FORMATION OF THE LATERAL SHEAR INTERFEROGRAMS WITH DIFFUSELY SCATTERED LIGHT FIELDS BASED ON THREE–EXPOSURE RECORDING OF A LENS FOURIER HOLOGRAM

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A shear interferometer is analyzed based on three–exposure recording of a lens Fourier hologram of a mat screen. It is shown that the spatial filtering in the hologram plane enables checking of wave aberrations of a lens or objective over the field to be done.

In the classical interferometry it is shown that a three–beam interferogram formed with diffraction gratings[1] with shifted wave fronts gives rise to moire bands the equation for which has a power by two units lower than that of a polynomial of wave aberrations. This makes it possible to readily and accurately find the coefficients specifying wave aberrations.[2] A method for obtaining three–exposure interferograms with moire bands to check the wave front is described based on three–exposure recordings of Fresnel holograms of a mat screen when it is illuminated with radiation with a quasiplanar wave front by superimposing the objective speckle–fields corresponding to the three exposures.

This paper considers the method of three–exposure recording of a lens Fourier hologram of a mat screen for checking wave aberration of a converging lens or an objective over the field.

According to Fig. 1*a* the mat screen *1* which lies in the plane (x_1, y_1) is illuminated by radiation with an aberrationless diverging spherical wave of the radius of curvature R which is formed with the lens L_0 and a circular point hole p_0 in the mat screen at its focus. In the plane (x_2, y_2) of the photographic plate *2* the Fourier transform of a mat screen is formed with the lens L_1 located immediately behind the mat screen when the condition[4] $R = f_1 l/(l - f_1)$ is fulfilled. Here f_1 is the focal length of a lens L_1 under control and l is the distance between the planes (x_1, y_1) and (x_2, y_2). In the plane of the photographic plate the recording of the Fourier hologram takes place during the first exposure using a diverging spherical reference wave of a radius of curvature $r = l$. Prior to the second exposure the mat screen and the lens L_r attached to the same shifting mechanism are displaced in the direction perpendicular to the optical axis, e.g., along the x axis by amount a. Prior to the third exposure they are displaced symmetrically to the optical axis by the same amount. At the reconstruction stage the hologram is illuminated with a small–aperture laser beam (Fig. 1*b*) at an angle $\theta = \arctan b/l$ with respect to the normal to the plane of the photographic plate, where b is the distance from the optical axis to the focal point of the lens L_r (Fig. 1*a*). An interference pattern is recorded in the focal plane (x_3, y_3) of the lens L_2 with focal length f_2.

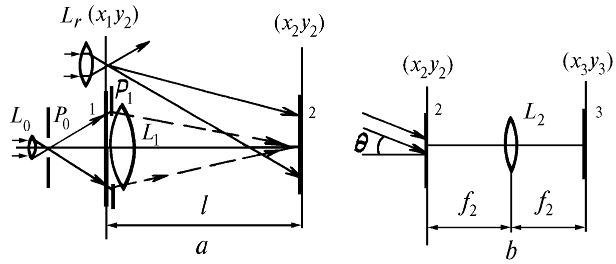

FIG. 1. The scheme for recording (a) and reconstructing (b) a three–exposure lens Fourier hologram: 1) mat screen; 2) photographic plate–hologram; 3) recording plane of the interference pattern; L_0, L_r, L_1, and L_2 are lenses; p_0 is a spatial filter; and p_1 is aperture diaphragm.

Based on Ref. 4 the complex amplitudes of fields reconstructed in the plane (x_2, y_2) at three exposures within a laser beam aperture characterized by the function $P_2(x_2, y_2)$ (see Ref. 5) can be represented as

$$u_0(x_2, y_2) \sim p_2(x_2, y_2)\,\{F[kx_2/l,\, ky_2/l] \otimes P_1(x_2, y_2)\}\,, \qquad (1)$$

$$u_{1,2}(x_2,y_2) \sim p_2(x_2,y_2)\{F[kx_2/l, ky_2/l] \otimes \exp(\mp ikax_2/l) P_1(x_2,y_2)\}. \qquad (2)$$

Here $\otimes$ is the symbol of convolution operation, k is the wave number,

$$F[kx_2/l, ky_2/l] = \int\limits_{-\infty}^{\infty}\int t(x_1,y_1)\exp[-ik(x_1 x_2 + y_1 y_2)/l] \times \mathrm{d}x_1 \mathrm{d}y_1$$

is the Fourier transform of the complex amplitude of the mat screen transmittance $t(x_1, y_1)$ which is a random function of coordinates, $P_1(x_2, y_2) = \int\limits_{-\infty}^{\infty}\int p_1(x_1, y_1) \exp[\,i\varphi(x_1, y_1) \times$
$\times \exp[-ik(x_1 x_2 + y_1 y_2)/l]\, \mathrm{d}x_1 \mathrm{d}y_1$ is the Fourier transform of the generalized pupil function $p_1(x_1, y_1) \exp i\varphi(x_1, y_1)$ of the lens L_1 under control (Fig. 1*a*) which takes into account its axial wave aberrations.

When deriving relations (1) and (2) it was assumed that the hologram is reconstructed at a point lying on the optical axis. Then in the range of spatial filtering the subjective speckle–fields of three exposures represented by relations (1) and (2) with corresponding angles between them, coincide. Moreover, the information about phase distortions produced in the light wave by a lens L_1 under control (Fig. 1*a*) is contained within an individual subjective speckle in the hologram plane. The amplitude–phase distribution of the field within this speckle is a result of the diffraction of a plane wave propagating along the optical axis on the pupil of the lens L_1.

