ON LIDAR RETURN POWER FROM A RANDOMLY ROUGH SURFACE WITH A COMBINED SCATTERING PHASE FUNCTION SOUNDED THROUGH THE ATMOSPHERE

M.L. Belov

All–Russian Scientific–Research Institute of Marine Fisheries and Oceanography, Moscow the Firm "EKKOS", Moscow

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Investigation presented in this paper concerns the lidar return power recorded in a bistatic optical arrangement from a rough surface with a combined scattering phase function of locally plane elements. An equation for that return power is derived for the case of sounding through an optically dense aerosol atmosphere of a surface with the scattering phase function including diffuse and quasispecular components. It is shown that the return power essentially depends on the diffuse–to–specular components ratio and on the roughness characteristics of the surface as well.

Power recorded by a lidar when sounding a plane surface with a combined scattering phase function was studied in Ref. 1. Power recorded by a laser sounding system in a bistatic arrangement of sounding (when the source and the receiver are spaced) of a rough surface with a combined scattering phase function of locally plane elementary portions (Fig. 1) is considered below.

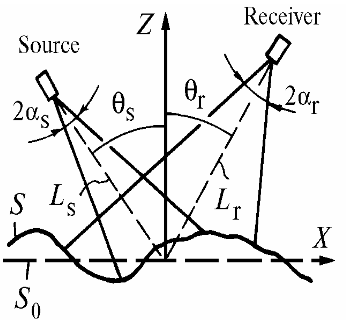

FIG. 1. Optical arrangement of sounding.

Let us assume that each locally plane element of the surface being sounded has a combined scattering phase function having the quasispecular and diffuse components.[1] The brightness $I(\mathbf{R}, \mathbf{m})$ of the radiation reflected from the elementary locally plane surface is equal to[1]

$$I(\mathbf{R}, \mathbf{m}) = \frac{E(\mathbf{R})}{\alpha \dfrac{2\pi}{n+2} + \beta\pi\Delta^2} \times$$

$$\times \left[\alpha \cos^n\theta + \beta \exp\left\{ -\frac{(\theta - \theta_0)^2\cos^2\theta_0 + (\varphi - \varphi_0)^2\sin^2\theta_0}{\Delta^2} \right\} \right], \quad (1)$$

where $E(\mathbf{R}) = AE_s(\mathbf{R})$, $E_s(\mathbf{R})$ is the irradiance of the surface produced by radiation coming from a source, A is the reflection coefficient of a local area, $\mathbf{R}$ is the spatial coordinate of an elementary scattering area, α and β are the coefficients determining the fractions of the diffusion and quasispecular reflection, (θ, θ_0) and (φ, φ_0) are the zenith and azimuthal angles of the observational direction and that of the reflected radiation maximum (the quasispecular component of the reflection) in the local system of coordinates related to an elementary reflecting surface. The angles θ_0 and φ_0 are related to the corresponding angles θ_S and φ_S, which characterize the direction of incident radiation, by the laws of geometric optics. Here n is the parameter characterizing the angular width of the scattering phase function of the diffusion component of reflection and Δ is the parameter characterizing the angular width of the scattering phase function of the quasispecular component of reflection. Formula (1) was derived for $\Delta \ll 1$.

Expression for the brightness of radiation arriving at the receiver and the integral relation for the power recorded by the receiver can be derived from the distribution of the brightness $I(\mathbf{R}, \mathbf{m})$ over the scattering surface S (we assume the shading produced by surface elements is negligible)[1]

$$P = \int \mathrm{d}\mathbf{R} \int \mathrm{d}\Omega(\mathbf{m}) \cos\theta_s\, I(\mathbf{R}, \mathbf{m})\, I_r(\mathbf{R}, \mathbf{m}) , \quad (2)$$

where $I_r(\mathbf{R}, \mathbf{m})$ is the brightness of radiation incident from a virtual source (with the parameters of the receiver) on the surface S at the point R and θ_s is the angle between the normal to the surface S at the point R and the direction towards the receiver.

