APPLICABILITY OF THE SIMPLEX METHOD TO DYNAMIC CORRECTION FOR THERMAL BLOOMING OF LIGHT BEAMS UNDER CONDITIONS OF FLUCTUATING MEDIUM PARAMETERS ALONG A PATH

I.V. Malafeeva, I.E. Tel'pukhovskii, and S.S. Chesnokov

M.V. Lomonosov State University, Moscow
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Applicability of the simplex search method to the problem of compensation for thermal blooming of laser radiation in a randomly inhomogeneous medium is studied using numerical simulations. An algorithm enabling one to improve the efficiency of the beam phase control is proposed. The algorithm is compared with the gradient technique. It is shown that the simplex search method is stable under conditions of moderate turbulence and fluctuating wind velocity.

The adaptive methods of light beam phase control[1] are now being widely used for compensating for light wave distortions caused by nonlinear refraction and turbulent fluctuations of the refractive index of the medium. Among these methods cross–aperture sounding has become of a wide use. Use of the cross–aperture sounding in combination with a gradient procedure for searching for the extremum in the irradiation of an object has been considered in Refs. 2 and 3 for the case of correcting for thermal defocusing of beams propagating through a medium with velocity pulsations. An effective control of a beam under conditions of high–frequency pulsations of the velocity is shown to be possible only with the help of rapid phase variations (sounding over unsteady parameters of a light field in the medium) what requires very high speed of operation of an adaptive system actuators.[3] In this connection for the control of light beams it is expedient to employ the methods which do not require calculations of the goal function gradient, and in particular, a simplex search method.[4] Comparative analysis of algorithms of compensating for stationary wind defocusing[5] reveals that the simplex method provides the highest rate of convergence of an iteration process of the phase optimization. In the regime of nonstationary wind refraction[6] a stepwise variation in controllable coordinates typical for the simplex method results in a forced scanning of the beam which improves the conditions of propagation.

The above said allows one to assume that the simplex method might be convenient for solving the problem of dynamic correction for distortions of a light beam propagating along the atmospheric path with pulsations of the wind velocity and fluctuations of the refractive index.

This paper concerns a numerical analysis of control of an intense light beam phase using an adaptive optical system. Nonstationary wind refraction in the turbulent atmosphere with random wind is considered. The efficiency of a simplex method of searching for maximum in the irradiation of an object in real time depending on the control base is studied. The comparison with the gradient method is made.

1. MODEL OF LIGHT BEAM PROPAGATION

When constructing a numerical model let us assume that a laser source delivers a single–mode beam of a Gaussian profile $E_0 = A_0 \exp(-(x^2 + y^2)/2a_0^2)$. The wave front of the beam controlled with a modal corrector (an elastic mirror) reflecting from which a collimated beam acquires the phase

$$U(x, y, t) = k \sum_{i=1}^{N} a_i(t)\, W_i(x, y) , \qquad (1)$$

where k is the wave number, a_i are the controllable coefficients, W_i are the basis modes, and N is the number of controllable coordinates. A complex amplitude of the light field E incident on the medium (in the plane $z = 0$) is determined as

$$E(x, y, 0, t) = E_0(x, y) \exp (iU(x, y, t)) . \qquad (2)$$

The goal function of the control is the focusing criterion J_f (see Ref. 1) calculated for the aperture of the radius a_0.

In the quasioptical approximation of the theory of diffraction the beam propagation in a weakly absorbing medium is described by the equation

$$2\, i\, k \frac{\partial E}{\partial z} = \Delta_{\perp} E + 2 \frac{k^2}{n_0} \left(\frac{\partial n}{\partial T} T + \tilde{n} \right) E , \qquad (3)$$

where $\tilde{n}$ is a random field describing natural fluctuations of the refractive index in the medium and $T = T(x, y, z, t)$ is the perturbation of the medium temperature along the path induced by a beam. For the latter the equation of heat transfer in a moving medium

$$\rho c_{\rm p} \left(\frac{\partial T}{\partial t} + (\mathbf{v}\nabla)\, T \right) = \alpha\, I\ , \ I = \frac{c n_0}{8\pi} E E^* \qquad (4)$$

is valid. Where ρ is the medium density, $c_{\rm p}$ is the specific heat, and $\mathbf{v}$ is the velocity of the medium motion, which is a random value in the atmosphere. Following the structure of the atmospheric turbulence,[7] $\mathbf{v}$ is assumed to have a constant component $\mathbf{v}_0$ and random pulsations δv_x, δv_y, and δv_z. Since the velocity components perpendicular to the OZ optical axis are decisive in the formation of a heat lens in the beam channel, v_0 is thought to be a velocity component lying in the plane XOY. If we assume the direction of the OX axis to be coincident with the mean velocity $\mathbf{v}_0$ of the medium motion we obtain that $\mathbf{v} = \{v_0 + \delta v_x, \delta v_y\}$.

