COLLECTIVE GENERATION OF DIFFRACTION–COUPLED LASERS UNDER CONDITIONS OF GAIN SATURATION

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This paper presents some results of a theoretical study of the effect of gain saturation of a diffraction–coupled laser array in an active medium on the structure of collective modes in the near and far diffraction zones.

INTRODUCTION

Modular multibeam laser systems have been increasingly employed in the last few years. Due to high quality of radiation and simple construction this approach makes it possible to enhance the total power of output radiation by increasing the number of modules.

Summation over incoherent fields of individual modules provides the linear increase in the intensity I with increase of the number of modules $N(I_{\mathrm{incoh}} \sim N)$, and the total divergence of radiation is determined by an aperture of a single module. In the case of coherent field summation the aperture of the entire array has a decisive impact on divergence of the output radiation and hence much larger values of peak intensity are attained ($I_{\mathrm{coh}} \sim N^2$).

An optical coupling shows a significant promise for frequency and phase synchronization of the fields of individual modules. A large number of papers are devoted to this problem, the basic results of these papers are described in Ref. 1. The efficiency of various methods of optical coupling and stability of a synchronous regime of generation of two lasers were analyzed in Refs. 2 and 3. The eigenmodes of collective generation by a large number of lasers were investigated in Ref. 4.

As a practical matter, the optical coupling arising from the diffraction exchange of radiation between the active elements placed inside the common cavity[3] is a subject of interest. It should be noted that for regular arrangement of lasers in the array the synchronization regime is effectively separated from the regime of independent generation by positioning a coupling mirror at the distance $z = z_{\mathrm{T}}/2$, where $z_{\mathrm{T}} = 2a^2/\lambda$ is the Talbot distance, a is the period of the array, and λ is the wavelength.[6,7,8] When radiation is reflected from the coupling mirror, the field is completely reproduced at the output ends of the active elements.

The theoretical studies and experimental observations indicate that the quality of alignment and the spread in the laser parameters strongly influence the efficiency of frequency and phase synchronization. Thus detuning of natural frequencies of the two lasers coupled through a semi–transparent mirror or an opening produces phase shift between the output fields[9] and at the exit from a locking region, this leads to a breakdown of synchronous regime and transition to an independent or chaotic generation.[2,3,10]

In large laser arrays random detunings of their natural frequencies result in formation of individual regions of cophased generation and hence degrade the coherence of output radiation of such systems.[11,12] Moreover, in the coupling arising from the Talbot effect there are several modes of collective generation with different field distributions but degenerated in their losses. On the one hand, this may result in the development of multimode generation and hence in the decrease of the total radiation divergence. On the other hand, this effect can be used for control of a directional pattern with relatively simple (binary) spatial or phase correctors.[13,14]

Thus the control of collective generation of the array of optically coupled lasers is of interest for both compensation for the random fluctuations of the array parameters and control of the parameters of output radiation in real time. Such an analysis was carried out for linear problem in Ref. 15.

In this paper we investigate the gain saturation effect in laser active media on the radiation profile of collective modes with the highest Q–factor. The field distribution in the near and far diffraction zones is examined as a function of the parameters of the finite laser array.

In contrast to Ref. 4 in which the effect of the active medium on the structure of the output radiation was considered based on a continuous model of the diffraction–coupled laser array, the present paper is concerned with an adequate discrete model in which a distributed nature of the medium is taken into account.

1. MATHEMATICAL MODEL

A one–dimensional periodic array of diffraction–coupled lasers is treated (Fig. 1). Individual active elements are coupled due to diffraction of radiation reflected from a coupling mirror M.

Let us assume that during collective generation of lasers the transverse modes of waveguides $f(x)$ are undistorted. Then the radiation field of the array in the plane $z = z_0$ is represented by superposition of the modes $f(x)$

$$E(x, z_0) = \sum_m e_m f(x - ma) . \qquad (1)$$

Under these assumptions the field $E(x, z)$ is unambiguously determined by complex amplitudes of waveguide modes e_m. The amplitudes e_m form the vector of the radiation profile in the regime of collective generation

$$\mathbf{E}^{\mathrm{T}} = (e_1, e_2, \ldots, e_N) ,$$

where N is the number of lasers in the array.

Transformation of the field for onore passage round the cavity can be represented in the operat form

$$E(x, z_0 + 2L) = \hat{U}\hat{P}\hat{G}E(x, z_0) \exp\{i\theta\} , \qquad (2)$$

where $\hat{G}$ is the linear diffraction operator describing the propagation of radiation in the coupling channel from z_0 to

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$z_0 + 2z_{co}$, $\hat{P}$ is the operator projecting the field $E(x, z_0 + 2z_{co})$ onto the transverse mode of the waveguide $f(x)$, $\hat{U}$ is the operator of radiation propagation along the waveguides taking into account the gain saturation, and θ is the geometric run–on of the phase accumulated for one passage round the cavity.

The action of the operators $\hat{G}$ and $\hat{P}$ on the field $E(x, z)$ is reduced to the multiplication of the vector **E** by the matrix of diffraction coupling M whose elements have the form[15]

$$M_{nm} = \int\int f(x - na)\, f(\xi - ma)\, G(x - \xi, 2z_{co})\, d\xi\, dx \,, \quad (3)$$

where $G(x, z)$ is the Green's function of quasioptical diffraction equation.