When the Fourier transform is performed with the lens L_2 (Fig. 1*b*) the diffraction field in the plane (x_3, y_3) is

$$u(x_3, y_3) \sim \{t(-\mu x_3, -\mu y_3)\,[\,p_1(-\mu x_3, -\mu y_3) \times \exp i\varphi(-\mu x_3, -\mu y_3) + p_1(-\mu x_3 - a, -\mu y_3) \times \exp i\varphi(-\mu x_3 - a, -\mu y_3) + p_1(-\mu x_3 + a, -\mu y_3) \times \exp i\varphi(-\mu x_3 + a, -\mu y_3)]\} \otimes P_2(x_3, y_3)\,, \quad (3)$$

where $\mu = l/f_2$;

$$P_2(x_3, y_3) = \int_{-\infty}^{\infty}\int p_2(x_2, y_2) \times \exp[-ik(x_2 x_3 + y_2 y_3)/f_2]\,\mathrm{d}x_2\mathrm{d}y_2$$

in the range, where images of the pupil of the lens L_1 overlap, is a superposition of the identical speckle–fields of three exposures. And the superposition of correlating speckle–fields results in the distribution of illumination

$$I(x_3, y_3) \sim \left\{1 + 4\cos\left[\varphi(-\mu x_3, -\mu y_3) - \frac{\varphi(-\mu x_3 + a, -\mu y_3) + \varphi(-\mu x_3 - a, -\mu y_3)}{2}\right] \times \cos\left[\frac{\varphi(-\mu x_3 + a, -\mu y_3) - \varphi(-\mu x_3 - a, -\mu y_3)}{2}\right] + 4\cos^2\left[\frac{\varphi(-\mu x_3 + a, -\mu y_3) - \varphi(-\mu x_3 - a, -\mu y_3)}{2}\right]\right\} \times \left|\,t(-\mu x_3, -\mu y_3) \otimes P_2(x_3, y_3)\right|^2. \quad (4)$$

Expression (4) describes a speckle–structure modulated by interference fringes of the lateral shear. The points of intersection of their maxima form the moire bands. If one neglects the scaling transformation for the first–order spherical aberrations, the equation describing the system of moire bands takes the form $[\partial^2\varphi(x_3, y_3)/\partial x_3^2]a = A(12x^2 + y^2)a = n\lambda$, where A is the coefficient of spherical aberration, n is the order of the interference band, and λ is the wavelength of a coherent light used for recording and reconstructing a hologram. This equation is quadratic and the shape of bands is a system of ellipses with the ratio of large to small axes equal to $\sqrt{3}$. Their large axes are parallel with the y axis.

When the hologram is displaced with respect to a small–aperture laser beam which reconstructs it along the direction of the shift axis the amplitude–phase distribution of the field within the subjective speckle in the vicinity of a point with coordinates x_2, $y_2 = 0$ is a result of diffraction of a plane wave propagating at an angle x_2/l with respect to the optical axis on the pupil of the lens L_1. Hence, an off–optical axis spatial filtration results in an interference pattern which specifies a combination of a spherical aberration and a coma. The equation describing the system of moire bands in this case takes the form $A(12x_3^2 + 4y_3^2)a + 6B\xi x_3 a = n\lambda$, where $\xi = x_2/\lambda l$ is the spatial frequency and B is the coefficient of the off–axis aberration of the coma type. The shape of moire bands also represents a system of ellipses but the position of their center is displaced along the shift axis by a value that depends on B, coordinate of a point, where the hologram is reconstructed, and on the value a of the shift.

In the experiment, required illumination was produced by a He–Ne laser at a wavelength of 0.63 μm. The three–exposure Fourier hologram of the mat screen was recorded using a lens with 90 mm focal length and 52 mm diameter for $R = 145$ mm, $l = 240$ mm, and $a = 0.6 \pm 0.002$ mm. The accuracy Δa of the shift prior to the exposures satisfied the condition $\Delta a \le \lambda l/d$ which follows from the requirement that the wave phase change within the pupil of the lens under control be not larger than π.

Depicted in Fig. 2 is a three–exposure shear interferogram recorded during spatial filtering on the optical axis performed by reconstructing the hologram using a small–aperture (≈ 2 mm in diameter) laser beam. The interference pattern characterizes the axis aberrations of a lens under control. The shear interferogram shown in Fig. 2*b* is related to the case of the hologram reconstruction at a point lying 8 mm aside from the optical axis. The moire bands describe a combination of a spherical aberration and a coma.

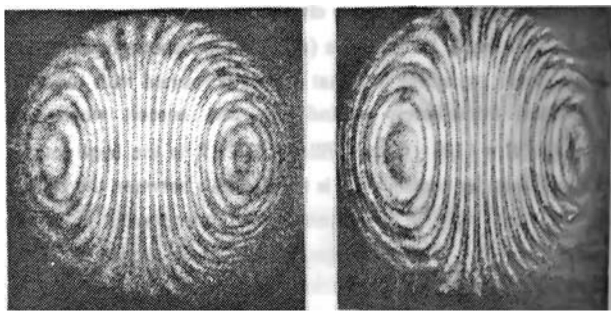

FIG. 2. Shear interferograms recorded during spatial filtering in the plane of the hologram: a) on the optical axis and b) off the optical axis.

Thus the three–exposure recording of a lens Fourier hologram of a mat screen results in the formation of moire bands which characterize the wave aberrations of a lens under control. Spatial filtration enables one to separate out the interference patterns corresponding to both spherical aberration and a combination of spherical and off–axis wave aberrations of the coma type. It should be noted that the three–exposure recording of the hologram based on superimposing of the subjective speckle of three exposures can be accomplished by displacing a mat screen and a lens under control with the help of one and the same mechanism[6] or by displacing the mat screen and the photographic plate.[7]

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