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Standard deviations of the fluctuating components are considered to be of the same value $\sigma_{v_x} = \sigma_{v_y} = \sigma_v$. A quantitative measure of the thermal distortions is the nonlinearity parameter $R = R_0 v_0 / v$, where $R_0 = \dfrac{2k^2 a_0^3 \mathrm{a} I_0}{n_0\, \mathrm{r}\, c_\mathrm{p} v_0}$ $\dfrac{\partial n}{\partial T}$ is the value determined based on the mean velocity v_0 of the medium. The parameter R_0 is proportional to the total power $P_0 = \pi a_0^2 I_0$ and the mean time of the radiation interaction with the medium $\tau_v = a_0 / v_0$.

To simulate fluctuations of the refractive index of the medium $\tilde{n}$ the modal representation[8] is used which allows one to essentially widen the inhomogeneity spectrum into the low–frequency spatial range. In this case, in each plane $z = \mathrm{const}$, the perturbations $\tilde{n}$ are expanded into a series over the orthogonal Zernike polynomials Z_i within some aperture of a radius R:

$$\tilde{n}(x, y) = \sum_{i=1}^{I} \tilde{\alpha}_i\, Z_i(x, y)\ , \qquad (5)$$

where the random coefficients $\tilde{a}_i$ are distributed according to the log–normal law with the zero mean, the variance being determined by the atmospheric conditions along the path. As shown in Ref. 8 in most practical cases it is quite sufficient to use only the first– and the second–order polynomials ($I = 5$) in expansion (5). In this case $R = a_0/2$.

Numerical solution of Eq. (3) has been obtained by the splitting method and using the fast Fourier transform algorithm. Material equation (4) was integrated using an explicit two–step Lax–Vendroff scheme.[9]

2. ALGORITHM OF A BEAM PHASE CONTROL

In the problem considered the main factors that worsen energy characteristics of a beam in the observational plane are its random wandering and defocusing. These factors must be taken into account first in developing the control strategy. Previously used modification of the algorithm of simplex search[6] cannot provide stable control under pulsations because it does not assume a movement of the simplex towards a "drifting" target. Therefore it would be reasonable to use search following a flexible strategy which can be divided into two stages. The first stage includes the control at the initial step of medium heating (during the time interval of the order of $2\,\tau_v$). Here, due to the properties of the simplex method and strongly pronounced transient processes it is necessary to use the algorithm from Ref. 6, which allows one to prevent the simplex from cycling. Then, at the second stage, when random wandering of the beam and transient processes occurring due to changes in a medium state are of particular importance one should employ the algorithm with the free reflection of vertices. Its basic rule is to reflect the worst vertex of the simplex without any additional conditions. As will be shown below such an organization of the control makes it possible to compensate for random wanderings of the beam and to avoid unstable regimes of search.

3. REGIME OF WEAK FLUCTUATIONS OF THE REFRACTIVE INDEX

First we consider propagation of a beam in a medium with the wind–induced velocity pulsations along the path, natural fluctuations of the refractive index ($\tilde{n} \approx 0$) being neglected. On the near–ground horizontal paths the regime of sufficiently frequent pulsations of the velocity can occur that makes the transitent processes in the beam–medium system be significantly strong. Let us assume for clarity that the mean time of pulsation freezing is $T_v = 2\,\tau_v$. Taking into account the character of nonlinear distortions of a beam it is natural to choose the controllable wave front in the form

$$U(x, y) = S_x \frac{x^2}{2} + S_y \frac{y^2}{2} + \theta_x x + \theta_y y\ , \qquad (6)$$

where S_x, S_y, θ_x, and θ_y are the wave front curvatures and tilts with respect to the OX and OY axes, respectively.

However with a large number of coordinates under control it is difficult to make an *a priori* analysis of the optimum search trajectory which is useful, in particular, for determining the initial simplex configuration. This is of particular importance in the presence of transitent processes: the first steps of search should be done in the true direction (e.g., the beam must start its focusing rather than defocusing). In the four–dimensional space there appear difficulties associated with the determination of this direction. Therefore it seems to be reasonable to decrease the number of controllable coordinates and thus to increase the operation rate of the adaptive system.