The square modulus of the element M_{nm} indicates the relative fraction of the energy of the mth laser which sustains generation in the nth laser when the phase shift between emissions of these lasers at an input window of the nth waveguide equals zero. The argument of the element M_{nm} bears the information about the run–on of the phase accumulated along the coupling channel.

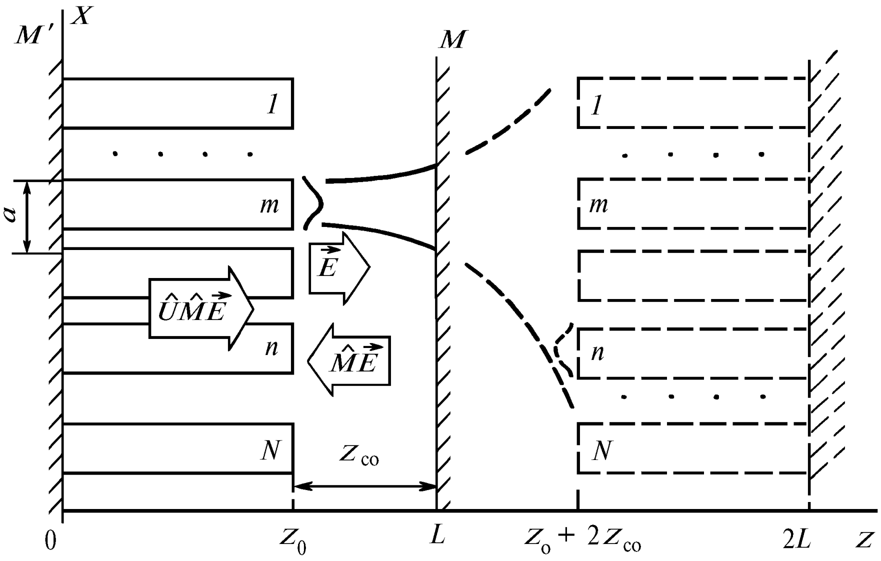

FIG. 1. Diffraction–coupled laser array. M is the coupling mirror.

To specify the operator $\hat{U}$ for the nth laser, we find the relation between the input $e_n^{\rm in}$ and output $e_n^{\rm out}$ amplitudes of the waveguide modes. The field in the nth laser is the superposition of waves propagating in opposite directions. The peak intensities of these waves I_+^n and I_-^n for a uniformly broadened gain line obey the equations

$$\frac{dI_+^n(z)}{dz} = \frac{G_0}{1 + I_+^n(z) + I_-^n(z)}\, I_+^n(z) \,, \quad (4a)$$

$$\frac{dI_-^n(z)}{dz} = -\frac{G_0}{1 + I_+^n(z) + I_-^n(z)}\, I_-^n(z) \,, \quad (4b)$$

where G_0 is the weak–signal gain. The intensities $I_+^n(z)$ and $I_-^n(z)$ are normalized to the saturation intensity I_s. The parameters G_0 and I_s are taken to be identical for all lasers.

The condition at the left boundary of the waveguide is

$$I_+^n(0) = R^2 I_-^n(0) \,, \quad (5)$$

where R is the coefficient of reflection of the field from the mirror M'.

For the system of equations (4) the relation

$$I_+^n(z)\, I_-^n(z) = J^2 \quad (6)$$

is valid, where J^2 is the constant independent of z.

Based on Eq. (6) by expressing $I_+^n(z)$ in terms of $I_-^n(z)$ and integrating Eq. (4b), we obtain

$$\ln u_n + I_{\rm in}^n (u_n - 1)(R^2 u_n + 1) = G_0 \,, \quad (7)$$

where $u_n = \dfrac{I_-^n(0)}{I_+^n(0)}$ and $I_{\rm in}^n = |e_n^{\rm in}|^2$ is the peak intensity of the wave entering the nth channel. Using the boundary condition given by Eq. (5) and relation (6), we can find the unknown relation between $e_n^{\rm in}$ and $e_n^{\rm out}$:

$$e_n^{\rm out} = R\, u_n\, e_n^{\rm in} \,, \quad (8)$$

where $u_n(|e_n^{\rm in}|^2)$ is the solution of Eq. (7) which is a function of the square modulus of the input wave amplitude.

Thus the action of the operator $\hat{U}$ is reduced to the multiplication by the diagonal matrix whose elements are

$$(\hat{U})_{nm} = R\, \delta_{nm}\, u_n \,. \quad (9)$$

In the regime of the collective radiation of diffraction–coupled lasers the field after passage round the cavity is reconstructed to within some complex coefficient γ. As a result we obtain the following eigenvalue problem:

$$\gamma\, \mathbf{E} = \hat{U}(|\mathbf{E}|^2)\, \hat{M}\mathbf{E} \,. \quad (10)$$

Nonlinear matrix equation (10) determines the losses and profiles of radiation of the array of the diffraction–coupled lasers under conditions of gain saturation.

