MODELING OF RADIATION OF THE ATMOSPHERE – OCEAN SYSTEM BY THE METHOD OF INFLUENCE FUNCTIONS

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Propagation of solar radiation in the "atmosphere — ocean" system is described using the model of two media with reflecting and transmitting smooth or wavy interface. The solution is represented in the form of linear and nonlinear functionals forming the optical transfer operator whose kernel is a vector function of the influence of the atmosphere and ocean. The vector function of the influence for horizontally homogeneous problem is the response of each medium to the external unidirectional radiation flux incident from the interface.

The present paper deals with the mathematical models constructed for the detailed study of the radiation field formation and image transfer in the atmosphere–ocean system based on the numerical experiments. The method of the influence functions (IF) or fundamental solutions was developed as applied to two–medium problems with reflecting and transmitting smooth or wavy interface.[1–4] The basis for mathematical apparatus for constructing the IF models and optical transfer operator (OTO) is provided with a series of perturbation theory, theory of generalized solutions of kinetic equations, and theory of fundamental solutions of equations with differential operators in terms of partial derivatives. The complete solution of the problem on account of nonlinear approximations in the multiplicity of radiation interaction (reflection and transmission) with the interface is reduced to finding of the influence functions of the atmosphere and ocean, i.e., to the fundamental solutions of linear problems of the transfer theory independently for each medium and to calculation of nonlinear functionals whose kernels are the influence functions of the atmosphere θ_a and ocean θ_{oc}. As a result, in addition to the complete solution, the explicit relation, which describes the optical transfer operator of the system, is determined between the measured radiative characteristics and the parameters of the interface between the media. The new results of the proposed approach are the reduction of numerical solution of a single boundary–value problem for two media to that of the two boundary–value problems independently for each medium and formulation of the OTO in the matrix form with the kernel being a two–component vector $\{\Theta = \theta_a \theta_{oc}\}$. The constructed mathematical models of the IF and OTO allow one to develop new algorithms for remote sensing of the atmosphere–ocean system and image transfer theory.

In this paper we restrict ourselves to consideration of a horizontally homogeneous problem, though the aforementioned approach was developed for the problem with inhomogeneities in horizontal planes. As we show below for homogeneous smooth or wavy interfaces the IF's of the atmosphere and ocean are the responses of the media to the propagation of an unidirectional wide beam.

PROBLEM FORMULATION

Propagation of solar radiation in the atmoshere–ocean system is described by two classes of problems:

1) problems with a nonorthotropic boundary in which the ocean is modeled as a reflecting base (Fig. 1) and

2) problems in the atmosphere–ocean system with an internal interface reflecting and transmitting the radiation (Fig. 2).

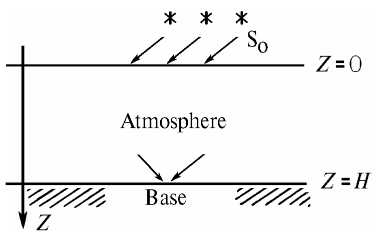

FIG. 1.

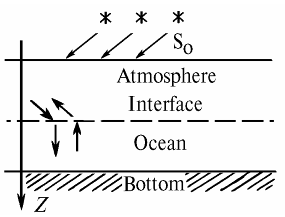

FIG. 2.

The problems with a nonorthotropic surface were considered in detail in Refs. 1–4. In this paper we call attention to the problems in the atmosphere–ocean system (Fig. 2).

The direction of radiation propagation is specified by the vector $s = \{\mu, \varphi\}$, $\mu = \cos \upsilon$, $\mu \in [-1, 1]$ on a unit sphere $\Omega = [-1, 1] \times [0, 2\pi]$, where $\upsilon \in [0, 180°]$ is the zenith angle counted off from the positive direction of the z axis and $\varphi \in [0, 2\pi]$ is the azimuth. The value $\varphi = 0$ is assumed to lie in a plane of solar vertical, i.e., a solar flux is incident on the layer boundary $z = 0$ in the direction $s_0 = \{\mu_0, \varphi_0\}$ at the zenith angle $\upsilon_0 \in [0, 90°]$, where $\mu_0 = \cos \upsilon_0$ and the azimuth $\varphi_0 = 0$.

For downward transmitted radiation we introduce the hemisphere of directions $\Omega^+ = \{(\mu, \varphi): \mu > 0\}$ and for upward reflected radiation we introduce $\Omega^- = \{(\mu, \varphi): \mu < 0\}$; $\Omega = \Omega^+ \cup \Omega^-$.

The boundary conditions are written down using the sets

$$\Gamma_0 = \{(z, s): z = 0, s \in \Omega^+\}, \quad \Gamma_H = \{(z, s): z = H, s \in \Omega^-\},$$

$$\Gamma_{h^+} = \{(z, s): z = h, s \in \Omega^+\}, \quad \Gamma_{h^-} = \{(z, s): z = h, s \in \Omega^-\}.$$

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At the altitude $z = h$ there is an interface between the two media. The radiation transmission through this interface is described by the reflection $\hat{R}_1$ and $\hat{R}_2$ and transmission $\hat{T}_{12}$ and $\hat{T}_{21}$ operators, where the subscript 1 corresponds to the upper layer (it is usually the atmosphere) and the subscript 2 – to the lower layer (ocean)

$$[\hat{R}_1\Phi^+](z = h, s) = \int_{\Omega^+} \Phi(z = h, s')\, \boldsymbol{R}_1(s, s')\, ds', \quad s \in \Omega^- ;$$

$$[\hat{R}_2\Phi^-](z = h, s) = \int_{\Omega^-} \Phi(z = h, s')\, \boldsymbol{R}_2(s, s')\, ds', \quad s \in \Omega^+ ;$$

$$[\hat{T}_{12}\Phi^+](z = h, s) = \int_{\Omega^+} \Phi(z = h, s')\, \boldsymbol{T}_{12}(s, s')\, ds', \quad s \in \Omega^+ ;$$

$$[\hat{T}_{21}\Phi^-](z = h, s) = \int_{\Omega^-} \Phi(z = h, s')\, \boldsymbol{T}_{21}(s, s')\, ds', \quad s \in \Omega^- ;$$