Taking into account the fact that the beam defocusing is in fact axisymmetric in the presence of random pulsations of the velocity it is natural to decrease the number of the controllable variables by introducing a combined mode $\left(\dfrac{x^2}{2} + \dfrac{y^2}{2}\right)$, i.e., to assume that

$$U(x, y) = S \left(\frac{x^2}{2} + \frac{y^2}{2}\right) + \theta_x x + \theta_y y\ . \qquad (7)$$

Figure 1 shows the efficiency of a beam control in basis (7) using the comparison of two strategies of search for the focusing criterion maximum – constant strategy "without cycling"[6] and a flexible strategy proposed in this paper (Sec. 2).

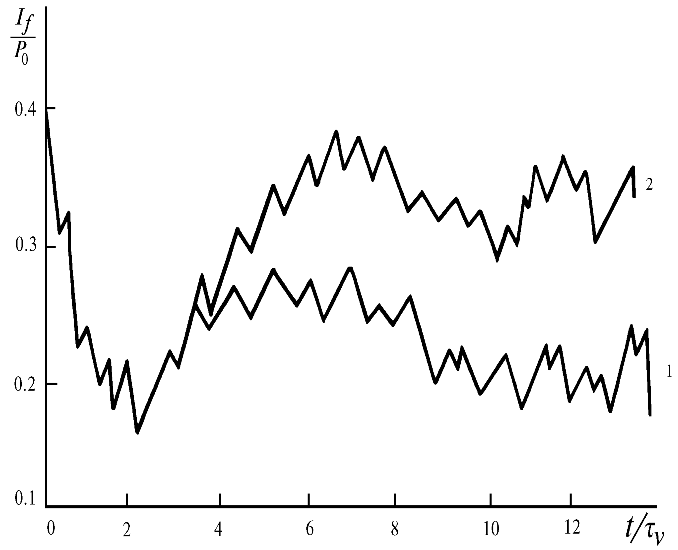

FIG. 1. The dependence of the normalized criterion of focusing J_f on the time t for the control being done in basis (7) with the constant (curve 1) and a flexible (curve 2) strategies of search. Time of the strategy change $t = 2\tau_v$. Conditions of propagation are $z_0 = 0.5\ k\alpha_0^2$, $R_0 = -20$, and $\sigma_v = 0.3\ v_0$.

In the subsequent numerical experiments the beam control was assumed to be done during a finite time interval $T = 12\ \tau_v$ after switching a laser source. The efficiency of search was estimated by the total light energy entering the receiving aperture during the time T. The results of numerical simulations have shown that the optimal size of the simplex L_{opt} is determined only by the average–over–the–path value of the nonlinearity parameter and could be estimated on the basis of the considerations given in Ref. 6.

Figure 2 shows a typical time dependence of J_f in the course of the control. Also represented here are the values of the standard deviation of the focusing criterion averaged over 120 realizations. It was found that for the values $< R_v > = -20 \dots -30$ the control based on the simplex method made it possible to increase the energy characteristics on the average by a factor of 1.4, as compared with the propagation of both collimated and focused beams.

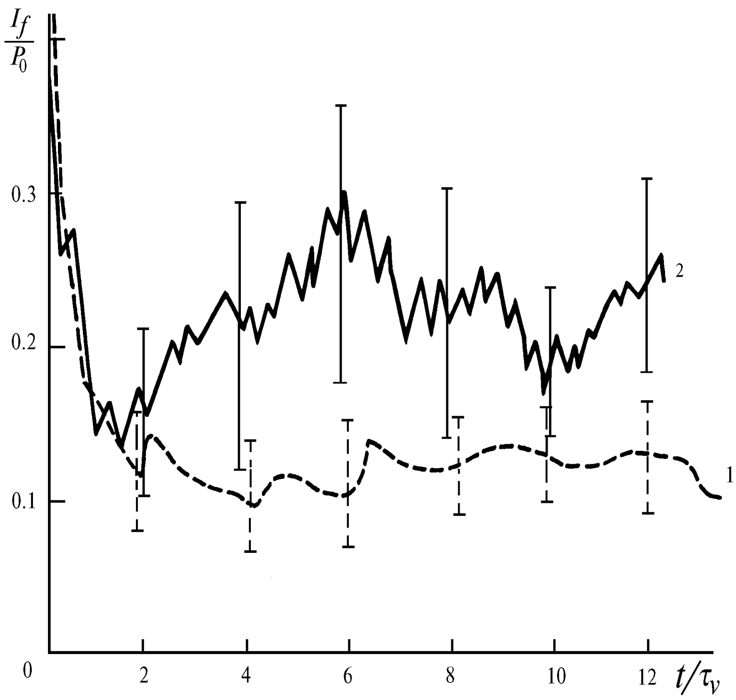

FIG. 2. The dependence of the normalized criterion of focusing J_f on time in one of the realizations of the wind velocity and standard deviation σ_j obtained by averaging over 120 realizations. Curves: 1) without a control and 2) with the control in the basis (Eq. (7)). Conditions of propagation: $z_0 = 0.5\ k\alpha_0^2$, $R_0 = -20$, and $\sigma_v = 0.3\ v_0$.