In the case of a homogeneous scattering atmosphere with a strongly forward–peaked scattering phase function, when the angle, at which the received aperture is observed from the points on the scattering surface, is much smaller than the angular width of the quasispecular component of the scattering phase function and the characteristic scale of the surface slope variations and the field of view angles of the receiver, the expression for the power recorded by the receiver can take the form (in the small angle approximation for the source and receiver directional patterns let us assume that a source, a receiver, and their optical axes are in the plane XOZ, and then pass from integrating over the rough surface S to integrating over its projection S_0 onto the plane $z = 0$, and use the results from Refs. 1–5)

$$P = \frac{A}{\pi} \frac{1}{\alpha \dfrac{2}{n+2} + \beta\Delta^2} \left[\alpha \int_{S_0} \frac{\mathrm{d}^2R_0}{n_z} \cos^n\theta_s\, E_s(\mathbf{R}'_{0\,\xi})\, E_r(\mathbf{R}'_{0\,\xi}) + \right.$$

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$$+ \beta \int\limits_{S_0} \frac{d^2R_0}{n_z} E_s(\mathbf{R}'_{0\xi}) E_r(\mathbf{R}''_{0\xi}) \exp\left\{ -\frac{K_y^2}{\Delta^2}(C - \gamma_x D)^2 - \right.$$

$$\left.\left. - \frac{K_x^2}{\Delta^2}\left[R_{0y}\, s - \gamma_y K_y (C\gamma_x + D)\right]^2 \right\}\right], \qquad (3)$$

where

$\mathbf{R}'_{0\xi} = \{(R_{0x}\cot\theta_s - \zeta(R_0))\sin\theta_s,\ R_{0y}\}$;

$\mathbf{R}''_{0\xi} = \{(R_{0x}\cot\theta_r - \zeta(R_0))\sin\theta_r,\ R_{0y}\}$;

$$K_y = \frac{n_z}{\sqrt{1 - n_z^2\gamma_y^2}}\ ;\ K_x = \frac{n_z}{\sqrt{1 - n_z^2\gamma_x^2}}\ ;\ s = \frac{A_r}{B_r} + \frac{A_s}{B_s}\ ;$$

$$C = (\sin\theta_s - \sin\theta_r) + R_{0x}\, t\ ;\ t = \frac{A_r\cos^2\theta_r}{B_r} + \frac{A_s\cos^2\theta_s}{B_s}\ ;$$

$$D = \sqrt{1 - \left(\sin\theta_r - \frac{R_{0x}A_r\cos^2\theta_r}{B_r}\right)^2 - \left(\frac{A_r R_{0y}}{B_r}\right)^2} +$$

$$+ \sqrt{1 - \left(\sin\theta_s + \frac{R_{0x}A_s\cos^2\theta_s}{B_s}\right)^2 - \left(\frac{A_s R_{0y}}{B_s}\right)^2}\ ;$$

$$A_{s,r} = 0.5\left(\alpha_{s,r}^2 + \sigma L_{s,r}\langle\gamma^2\rangle\right)^{1/2}\ ;$$

$$B_{s,r} = \frac{0.5L_{s,r}\left(\alpha_{s,r}^2 + 0.5\,\sigma L_{s,r}\langle\gamma^2\rangle\right)}{\left(\alpha_{s,r}^2 + \sigma L_{s,r}\langle\gamma^2\rangle\right)^{1/2}}\ ;$$

$E_s(\mathbf{R})$ and $E_r(\mathbf{R})$ are the irradiances on the surface S produced by the radiation incident on the surface S from a real and virtual sources in the atmosphere, respectively,[2,3] L_s and L_r are the distances from the source and receiver to the surface; $2\alpha_s$ and $2\alpha_r$ are the divergence angles of the source and the field of view angle of the receiver, σ is the scattering coefficient of the atmosphere; $\langle\gamma^2\rangle$ is the variance of the deflection angle appearing during an elementary scattering act, ζ and $\gamma = \{\gamma_x, \gamma_y\}$ are the height and the vector of slopes of a rough surface, $\mathbf{n} = \{n_x, n_y, n_z\}$ is the unit vector of the normal to the elementary surface, and θ_s and θ_r are the angles between the normal to the surface S_0 and the direction towards the source and receiver, respectively.