The optical properties of the atmosphere and ocean are determined by vertical distributions of the coefficients of extinction $\sigma_e(z) = \sigma_s(z) + \sigma_{abs}(z)$, absorption $\sigma_{abs}(z)$, total scattering $\sigma_s(z) = \sigma_a(z) + \sigma_m(z)$ including aerosol $\sigma_a(z)$ and molecular $\sigma_m(z)$ components as well as by the total scattering phase function

$$\gamma(z, \chi) = \frac{\sigma_a(z)}{\sigma_s(z)}\, \gamma_a(z, \chi) + \frac{\sigma_m(z)}{\sigma_s(z)}\, \gamma_m(\chi)$$

which, in the general case, incorporates the aerosol $\gamma_a(z, \chi)$ and molecular $\gamma_m(\chi) = 3(1 - \cos^2\chi)/(16\pi)$ components.

The integro–differential operator of the kinetic equation $\hat{K} \equiv \hat{D} - \hat{S}$ contains the transfer operator $\hat{D} \equiv (s, \text{grad}) + \sigma_e(z)$ and the collision integral $\hat{S}\Phi = \sigma_s(z)\int_{\Omega} \Phi\gamma ds'$. For the one–dimensional plane problem (with horizontal homogeneity) the transfer operator is

$$\hat{D}_z \equiv \mu \frac{\partial}{\partial z} + \sigma_e(z) , \quad \hat{K}_z \equiv \hat{D}_z - \hat{S} .$$

ON SEPARATION IN THE CONTRIBUTIONS OF THE ATMOSPHERE AND OCEAN

Let us consider the boundary–value problem for the radiative transfer equation in the atmosphere–ocean system with the interface

$$\begin{cases} \hat{K}_z\Phi = 0 , \quad \Phi\big|_{\Gamma_0} = f_0 , \quad \Phi\big|_{\Gamma_H} = q\hat{R}_H\Phi , \\ \Phi\big|_{\Gamma_{h^+}} = \hat{R}_2\Phi^- + \hat{T}_{12}\Phi^+ , \quad \Phi\big|_{\Gamma_{h^-}} = \hat{R}_1\Phi^+ + \hat{T}_{21}\Phi^- \end{cases} \tag{1}$$

without comprehensive analysis of the forms of the reflection and transmission operators. We make use of linear properties of the boundary–value problem in the form of Eq. (1) with respect to the sources and represent the total radiation field of the system in the form of a superposition

$$\Phi = \Phi^0 + \Phi_a + \Phi_{aR} + \Phi_{oc} + \Phi_q ,$$

whose components are the solutions of the following problems.

The direct attenuated solar radiation Φ^0 is the solution of the problem

$$\begin{cases} \hat{D}_z\Phi^0 = 0 , \quad \Phi^0\big|_{\Gamma_0} = [\pi\Sigma_\lambda(s - s_0)] , \\ \Phi^0\big|_{\Gamma_H} = 0 , \quad \Phi^0\big|_{\Gamma_{h^+}} = 0 , \quad \Phi^0\big|_{\Gamma_{h^-}} = 0 \end{cases}$$

for the upper layer $z \in [0, h]$ and $\Phi^0 \neq 0$ only for $s = s_0$.

The background radiation of the atmosphere Φ_a is the solution of the problem with null boundary conditions for the layer $z \in [0, h]$

$$\begin{cases} \hat{K}_z\Phi_a = [\hat{S}\Phi^0] , \quad \Phi_a\big|_{\Gamma_0} = 0 , \quad \Phi_a\big|_{\Gamma_H} = 0 , \\ \Phi_a\big|_{\Gamma_{h^+}} = 0 , \quad \Phi_a\big|_{\Gamma_{h^-}} = 0 . \end{cases}$$

The radiation of the atmosphere reflected from the interface is the solution of the boundary–value problem for the layer $z \in [0, h]$ with the source located at $z = h$

$$\begin{cases} \hat{K}_z\Phi_{aR} = 0 , \quad \Phi_{aR}\big|_{\Gamma_0} = 0 , \quad \Phi_{aR}\big|_{\Gamma_H} = 0 , \\ \Phi_{aR}\big|_{\Gamma_{h^+}} = 0 , \quad \Phi_{aR}\big|_{\Gamma_{h^-}} = \hat{R}_1\Phi^+_{aR} + [\hat{R}_1(\Phi^0 + \Phi^+_a)] \end{cases} \tag{2}$$

and can be found as the sum of two components

$$\Phi_{aR} = \Phi^0_{aR} + \Phi^{hz}_{aR} .$$

The component Φ^0_{aR} is the contribution to atmospheric haze due to scattering in the upper layer of the direct attenuated radiation reflected from the interface ($z \in [0, h]$):

$$\begin{cases} \hat{K}_z\Phi^0_{aR} = 0 , \quad \Phi^0_{aR}\big|_{\Gamma_0} = 0 , \quad \Phi^0_{aR}\big|_{\Gamma_H} = 0 , \\ \Phi^0_{aR}\big|_{\Gamma_{h^+}} = 0 , \quad \Phi^0_{aR}\big|_{\Gamma_{h^-}} = \hat{R}_1\Phi^0_{aR} + [\hat{R}_1F^0] . \end{cases} \tag{3}$$

Scattering of a diffuse component of haze reflected from the interface produces the component Φ^{hz}_{aR} being the solution of the problem in the atmosphere ($z \in [0, h]$)

$$\begin{cases} \hat{K}_z\Phi^{hz}_{aR} = 0 , \quad \Phi^{hz}_{aR}\big|_{\Gamma_0} = 0 , \quad \Phi^{hz}_{aR}\big|_{\Gamma_H} = 0 , \\ \Phi^{hz}_{aR}\big|_{\Gamma_{h^+}} = 0 , \quad \Phi^{hz}_{aR}\big|_{\Gamma_{h^-}} = \hat{R}_1\Phi^{hz^+}_{aR} + [\hat{R}_1\Phi^+_a] . \end{cases} \tag{4}$$

