>/<I'(\theta = -\theta_0')>$ and $<I(\theta/\theta_0)>/<I_0(\theta = -\theta_0)>$ ($I_0(\theta)$ is the intensity distribution of the waves propagated through the homogeneous medium) for $\mu = 0.3$ (Fig. 1) and $\mu_r = \mu = 1.2$ (Fig. 2) are shown in Figs. 1 and 2, where $m = n = 1$ for the coherent ($l = 0$, solid line) and partially coherent ($l = 1$, dashed line) radiated fields. It can be seen from the figures that an enhancement of the backscatter is accompanied by the growth of the bias of the estimate of the angular position of the reflector. The equation for the amplification factor N has the form

$$N = \frac{1 + 2(l + \Omega_r^2)}{[1 + 2(l + \Omega_r^2)](1 + 2n) + 2m} \times$$

$$\times \exp\frac{8\mu m}{[1 + 2(l + \Omega_r^2)][1 + 2(l + m + \Omega_r^2)]}. \quad (5)$$

It follows from this equation that there is a certain threshold value m_r^* such that for $\mu_r > m_r^*$ the value of N starts to increase rapidly, i.e., in the turbulent medium the maximum in the average intensity distribution decays slower than in the homogeneous medium.

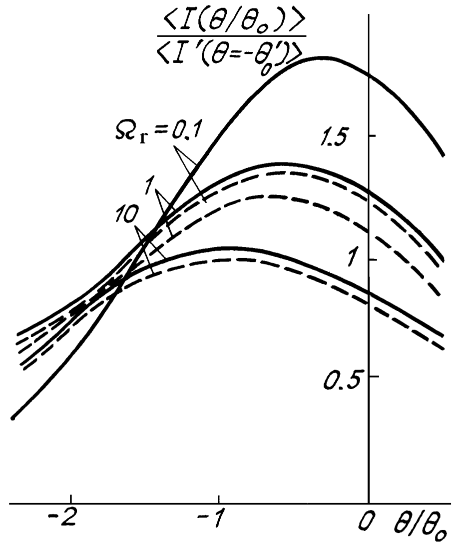

FIG. 1.

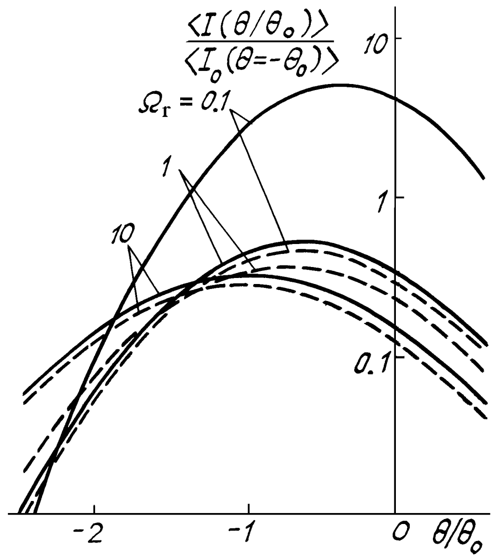

FIG. 2.

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