PROBABILITY DENSITY OF THE SATURATED INTENSITY FLUCTUATIONS OF OPTICAL WAVES IN THE TURBULENT ATMOSPHERE

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A model probability density and experimental data on the saturated intensity fluctuations are compared. It is shown that experimental data on the saturated wave intensity fluctuations are definitely indicative of deviations from the lognormal distribution and tend to the K–distribution, which should be considered as an asymptotic approximation for the probability density of the saturated intensity fluctuations.

Modern theory of optical wave propagation through a turbulent atmosphere based on the parabolic approximation of the wave equation describes incompletely the probability density of the intensity fluctuations as a function of conditions even for the free propagation.[1] It is theoretically shown that one of the universal dimensionless parameters determining the functional form of the probability density of the intensity is the parameter β_0.

$$\beta_0^2 = 1.23\, C_n^2\, k^{7/6}\, L^{11/6}\ ,$$

where C_n^2 is the structure constant of the refractive index field, L is the path length, and $k = 2\pi/\lambda$ is the wave number.

The theory and experiment give a lognormal distribution in the limiting case of the weak fluctuations when $\beta_0 < 1$. In the other limiting case (the values of β_0 up to 10 were realized, as a rule, on the sufficiently long paths) the inference on applicability of the exponential distribution was made based on the asymptotic analysis of the behavior of the normalized moments of the intensity $<I^m>$

$$<I^m> = m![1 + 0.21\beta_0^{-4/5} m(m-1)]\ ,\ \beta_0 \to \infty\ .$$

In the real atmosphere β_0 is finite. That is why the question on the arising deviations from the exponential distribution or on the accuracy of the asymptotic analysis itself was not discussed.

From the physical viewpoint the lognormal distribution corresponds to the single–ray propagation of radiation from a source to a receiver whereas the exponential distribution corresponds to the multiray propagation. This was one of the premises for the approximation of the distribution density by a superposition of the lognormal and Rayleigh distributions[2] associated with the limiting cases of propagation for the arbitrary parameter β_0. A correct comparison of the model data being obtained in this way with the experimental results is the subject for separate consideration. It should be noted here only that the exponential distribution was not experimentally observed even for large β_0 (Ref. 3).

The probability density of the saturated intensity fluctuations is proposed to be described by the K–distribution[4] in a number of papers. The model and experimental data were compared for the normalized moments of the intensity without an account of the real instrumental and statistical measurement errors except Refs. 3 and 5. In the real atmosphere the evaluations of the higher–order moments are accompanied by large errors.[6] Therefore, the agreement between experimental data and any distribution on the basis of coincidence of the first few moments (up to the fifth moment, as a rule) should be considered insufficiently grounded when we are interested in such details as a probability of deep fading, a position of the distribution mode, etc.

This paper presents the results of comparison of the model probability density with experimental data for the saturated intensity fluctuations.

The experiment was carried out with the help of the equipment and technique described in Refs. 5 and 7 in detail. A quasiplane wave of the source was formed with a lens objective 500 mm in diameter (the effective beam radius was $\alpha_0 \sim 8$ cm). The total length of V–shaped path with reflection was 2.5 km, and the parameter β_0 varied in the range $10 \le \beta_0 \le 13$. The requirements imposed by Ref. 6 on the measurement accuracy were taken into account in signal recording and data processing.

Figure 1 shows the typical histogram $P(I)$ of the instantaneous values of the intensity I ($\beta_0 = 11.5$ and the scintillation index $\beta_0 = 1.18$) and its comparison with the model values for the lognormal distribution

$$P(I) = (\sqrt{2\pi\sigma I})^{-1} \exp[-1/2\sigma^2(\ln I - \xi)^2]\ ;$$
$$\sigma = \ln(1 + \beta^2)\ ,\ \ \xi = \ln<I>/(1 + \beta^2)^{1/2}\ , \qquad (1)$$

the K–distribution

$$<I>P(I) = \frac{2}{\Gamma(y)}\, y^{(y+1)/2} I^{(y-1)/2} K_{y-1}[2(Iy)^{1/2}]\ ; \qquad (2)$$

$$y = 2/(\beta^2 - 1)\ ,\ \ y > 0\ ,$$

and the exponential distribution

$$P(I) = <I>^{-1}\exp(-I/<I>)\ . \qquad (3)$$

The variance of the histogram estimate evaluated according to Ref. 8 is indicated by the vertical bar. The bias of the histogram estimate in the region of deep fading is insignificant and can be neglected.[8]