The radiation produced in the atmosphere and incident on the interface $z = h$ is the source of the light field component of the system Φ_{oc} in the formation of which the ocean is directly involved ($\Phi_{oc} \neq 0$ for $z \in [0, H]$)

$$\begin{cases} \hat{K}_z\Phi_{oc} = 0\,, \quad \Phi_{oc}\big|_{\Gamma_0} = 0\,, \quad \Phi_{oc}\big|_{\Gamma_H} = 0\,, \\ \Phi_{oc}\big|_{\Gamma_{h^+}} = \hat{R}_2\Phi^-_{oc} + \hat{T}_{12}\Phi^+_{oc} + [\hat{T}_{12}(\Phi^0 + \Phi^+_{a} + \Phi^+_{aR})], \\ \Phi_{oc}\big|_{\Gamma_{h^-}} = \hat{R}_1\Phi^+_{oc} + \hat{T}_{12}\Phi^-_{oc}. \end{cases} \quad (5)$$

After detailed examination we can introduce the superposition

$$\Phi_{oc} = \Phi^0_{oc} + \Phi^{hz}_{oc}$$

with separation of the brightness field components associated with the direct solar radiation Φ^0_{oc}

$$\begin{cases} \hat{K}_z\Phi^0_{oc} = 0\,, \quad \Phi^0_{oc}\big|_{\Gamma_0} = 0\,, \quad \Phi^0_{oc}\big|_{\Gamma_H} = 0\,, \\ \Phi^0_{oc}\big|_{\Gamma_{h^+}} = \hat{R}_2\Phi^{0-}_{oc} + \hat{T}_{12}\Phi^{0+}_{oc} + [\hat{T}_{12}(\Phi^0 + \Phi^0_{aR})]\,, \\ \Phi^0_{oc}\big|_{\Gamma_{h^-}} = \hat{R}_1\Phi^{0+}_{oc} + \hat{T}_{21}\Phi^{0-}_{oc} \end{cases} \quad (6)$$

and with the atmospheric haze Φ^{hz}_{oc}

$$\begin{cases} \hat{K}_z\Phi^{hz}_{oc} = 0\,, \quad \Phi^{hz}_{oc}\big|_{\Gamma_0} = 0\,, \quad \Phi^{hz}_{oc}\big|_{\Gamma_H} = 0\,, \\ \Phi^{hz}_{oc}\big|_{\Gamma_{h^+}} = \hat{R}_2\Phi^{hz-}_{oc} + \hat{T}_{12}\Phi^{hz+}_{oc} + [\hat{T}_{12}(\Phi^+_{a} + \Phi^{hz+}_{aR})]\,, \\ \Phi^{hz}_{oc}\big|_{\Gamma_{h^-}} = \hat{R}_1\Phi^{hz+}_{oc} + \hat{T}_{21}\Phi^{hz-}_{oc}\,. \end{cases} \quad (7)$$

The contribution of the illumination from the reflecting ocean bottom is found as a solution of the boundary–value problem

$$\begin{cases} \hat{K}_z\Phi_q = 0\,, \quad \Phi_q\big|_{\Gamma_0} = 0\,, \quad \Phi_q\big|_{\Gamma_H} = q\hat{R}_H\Phi_q + [qE]\,, \\ \Phi_q\big|_{\Gamma_{h^+}} = \hat{R}_2\Phi^-_q + \hat{T}_{12}\Phi^+_q\,, \quad \Phi_q\big|_{\Gamma_{h^-}} = \hat{R}_1\Phi^+_q + \hat{T}_{21}\Phi^-_q\,, \end{cases} \quad (8)$$

in which the source of radiation is the illuminance of the ocean bottom $E = \hat{R}_H\Phi^-_{oc}$.

For the Lambertian ocean bottom in the case of one–dimensional plane problem E = const and the solution of boundary–value problem (8) can be sought based on the formula

$$\Phi_q(z, \mu, \varphi) = qE\Psi(z, \mu, \varphi)/(1 - qc_0)\,, \quad c_0 \equiv \hat{R}\Psi$$

determining the explicit dependence of the illumination on the albedo of the ocean bottom q in terms of the transmission function of the Ψ–solution of the problem with isotropic insolation at $z = H$

$$\begin{cases} \hat{K}_z\Psi = 0\,, \quad \Psi\big|_{\Gamma_0} = 0\,, \quad \Psi\big|_{\Gamma_H} = 1\,, \\ \Psi\big|_{\Gamma_{h^+}} = \hat{R}_2\Psi^- + \hat{T}_{12}\Psi^+\,, \quad \Upsilon\big|_{G_{h^-}} = \hat{R}_1\Psi^+ + \hat{T}_{21}\Psi^-\,. \end{cases} \quad (9)$$

EQUATIONS FOR THE INFLUENCE FUNCTIONS OF THE ATMOSPHERE AND OCEAN AND THE OPTICAL TRANSFER OPERATOR

The solution of the problems for the atmospheric radiation components given by Eqs. (2)–(4) was studied in Refs. 1–4, where the ocean was taken into account as a reflecting nonorthotropic or Lambertian surface. The components of the field Φ^0_{aR}, Φ^{hz}_{aR}, and Φ_{aR} were calculated in terms of the influence function of the atmosphere, i.e., the solution of the boundary–value problem

$$\{K_z\theta_a = 0\,, \quad \theta_a\big|_{\Gamma_0} = 0\,, \qquad \theta_a\big|_{\Gamma_{h^-}} = \delta(s - s^-)\,. \quad (10)$$