When the heights and slopes of the rough surface S vanish formula (3) becomes an expression for the power received from the plane surface with a combined scattering phase function.[1]

Assuming the distribution of the heights and slopes of the surface S to be Gaussian and averaging Eq. (3) over ζ and γ we can derive the expression for $\overline{P}$, i.e., the average (over the ensemble of surfaces) power recorded by the receiver (we assume also that the surface S is smoothly rough, $\gamma_x^2, \gamma_y^2 \ll 1$)

$$\overline{P} \simeq \frac{A}{\pi}\frac{\nu^{-1/2}}{\alpha\dfrac{2}{n+2} + \beta\Delta^2}\left[\alpha F(\gamma_0) \int\limits_{S_0} d^2R_0\, E_s(\mathbf{R}'_0)\ \times\right.$$

$$\times\ E_r(\mathbf{R}''_0)\exp\{ER_{0,x}^2\} + \beta\frac{G}{\mu}\int\limits_{S_0} d^2R_0\, E_s(\mathbf{R}'_0)\, E_r(\mathbf{R}''_0)\ \times$$

$$\left.\times \exp\left\{ER_{0,x}^2 - \frac{R_{0,y}^2 s^2}{\Delta^2\mu} - \frac{1}{\Delta^2\mu}(q_x + R_{0x}\, t)^2\right\}\right], \qquad (4)$$

where

$$\nu = 1 + 2\sigma_0^2\left(\frac{\sin^2\theta_s}{4B_s^2} + \frac{\sin^2\theta_s}{4B_s^2}\right);$$

$$E = \frac{2\sigma_0^2}{\nu}\left[\frac{\sin\theta_s\cos\theta_s}{4B_s^2} + \frac{\sin\theta_r\cos\theta_r}{4B_r^2}\right]^2;$$

$$F(\gamma_0) = m_{rz}^n\,(2\gamma_0^2)^{-n/4}\exp\left(\frac{1}{4\gamma_0^2}\right)\times$$

$$\times\left[(2\gamma_0^2)^{-1/4}W_{-\frac{(n+1)}{4},\,-\frac{(n-1)}{4}}\left(\frac{1}{2\gamma_0^2}\right) + \right.$$

$$\left. + \frac{m_{sx}\, m_{rx}}{2m_{rz}\, m_{sz}}(n+1)(2\gamma_0^2)^{-1/4}W_{-\frac{(n+3)}{4},\,-\frac{(n-3)}{4}}\left(\frac{1}{2\gamma_0^2}\right)\right];$$

$$\mu = 1 + \frac{2\gamma_0^2\, q_z^2}{\Delta^2}\ ;\ \ G = \frac{(m_{sz} - \gamma_{mx}m_{sx})(m_{rz} - \gamma_{mx}m_{rx})}{m_{sz}\, m_{rz}}\ ;$$

$$\gamma_{mx} = -\frac{q_x\, q_z}{\dfrac{\Delta^2}{2\gamma_0^2} + q_z^2}\ ;\ \ q_z = -(\cos\theta_s + \cos\theta_r)\ ;$$

$$q_x = (\sin\theta_s - \sin\theta_r)\ ;$$

$$R'_0 = \{R_{0x}\cos\theta_s,\ R_{0y}\}\ ;\ R''_0 = \{R_{0x}\cos\theta_r,\ R_{0y}\}\ ,$$

σ_0^2 and γ_0^2 are the variances of the heights and slopes of the randomly rough surface S, $\mathbf{m}_s = \{m_{sx}, m_{sz}\}$ and $\mathbf{m}_r = \{m_{rx}, m_{rz}\}$ are the unit vectors indicating the direction of the radiation incident on the surface and the direction from the surface towards the receiver, $W_{n,m}(x)$ is the Whittaker function.