As can be seen from the comparison with the gradient method the use of both these methods enables one to reach approximately the same average over time values of the focusing criterion $< J_f >$ (see Fig. 3). Under conditions of wind velocity pulsations in the range of σ_v up to $\sigma_v \le 0.5\, v_0$ the algorithm of simplex search is stable. It should be noted that with the σ_v increase the standard deviations of focusing criterion in fact does not increase.

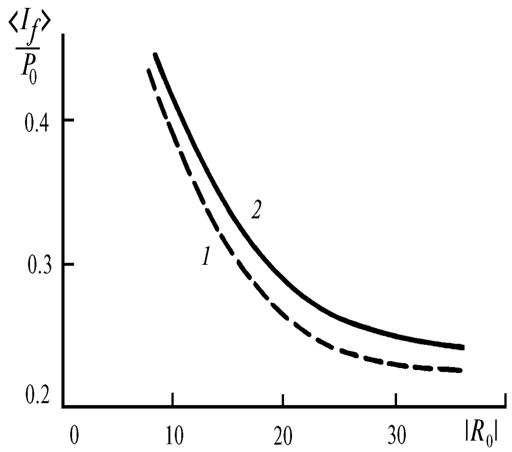

FIG. 3. Mean values of the normalized criterion of focusing as a function of the mean parameter of nonlinearity along the path during the control based on the gradient method in the basis (Eq. (6)) (curve 1) and the simplex method in the basis (Eq. (7)) (curve 2). Conditions of propagation: $z_0 = 0.5\, k\alpha_0^2$ *and* $\sigma_v = 0.3\, v_0$.

This may be accounted for by the fact that the algorithm under study provides uniform scanning with a beam over mutually perpendicular planes so that the mean deviation of the center of gravity of the beam $< r_c > \le a_0/2$. It should be noted that the use of the gradient method under conditions when $\sigma_v \ge 0.3\, v_0$ would require a more complicated control procedure to provide the stability, for example, to use separate soundings for the focusing and tilts.[3]

4. TURBULENT ATMOSPHERE WITH A RANDOM WIND ALONG THE PATH

We consider here the problem of propagation of a beam in a randomly inhomogeneous medium described by Eqs. (3) and (4) with all terms taken into account. Let us assume that the mean times of frozen wind velocity pulsations as well as of each realization of the random field of the refractive index fluctuations $\tilde{n}$ are the same and equal to $2\,\tau_v$, their change occurring at the same instant of time.

The quality of the control is studied as a function of the parameter $D_s(2a)$, Ref. 10, which specifies turbulence of the atmosphere along the path. The efficiency of the correction W is estimated based on the ratio of the total energy incident on the receiving aperture during the time of the control $T = 12\,\tau_v$ to the same value but in the case of propagation of a focused uncontrollable beam.

The calculational results for a single concrete set of realizations replacing each other in a time interval T are depicted in Fig. 4. As can be seen from this figure, the control based on the simplex method is stable and reasonably efficient within a wide range of the parameter $D_s(2a)$. However, with the increase of this parameter the quality of the correction decreases. It is obvious that under these conditions the optimal size of a simplex must be reconsidered.

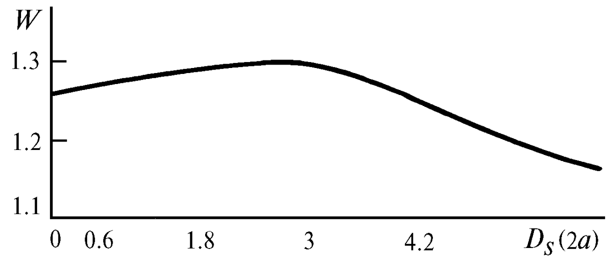

FIG. 4. The dependence of the control efficiency W on the structure function of the spherical wave phase $D_s(2a)$. *Conditions of propagation:* $z_0 = 0.5\, k\alpha_0^2$, $R_0 = -20$, *and* $\sigma_v = 0.3\, v_0$.

In conclusion it should be noted once more that in parallel with the known useful properties of the simplex search in the above–considered problems this method improves the possibilities of a beam control in a real–time scale since it requires approximately a two times lower operation rate of the system compared to that in the gradient techniques.

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