Aforementioned problems (5)–(7) for determining the individual components of radiation formed in the ocean can be written down in the general form

$$\begin{cases} \hat{K}_z\Phi_{oc} = 0\,, \quad \Phi_{oc}\big|_{\Gamma_0} = 0\,, \quad \Phi_{oc}\big|_{\Gamma_H} = 0\,, \\ \Phi_{oc}\big|_{\Gamma_{h^+}} = \eta(\hat{R}_2 F^-_{oc} + \hat{T}_{12}\Phi^+_{oc} + E_{oc}(s))\,, \\ \Phi^0_{oc}\big|_{\Gamma_{h^-}} = \eta(\hat{R}_1\Phi^-_{oc} + \hat{T}_{21}\Phi^+_{oc}\; E_a(s))\,, \end{cases} \quad (11)$$

where the radiation sources are the illumination of the ocean from above, i.e., from the atmosphere, $E_{oc}(s)$ and the illumination of the atmosphere from below, i.e., from the ocean, $E_a(s)$. In particular, for problem (5) we have

$$E_{oc}(s) \equiv \hat{T}_{12}(\Phi^0 + \Phi^+_a + \Phi^+_{aR})\,,$$

for problem (6)

$$E_{oc}(s) \equiv \hat{T}_{12}(\Phi + \Phi^0_{aR})\,,$$

for problem (7)

$$E_{oc}(s) \equiv \hat{T}_{12}\big(\Phi^+_a + \Phi^{hz+}_{aR}\big)\,,$$

and for all three problems (5)–(7) $E_a(s) \equiv 0$.

Let us introduce a perturbation series for solving problem (11)

$$\Phi_{oc} = \sum_{n=1}^{\infty} \eta^n\Phi_n \quad (12)$$

with the parameter η, which is indicative of the event of passing through the interface, and the two–component vectors

$$\mathbf{\Phi}_n = \{\Phi_{a\,n}, \Phi_{oc\,n}\}, \; \mathbf{E} = \{E_a, E_{oc}\}\,, \; \mathbf{\Theta} = \{\theta_a, \theta_{oc}\}\,. \quad (13)$$

In the linear approximation (n = 1) the problem with two sources $E_a(s)$ and $E_{oc}(s)$

$$\begin{cases} \hat{K}_z\Phi_1 = 0\,, \quad \Phi_1\big|_{\Gamma_0} = 0\,, \quad \Phi_1\big|_{\Gamma_H} = 0\,, \\ \Phi_1\big|_{\Gamma_{h^+}} = E_{oc}(s)\,, \quad \Phi_1\big|_{\Gamma_{h^-}} = E_a(s) \end{cases}$$

separates into two problems: for the ocean ($z \in [h, H]$)

$$\{\hat{K}_z \Phi_{oc\,1} = 0\ ,\quad \Phi_{oc\,1}\big|_{\Gamma_H} = 0\ ,\quad \Phi_{oc\,1}\big|_{\Gamma_h^+} = E_{oc}(s) \qquad (14)$$

and for the atmosphere ($z \in [0, h]$)

$$\{\hat{K}_z \Phi_{a\,1} = 0\ ,\quad \Phi_{a\,1}\big|_{\Gamma_0} = 0\ ,\quad \Phi_{a\,1}\big|_{\Gamma_h^-} = E_a(s)\ ,$$

where $\Phi_{a\,1} \equiv 0$ since $E_a(s) \equiv 0$.

Let us represent the illumination as a functional

$$E_{oc}(s) = \frac{1}{2\pi}\int_{\Omega^+} E_{oc}(s^+)\,(s - s^+)\,ds^+,$$

then the solution of problem (14) can be written down as a linear functional ($s \in \Omega$, $z \in [h, H]$

$$\Phi_{oc\,1}(z, s) = (\theta_{oc}, E_{oc}) = \frac{1}{2\pi}\int_{\Omega^+} E_{oc}(s^+)\theta_{oc}(z, s, s^+)\,ds^+,$$

whose kernel is the influence function of the ocean, i.e., the solution of the problem for the layer $z \in [h, H]$

$$\{\hat{K}_z \theta_{oc} = 0\ ,\quad \theta_{oc}\big|_{\Gamma_H} = 0\ ,\quad \theta_{oc}\big|_{\Gamma_h^+} = \delta(s - s^+)\ . \qquad (15)$$

In the second approximation ($n = 2$) the problem ($z \in (0, H]$)

$$\begin{cases} \hat{K}_z \Phi_2 = 0\ ,\quad \Phi_2\big|_{\Gamma_0} = 0\ ,\quad \Phi_2\big|_{\Gamma_H} = 0\ , \\ \Phi_2\big|_{\Gamma_h^+} = \hat{R}_2\Phi_1^- + \hat{T}_{12}\Phi_1^+\ , \\ \Phi_2\big|_{\Gamma_h^-} = \hat{R}_1\Phi_1^+ + \hat{T}_{21}\Phi_1^- \end{cases} \qquad (16)$$

separates into two problems ($\hat{R}_1\Phi^+_{a\,1} = 0$, $\hat{T}_{12}\Phi^+_{a\,1} = 0$): for the layer $z \in [0, h]$

$$\{\hat{K}_z \Phi_{a\,2} = 0\ ,\quad \Phi_{a\,2}\big|_{\Gamma_0} = 0\ ,\quad \Phi_{a\,2}\big|_{\Gamma_h^-} = \hat{T}_{21}\Phi^-_{oc\,1}$$

and for the layer $z \in [h, H]$

$$\{\hat{K}_z \Phi_{oc\,2} = 0\ ,\quad \Phi_{oc\,2}\big|_{\Gamma_H} = 0\ ,\quad \Phi_{oc\,2}\big|_{\Gamma_h^+} = \hat{R}_2\Phi^-_{oc\,1}\ .$$