By calculating the integrals entering into Eq. (4) for the average power received when a randomly rough surface with a combined local scattering phase function is sounded, we derive the following analytical expression:

$$\overline{P} \simeq \frac{A\,\nu^{-1/2}}{\alpha\dfrac{2}{n+2} + \beta\Delta^2}\cdot\frac{r_r^2\alpha_r^2\, m_{rz}\, m_{sz}\, P_0\exp\{-(\varepsilon - \sigma)(L_s + L_r)\}}{16\,B_s^2\,B_r^2}\times$$

$$\times\left[\alpha F(\gamma_0)\, p^{-1/2}(q - E)^{-1/2} + \beta\frac{G}{\mu}\left(p + \frac{s^2}{\Delta^2\mu}\right)^{-1/2} H^{-1/2}\ \times\right.$$

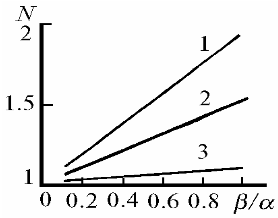
$$\left.\times\exp\left\{-\frac{q_x^2}{\Delta^2\mu}\left(1 - \frac{t^2}{\Delta^2\mu H}\right)\right\}\right], \qquad (5)$$

where $H = q - E + \dfrac{t^2}{\Delta^2\mu}$; $p = \dfrac{1}{4B_s^2} + \dfrac{1}{4B_r^2}$; $q = \dfrac{\cos^2\theta_r}{4B_r^2} + \dfrac{\cos^2\theta_s}{4B_s^2}$;

P_0 is the power emitted by a source, r_r is the effective radius of the received aperture, and ε is the extinction coefficient of the atmosphere.

For σ_0 and as $\gamma_0 \to 0$ formula (5) is identical to the formula for the power received for the case of sounding of a plane surface with a combined scattering phase function.[1] For $\beta = 0$, $n = 0$, $\langle\gamma^2\rangle = 0$, and $\sigma = 0$ formula (5) becomes an expression for the average power received from a randomly rough, locally Lambertian surface in a transparent atmosphere.[6]

For $\alpha = 0$, $\Delta \to 0$, $\sigma = 0$, and $<\gamma^2> = 0$, formula (5) becomes an expression for the average power received from the randomly rough locally specular surface in a transparent atmosphere.[7]



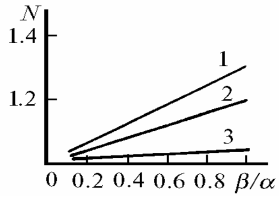


FIG. 2. Received radiation power as a function of the diffuse–to–specular components of the scattering phase function ratio for a surface in a transparent atmosphere.

FIG. 3. The same as in Fig. 2 but for an optically dense atmosphere.

Figures 2 and 3 show the dependences N (the ratio of the power $\overline{P}$ to the power $P(\beta = 0,\ n = 0,\ \sigma = 0,$ and $\gamma_0 = 0)$ calculated for a plane Lambertian surface) on the parameter β/α. Calculations were performed using formula (5) for the following values of the parameters: $L_s = L_r = 10^3$ m, $\alpha_s = 10^{-2}$, $\alpha_r = 10^{-1}$, $\Delta = 10^{-1}$, $n = 0$, $\theta_s = \theta_r = 0$, $\sigma<\gamma^2> = 0$ (Fig. 2); $\sigma<\gamma^2> = 10^{-5}$ m^{-1} (Fig. 3); for $\gamma_0^2 = 0$ and $\sigma_0^2 = 0$ (curve *1*), for $\gamma_0^2 = 10^{-3}$ and $\sigma_0^2 = 2$ m^2 (curve *2*), for $\gamma_0^2 = 10^{-2}$ and $\sigma_0^2 = 2$ m^2 (curve *3*).

The figures show that an increase of the fraction of the quasispecular component of the scattering phase function (the increase of the parameter β/α) leads to the increase in the received power. Virtually this can be explained by the fact that with increasing β/α the radiation reflected by the surface is to a greater degree concentrated in the vicinity of the specular reflection direction. Random roughness of the surface leads to weakening of this effect that is associated with the "spreading" of the quasispecular scattering phase function of the surface.

The atmospheric turbidity increase leads to the smoothing effects associated with the dependence of the scattering properties of a surface on the received power.

The results obtained in this paper could be useful in the analysis of remote sensing system functioning.

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