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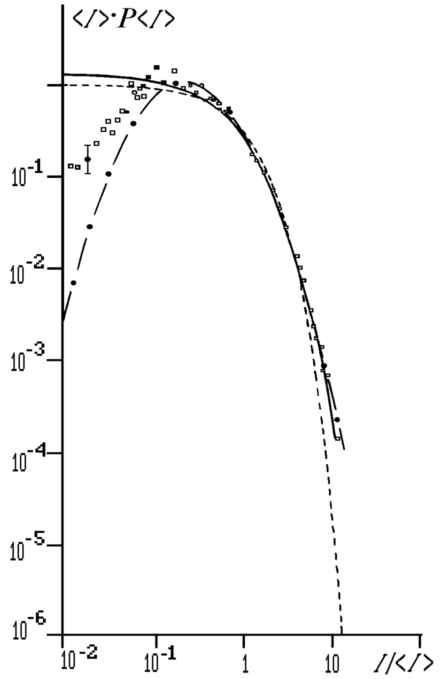

FIG. 1. A comparison of the histogram of the normalized intensity with the lognormal distribution (curve 1), exponential distribution (curve 2), and K–distribution (curve 3) for $\beta_0 = 11.5$ and $\beta = 1.18$ *in the range* $0.01 \le I/\langle I\rangle \le 15$ *for a plane wave.*

In spite of the fact that the value of β_0 is considerably greater than in the experiment of Ref. 3, the pronounced deviation of the histogram from distribution (3) can be seen in the regions of spikes $I \approx 10\langle I\rangle$ and of deep fading. This circumstance was noted previously in Ref. 3 for $I > \langle I\rangle$ and $\beta_0 = 5$.

It is clear from the results of comparison of the histogram with the lognormal and K–distributions that these distributions are sufficiently close for $I > \langle I\rangle$ and the difference between them is in the limits of the statistical measurement errors. In the region of deep fading the experimental values lie within these distributions and are closer to the K–distribution in value of the probability. The inference on the applicability of the lognormal distribution for the approximation of the probability density of the intensity fluctuations for a plane wave was drawn in Ref. 3 for the parameter $\beta_0 = 5$. As can be seen from our data, this is typical of $I > I_m$, where I_m is the modal value of the lognormal distribution. The inference of Ref. 3 on the applicability of the lognormal distribution for the entire range of the intensity values was drawn based on an analysis of the histogram with the linear scale of abscissa while the data were obtained with the help of the equipment whose dynamic range was insufficient for such measurements. All these resulted in the large bias of the histogram in the region of fading. This bias was not evaluated in Ref. 3. Really, as could be seen from the plot shown in Ref. 3, only the intensities $I > I_m$ were studied in detail.

To demonstrate the importance of this fact, Fig. 2 shows the same values as in Fig. 1 but for the narrower dynamic range $0.1 \le I/\langle I\rangle \le 15$. In this case the experimental values are fitted well not only by the K– and lognormal distributions but also by the Weibull distribution[9]

$$P(I) = \beta b(bI)^{b-1}\exp[-(bI)^{b}]\ ; \qquad b = \Gamma(1+\beta^{-1})/\langle I\rangle\ . \tag{4}$$

The moments of distribution (4) practically coincide with the moments of the K–distribution.[9]

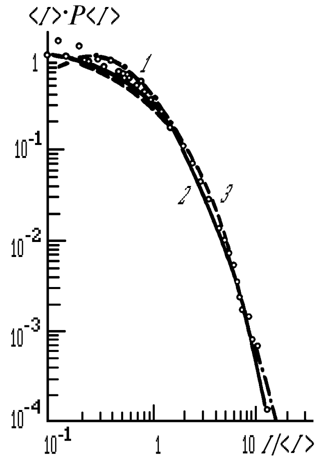

FIG. 2. A comparison of the histogram of the normalized intensity with the lognormal distribution (curve 1), K–distribution (curve 2), and Weibull distribution (curve 3) for $\beta_0 = 11.5$ and $\beta = 1.18$ *in the range* $0.1 \le I/\langle I\rangle \le 15$ *for a plane wave.*

Thus, the experimental data on a plane wave for the saturated intensity fluctuations are definitely indicative of deviations from the lognormal distribution, moreover, these deviations are such that the histograms tend to the K–distribution. Apparently, a tendency to the K–distribution is asymptotic. It is associated with the fact that formula (2) corresponding to the model of the multiray propagation still preassumes the independence of the phase fluctuations of the partial waves (rays). At the same time, in the turbulent atmosphere the fluctuations of the phase difference of the optical waves are correlated at distances up to the outer scale of turbulence, along which very many rays are present. Therefore, the K–distribution should be considered as an asymptotic approximation for the probability density of the saturated intensity fluctuations.

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