The solution of problem (16) is written down for two components in the form of linear functionals

$$\Phi_{a\,2}(z, s) = (\theta_a, \hat{T}_{21}\Phi^-_{oc\,1}) =$$

$$= \frac{1}{2\pi}\int_{\Omega^-} [\hat{T}_{21}\Phi^-_{oc\,1}](s^-)\,\theta_a(z, s, s^-)ds^- =$$

$$= \frac{1}{2\pi}\int_{\Omega^-} \theta_a(z, s, s^-)ds^- \frac{1}{2\pi}\int_{\Omega^+} [\hat{T}_{21}\theta^-_{oc}](s_1^+)\,E_{oc}(s_1^+)\,ds_1^+\ ,$$

$$\Phi_{oc\,2}(z, s) = (\theta_{oc}, \hat{R}_2\Phi^-_{oc\,1}) =$$

$$= \frac{1}{2\pi}\int_{\Omega^+} [\hat{R}_2\Phi^-_{oc\,1}](s^+)\,\theta_{oc}(z, s, s^+)\,ds^+ =$$

$$= \frac{1}{2\pi}\int_{\Omega^+} \theta_{oc}(z, s, s^+)\,ds^+ \frac{1}{2\pi}\int_{\Omega^+} [\hat{R}_2\theta^-_{oc}](s_1^+)\,E_{oc}(s_1^+)\,ds_1^+\ ,$$

or

$$\begin{cases} \Phi_{a2} = (\theta_a, (\hat{T}_{21}\theta^-_{oc}, E_{oc})) = (\theta_a, \hat{T}_{21}(\theta_{oc}, E_{oc}))\ , \\ \Phi_{oc\,2} = (\theta_{oc}, (\hat{R}_2\theta^-_{oc}, E_{oc})) = (\theta_{oc}, \hat{R}_2(\theta_{oc}, E_{oc}))\ . \end{cases}$$

For the third and subsequent approximations ($n \ge 3$) the total problem ($z \in [0, H]$)

$$\begin{cases} \hat{K}_z \Phi_n = 0\ ,\quad \Phi_n\big|_{\Gamma_0} = 0\ ,\quad \Phi_n\big|_{\Gamma_H} = 0\ , \\ \Phi_n\big|_{\Gamma_h^+} = \hat{R}_2\Phi^-_{n-1} + \hat{T}_{12}\Phi^+_{n-1}\ ,\quad \Phi_n\big|_{\Gamma_h^-} = \hat{R}_1\Phi^+_{n-1} + \hat{T}_{21}\Phi^-_{n-1} \end{cases}$$

separates into two problems according to the sources: for the layer $z \in [0, h]$

$$\{\hat{K}_z \Phi_{a\,n} = 0,\quad \Phi_{a\,n}\big|_{\Gamma_0} = 0,\quad \Phi_{a\,n}\big|_{\Gamma_h^-} = \hat{R}_1\Phi^+_{a\,n-1} + \hat{T}_{21}\Phi^-_{oc\,n-1}$$

and for the layer $z \in [h, H]$

$$\begin{cases} \hat{K}_z \Phi_{oc\,n} = 0\ ,\quad \Phi_{oc\,n}\big|_{\Gamma_H} = 0\ , \\ \Phi_{oc\,n}\big|_{\Gamma_h^+} = \hat{R}_2\Phi^+_{oc\,n-1} + \hat{T}_{12}\Phi^+_{a\,n-1}\ . \end{cases}$$

Let us write down the linear functionals for several successive approximations including the terms engendered by the source E_a

$n = 1$

$$\Phi_{a\,1} \equiv (\theta_a, E_a)\ ,$$
$$\Phi_{oc\,1} = (\theta_{oc}, E_{oc})\ ;$$

$n = 2$

$$\Phi_{a\,2} = (\theta_a, \hat{R}_1\Phi^+_{a\,1} + \hat{T}_{21}\Phi^-_{oc\,1})\ ,$$

$$\Phi_{oc\,2} = (\theta_{oc}, \hat{R}_2\Phi^-_{oc\,1} + \hat{T}_{12}\Phi^-_{a\,1})\ ;$$

$n \ge 3$

$$\Phi_{a\,n} = (\theta_a, \hat{R}_1\Phi^+_{a\,n-1} + \hat{T}_{21}\Phi^-_{oc\,n-1})\ ;$$

$$\Phi_{oc\,n} = (\theta_{oc}, \hat{R}_2\Phi^-_{oc\,n-1} + \hat{T}_{12}\Phi^-_{a\,n-1})\ .$$

We now determine the linear vector functional

$$(\mathbf{\Theta}, \mathbf{f}) = \begin{cases} (\theta_a, f_a) = \dfrac{1}{2\pi}\displaystyle\int_{\Omega^-} \theta_a(z, s, s^-) f_a(s^-)\,ds^-, \\ (\theta_{oc}, f_{oc}) = \dfrac{1}{2\pi}\displaystyle\int_{\Omega^+} \theta_{oc}(z, s, s^+) f_{oc}(s^+)\,ds^+ \end{cases}$$

and the operation at the interface $z = h$

$$\hat{P}\mathbf{f} \equiv \hat{P}(\Theta, \mathbf{f}) = \begin{bmatrix} \hat{R}_1 & \hat{T}_{21} \\ \hat{T}_{12} & \hat{R}_2 \end{bmatrix} \begin{bmatrix} (\theta_a, f_a) \\ (\theta_{oc}, f_{oc}) \end{bmatrix} =$$

$$= \begin{bmatrix} \hat{R}_1(\theta_a, f_a) + \hat{T}_{21}(\theta_{oc}, f_{oc}) \\ \hat{T}_{12}(\theta_a, f_a) + \hat{R}_2(\theta_{oc}, f_{oc}) \end{bmatrix} = \begin{bmatrix} (\hat{R}_1\theta_a^+, f_a) + (\hat{T}_{21}\theta_{oc}^-, f_{oc}) \\ (\hat{T}_{12}\theta_a^+, f_a) + (\hat{R}_2\theta_{oc}^-, f_{oc}) \end{bmatrix},$$

where

$$[\hat{R}_1\theta_a^+](s, s^-) = \int_{\Omega^+} \boldsymbol{R}_1(s, s')\, \theta_a^+(h, s', s^-)\, ds', \quad s, s^- \in \Omega^- ;$$

$$[\hat{T}_{12}\theta_a^+](s, s^-) = \int_{\Omega^+} \boldsymbol{T}_{12}(s, s')\theta_a^+(h, s', s^-)\, ds', \; s \in \Omega^+, \; s^- \in \Omega^- ;$$

$$[\hat{R}_2\theta_{oc}^-](s, s^+) = \int_{\Omega^-} \boldsymbol{R}_2(s, s')\, \theta_{oc}^-(h, s', s^+)\, ds', \quad s, s^+ \in \Omega^+ ;$$

$$[\hat{T}_{21}\theta_{oc}^-](s, s^+) = \int_{\Omega^-} \boldsymbol{T}_{21}(s, s')\, \theta_{oc}^-(h, s', s^+)\, ds', \; s \in \mathrm{X}^-, \; s^+ \in \mathrm{X}^+ ,$$

so that

$$[\hat{P}\mathbf{f}](s) \equiv \hat{P}(\Theta, \mathbf{f})(s) =$$

$$= \begin{cases} \dfrac{1}{2\pi} \displaystyle\int_{\Omega^-} \Big\{[\hat{R}_1\theta_a^+](s, s^-)\, f_a(s^-) + \\ \quad + [\hat{T}_{21}\theta_{oc}^-](s, s^-)\, f_{oc}(s^-)\Big\}\, ds^-, \; s \in \Omega^- , \\ \dfrac{1}{2\pi} \displaystyle\int_{\Omega^-} \Big\{[\hat{T}_{12}\theta_a^+](s, s^+)\, f_a(s^+) + \\ \quad + [\hat{R}_2\theta_{oc}^-](s, s^+) f_{oc}(s^+)\Big\}\, ds^+, \; s \in \Omega^+ . \end{cases}$$

(17)

Let us write the nth approximation in the vector form and make use of definition (17):

$$\boldsymbol{\Phi}_1 = \begin{bmatrix} 0 \\ \Phi_{oc\,1} \end{bmatrix} = \begin{bmatrix} (\theta_a, E_a) \\ (\theta_{oc}, E_{oc}) \end{bmatrix} = (\Theta, \mathbf{E}) ,$$

$$\mathbf{F}_1 = \hat{P}\boldsymbol{\Phi}_1 = \hat{P}(\Theta, \mathbf{E}) = \hat{P}\mathbf{E} = \begin{bmatrix} \hat{T}_{21}\, \Phi_{oc\,1}^- \\ \hat{R}_2\, \Phi_{oc\,1}^- \end{bmatrix},$$

$$\boldsymbol{\Phi}_2 = (\Theta, \mathbf{F}_1) = (\Theta, \hat{P}\mathbf{E}) = (\Theta, \hat{P}\boldsymbol{\Phi}_1) = (\Theta, \hat{P}(\Theta, \mathbf{E})) ,$$

$$\mathbf{F}_2 = \hat{P}\boldsymbol{\Phi}_2 = \hat{P}(\Theta, \mathbf{F}_1) = \hat{P}\mathbf{F}_1 = \hat{P}^2\mathbf{E} = \begin{bmatrix} \hat{R}_1\Phi_{a\,2}^+ + \hat{T}_{21}\Phi_{oc\,2}^- \\ \hat{R}_2\, \Phi_{oc\,2}^- + \hat{T}_{12}\Phi_{oc\,2}^+ \end{bmatrix},$$

$$\Phi_3 = (\Theta, \mathbf{F}_2) = (\Theta, \hat{P}\Phi_2) = (\Theta, \hat{P}\Phi_1) = (\Theta, \hat{P}^2\mathbf{E}) ,$$

$$\mathbf{F}_3 = \hat{P}\Phi_3 = \hat{P}(\Theta, \mathbf{F}_2) = \hat{P}\mathbf{F}_2 = \hat{P}(\Theta, \hat{P}^2\mathbf{E}) = \hat{P}^3\mathbf{E} .$$

It can clearly be seen that two successive approximations are related by the recurrent formula

$$\Phi_n = (\Theta, \hat{P}\Phi_{n-1}) ,$$

which comprises the matrix operator describing the single passage through the interface $z = h$. Thus, the two–component vector specifies the source at the interface $z = h$ in the problem for the nth approximation

$$\mathbf{F}_{n-1} = \hat{P}\Phi_{n-1} = \begin{bmatrix} \hat{R}_1 & \hat{T}_{21} \\ \hat{T}_{12} & \hat{R}_2 \end{bmatrix} \begin{bmatrix} \Phi_{a\,n-1}^+(z = h, s) \\ \Phi_{oc\,n-1}^-(z = h, s) \end{bmatrix} =$$

$$= \begin{bmatrix} \hat{R}_1\Phi_{a\,n-1}^+ + \hat{T}_{21}\Phi_{oc\,n-1}^- \\ \hat{R}_2\Phi_{oc\,n-1}^- + \hat{T}_{12}\Phi_{a\,n-1}^+ \end{bmatrix}.$$

Let for $n \ge 2$

$$\Phi_n = (\Theta, \hat{P}^{\,n-1}\mathbf{E}) .$$

Then for the $(n + 1)$th approximation the problem

$$\begin{cases} \hat{K}_z\Phi_{n+1} = 0 , \quad \Phi_{n+1}\big|_{\Gamma_0} = 0 , \quad \Phi_{n+1}\big|_{\Gamma_H} = 0 , \\ \Phi_{n+1}\big|_{\Gamma_h^+} = \hat{R}_2\Phi_n^- + \hat{T}_{12}\Phi_n^+ , \\ \Phi_{n+1}\big|_{\Gamma_h^-} = \hat{R}_1\Phi_n^+ + \hat{T}_{21}\Phi_n^- \end{cases}$$

separates into two problems: for the layer $z \in [0, h]$

$$\begin{cases} \hat{K}_z\Phi_{a\,n+1} = 0 , \quad \Phi_{a\,n+1}\big|_{\Gamma_0} = 0 , \\ \Phi_{a\,n+1}\big|_{\Gamma_h^-} = \hat{R}_1\Phi_{a\,n}^+ + \hat{T}_{21}\Phi_{oc\,n}^- \end{cases}$$

and for the layer $z \in [h, H]$

$$\begin{cases} \hat{K}_z\Phi_{oc\,n+1} = 0 , \quad \Phi_{oc\,n+1}\big|_{\Gamma_H} = 0 , \\ \Phi_{oc\,n+1}\big|_{\Gamma_h^+} = \hat{R}_2\Phi_{oc\,n}^- + \hat{T}_{12}\Phi_{a\,n}^+ , \end{cases}$$

and the two–component solution is obtained in the form of linear functionals

$$\Phi_{a\,n+1}(z, s) = (\theta_a, \hat{R}_1\Phi_{a\,n}^+ + \hat{T}_{21}\Phi_{oc\,n}^-) , \; z \in [0, h] ,$$

$$\Phi_{oc\,n+1}(z, s) = (\theta_{oc}, \hat{R}_2\Phi_{oc\,n}^- + \hat{T}_{12}\Phi_{a\,n}^+ , \; z \in [h, H] ,$$

or in the vector form

$$\Phi_{n+1} = (\Theta, \hat{P}_n\Phi_n) = (\Theta, \hat{P}(\Theta, \hat{P}^{n-1}\mathbf{E})) = (\Theta, \hat{P}^n\mathbf{E}) .$$

So, for $n \geq 1$ ($\mathbf{F}_0 \equiv \mathbf{E}$)

$$\mathbf{F}_n = \hat{P}\mathbf{F}_{n-1}) = \hat{P}\Phi_n = \hat{P}^n\mathbf{E} ,$$

$$\Phi_n = (\Theta, \mathbf{F}_{n-1}) = (\Theta, \hat{P}\Phi_{n-1}) = (\Theta, \hat{P}^{n-1}\mathbf{E}) ,$$

$$\Phi = \sum_{n=1}^{\infty} \Phi_n = (\Theta, \mathbf{E}) + \sum_{n=2}^{\infty} (\Theta, \hat{P}^{n-1}\mathbf{E}) = (\Theta, \mathbf{E}) +$$

$$+ \left(\Theta, \sum_{n=2}^{\infty} \hat{\boldsymbol{P}}^{n-1}\mathbf{E}\right) = (\Theta, \mathbf{E}) + \left(\Theta, \sum_{n=1}^{\infty} \hat{\boldsymbol{P}}^{n}\mathbf{E}\right) =$$

$$= \left(\Theta, \sum_{n=0}^{\infty} \hat{\boldsymbol{P}}^{n}\mathbf{E}\right) = (\Theta, \hat{Z}\mathbf{E}) ,$$

where

$$\hat{Z} \equiv \sum_{n=0}^{\infty} \hat{\boldsymbol{P}}^n .$$

Thus, the terms of the parametric series are the terms of the Neumann series in multiplicity of radiation passage through the interface.

By separating out a background component $\Phi^0 + \Phi_a$, caused by radiation propagation only in the atmosphere, from the total radiation of the system, the contribution of the ocean influence can be described by boundary–value problem (11) with two nonzero sources

$$E_a = \hat{R}_1(\Phi^0 + \Phi_a^+) , \quad E_{oc} = \hat{T}_{12}(\Phi^0 + \Phi_a^+) .$$

Before calculation of functionals of any approximation, we must calculate the expressions

$$[\hat{R}_1\theta_a](s, s^-) , \hat{T}_{12}\,\theta_a](s, s^-) , [\hat{T}_{21}\theta_{oc}](s, s^+) , \text{and } [\hat{R}_2\theta_{oc}](s, s^+)$$

in discrete or analytical form as functions of the direction s and the parameters s^- and s^+.

ON ACCOUNT OF THE CONTRIBUTION OF REFLECTING OCEAN BOTTOM

At the lower boundary of the system ($z = H$) the law of reflection is prescribed by the operator

$$[\hat{R}_H\Phi] = \frac{1}{\pi} \int_{\Omega^+} \Phi(z = H, s')\mu' \mathrm{d}s'$$

for the Lambertian orthotropic surface or by the operator

$$[\hat{R}_H\Phi](s) = \int_{\Omega^+} \Phi(z = H, s')\eta(s, s') \, \mathrm{d}s'$$

for the nonorthotropic (e.g., Fresnel) surface.

The solution of the problem for the illumination produced by the ocean bottom located at the altitude $z = H$

$$\begin{cases} \hat{K}_z\Phi_q = 0 , \quad \Phi_q\big|_{\Gamma_0} = 0 , \quad \Phi_q\big|_{\Gamma_H} = q\hat{R}_H\Phi_q + qE_H , \\ \Phi_q\big|_{\Gamma_{h^+}} = (\hat{R}_2\Phi_q^- + \hat{T}_{12}\Phi_q^+)\varepsilon , \\ \Phi_q\big|_{\Gamma_{h^-}} = (\hat{R}_1\Phi_q^+ + \hat{T}_{21}\Phi_q^-)\varepsilon \end{cases}$$

is found in the form of a parametric perturbation series

$$\Phi_q(z, s) = \sum_{k=1}^{\infty} \varepsilon^k \Phi_{qk} .$$

In the zeroth approximation ($k = 0$) the radiation field is formed due to the illumination E_H

$$\begin{cases} \hat{K}_z\Phi_{q0} = 0 , \quad \Phi_{q0}\big|_{\Gamma_0} = 0 , \\ \Phi_{q0}\big|_{\Gamma_H} = q\hat{R}_H\Phi_{q0} + qE_H , \\ \Phi_{q0}\big|_{\Gamma_{h^+}} = 0 , \quad \Phi_{q0}\big|_{\Gamma_{h^-}} = 0 \end{cases}$$

and is considered only in the ocean ($z \in [h, H]$). Taking into account the n–fold interaction with the interface we can obtain the problem for the system ($z \in [0, H]$)

$$\begin{cases} \hat{K}_z\Phi_{qn} = 0 , \quad \Phi_{qn}\big|_{\Gamma_0} = 0 , \quad \Phi_{qn}\big|_{\Gamma_H} = q\hat{R}_H\Phi_{qn} , \\ \Phi_{qn}\big|_{\Gamma_{h^+}} = \hat{R}_2\Phi_{q\,n-1}^- + \hat{T}_{12}\Phi_{q\,n-1}^+ , \\ \Phi_{qn}\big|_{\Gamma_{h^-}} = \hat{R}_1\Phi_{q\,n-1}^+ + \hat{T}_{21}\Phi_{q\,n-1}^- , \end{cases} \quad (18)$$

which can be separated into individual problems for the layers $z \in [0, h]$ and $z \in [h, H]$. Let us introduce the superposition of radiation components produced by the rereflection from the ocean bottom and passage through the interface

$$\Phi_{qn} = \Phi_{qn}^q + \Phi_{qn}^0 .$$

Then we obtain the problem for the system $z \in [0, H]$

$$\begin{cases} \hat{K}_z\Phi_{qn}^0 = 0 , \quad \Phi_{qn}^0\big|_{\Gamma_0} = 0 , \quad \Phi_{qn}^0\big|_{\Gamma_H} = 0 , \\ \Phi_{qn}^0\big|_{\Gamma_{h^+}} = \hat{R}_2\Phi_{q\,n-1}^- + \hat{T}_{12}\Phi_{q\,n-1}^+ , \\ \Phi_{qn}^0\big|_{\Gamma_{h^-}} = \hat{R}_1\Phi_{q\,n-1}^+ + \hat{T}_{21}\Phi_{q\,n-1}^- \end{cases} \quad (19)$$

and for the layer $z \in [h, H]$

$$\{\hat{K}_z\Phi_{qn}^q = 0 , \quad \Phi_{qn}^q\big|_{\Gamma_{h^+}} = 0 , \quad \Phi_{qn}^q\big|_{\Gamma_H} = q\hat{R}_H\Phi_{qn}^q + q\hat{R}_H\Phi_{qn}^0 . \quad (20)$$

The solution of problem (19) is reduced to the solution of two problems: for the layer $z \in [0, h]$

$$\{\hat{K}_z\Phi_{qna}^0 = 0 , \quad \Phi_{qna}^0\big|_{\Gamma_0} = 0 , \quad \Phi_{qna}^0\big|_{\Gamma_{h^-}} = E_{a\,n-1}$$

with the source

$$E_{\text{a}\, n-1} = \hat{R}_1 \Phi^+_{q\, n-1} + \hat{T}_{21} \Phi^-_{q\, n-1}$$

and for the layer $z \in [h, H]$

$$\{\hat{K}_z \Phi^0_{qn\text{oc}} = 0\ ,\ \ \Phi^0_{qn\text{oc}}\Big|_{\Gamma_H} = 0\ ,\ \ \Phi^0_{qn\text{oc}}\Big|_{\Gamma_{h^+}} = E_{\text{oc}\, n-1}$$

with the source

$$E_{\text{oc}\, n-1} = \hat{R}_2 \Phi^-_{q\, n-1} + \hat{T}_{12} \Phi^+_{q\, n-1}\ .$$

Using the obtained results we find

$$\Phi^0_{qn\text{a}} = (\theta_\text{a},\ E_{\text{a}\, n-1})\ ,\ \ \Phi^0_{qn\text{oc}} = (\theta_\text{oc},\ E_{\text{oc}\, n-1})\ .$$

Problem (20) is the problem for the layer with the reflecting Lambertian or nonorthotropic surface[1–4] and its solution can be written in the form of the functional

$$\Phi^0_{qn}(z,\ s) = \frac{q}{2\pi} \int\limits_{\Omega^-} \theta_H(z,\ s,\ s_H)\, \text{d}s_H \times$$

$$\times \Bigg\{ E^0_{qn}(s_H) + \frac{q}{2\pi} \int\limits_{\Omega^-} [\hat{R}\theta_H](s_H,\ s_1)\, E^0_{qn}(s_1)\, \text{d}s_1 +$$

$$+ \sum_{k=3}^{\infty} \frac{q^{\,k-1}}{(2\pi)^{k-1}} \int\limits_{\Omega^-} [\hat{R}\theta_H](s_H,\ s_1)\, \text{d}s_1 \ldots$$

$$\ldots \int\limits_{\Omega^-} [\hat{R}\theta_H](s_{k-3},\ s_{k-2})\, \text{d}s_{k-2} \times$$

$$\times \int\limits_{\Omega^-} [\hat{R}\theta_H](s_{k-2},\ s_{k-1})\, E^0_{qn}(s_{k-1})\, \text{d}s_{k-1} \Bigg\}\ .$$

This explicit expression determines the relation between the illumination $E^0_{qn} \equiv \hat{R}_H \Phi^0_{qn}$ and the law of reflection R_H in terms of the influence function of the ocean $\theta_H(z,\ s,\ s_H)$ being the solution of the problem.

$$\{\hat{K}\theta_H = 0\ ,\ \ \theta_H\Big|_{\Gamma_{h^+}} = 0\ ,\ \ \theta_H\Big|_{\Gamma_H} = \delta(s - s_H)\ .$$

This method of reduction of the boundary–value problem for two media with the interface to two boundary–value problems for each medium separately provides more comprehensive study of the radiation transfer in such a complex system as atmosphere–ocean. The constructed models of the influence functions of the atmosphere and ocean are general–purpose and invariant under the properties of the boundaries. New formulation of the optical transfer operator of the atmosphere–ocean system turns out to be effective for the problems of remote sensing. The results presented here for a horizontally homogeneous problem are generalized for a three–dimensional problem with horizontal inhomogeneity of the interface and will be published in future.

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