The modulus of the eigenvalue γ characterizes the relative losses of radiation for one passage round the cavity which are equal to $1 - |\gamma|$. Hence the amplitude condition of generation of the laser array can be written down as

$$|\gamma| \exp\{G_{\text{thr}}\} = 1 , \qquad (11)$$

where G_{thr} is the threshold gain.

The argument of γ characterizes the fulfilment of phase condition of generation at the wavelength λ_{gen}

$$\arg \gamma + \theta\,(\lambda_{\text{gen}}) = 2\pi p , \qquad (12)$$

where p is the integer.

The run–on of the phase θ, being equal for all channels and associated with radiation propagation in the coupling channel and waveguide, does not affect the profile of the output radiation $\mathbf{E}$. It can be neglected in the solution of problem (10).

2. LINEAR PROBLEM

The linear eigenvalue problem

$$\gamma_0\, \mathbf{E} = \hat{M}\, \mathbf{E} , \qquad (13)$$

which follows from Eq. (10), corresponds to the system of waveguides without the active medium and with ideal mirrors ($G_0 = 0$, $u_n = 1$, and $R = 1$).

The fulfilment of the condition of complete reproduction of the field for one passage round the cavity given by Eq. (11) yields the expression for the threshold gain of collective generation of the kth eigenmode

$$G_{\text{thr}}^{(\kappa)} = -\ln |\gamma_0^{(\kappa)}| . \qquad (14)$$

In the hypothetical case of an infinite array in which the coupling mirror is positioned at a distance being equal to half the distance of reproduction ($z_{\text{co}} = z_{\text{T}}/2$), the cophased ($e_n = e_0$) and antiphased ($e_n = (-1)^n e_0$) collective modes[1] have zero losses ($|\gamma_0| = 1$, $G_{\text{thr}} = 0$). For a large but finite number of lasers we failed to obtain rigourous analytical solution of eigenproblem (13) with complex matrix.

Let the transverse modes of waveguides $f(x)$ have the form

$$f(x) = \sqrt[4]{\frac{2}{\pi\sigma^2}} \exp\left\{-\frac{x^2}{\sigma^2}\right\} , \qquad (15)$$

where σ is the characteristic scale of a mode. The period of the array is a.

The dependence of the threshold gain G_{thr} of cophased and antiphased collective modes on the distance z_{co} to the coupling mirror is shown in Fig. 2a. It was obtained from numerical solution of problem (13). The threshold gain of both these modes is seen to possess minimum at the same distance $z_{\text{co}} = z_{\text{T}}/2$ as for the infinite array. The threshold gain of the other modes of collective generation (with the same position of the mirror) is higher than the threshold for cophased and antiphased modes. The nonzero threshold gain in the minimum arises from the array finiteness and, consequently, from incomplete reproduction of the fields of cophased and antiphased modes due to diffraction losses at the edges of the array.

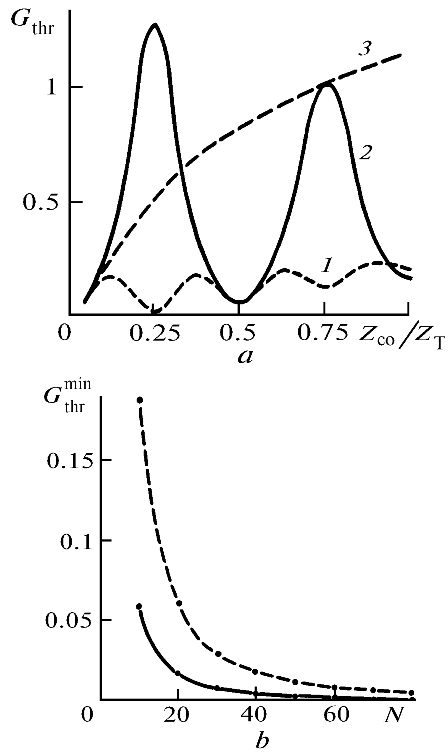

FIG. 2. Threshold gains G_{thr} for cophased and antiphased collective modes: a) G_{thr} vs the distance z_{co} to the coupling mirror for $a/\sigma = 4$ and $N = 10$ for antiphased (1), cophased (2), and independent (3) generations of the laser array and b) G_{thr} vs the number of lasers N in the array for $z_{\text{co}} = z_{\text{T}}/2$, $a/\sigma = 4$ (solid curve), and $a/\sigma = 8$ (dashed curve).

The decrease of the generation threshold of the lowest and higher–order modes with increase of the number of lasers N in the array is shown in Fig. 2b. The increase of the threshold gain G_{thr} with increase of a/σ is due to the increasing losses in the coupling channel.

For cophased and antiphased collective modes the absolute values of radiation amplitudes e_n in each laser coincide. These modes differ only in phase relations between the fields in the modules. The distribution of e_n over the channels is depicted in Fig. 3a. Due to uncompensated diffraction losses at the edges of array the field amplitude moduli in the peripheral lasers are smaller than those in the central ones. The phase shift φ_n between the phase of peripheral lasers and those located at the center of the array appears simultaneously (Fig. 3b).

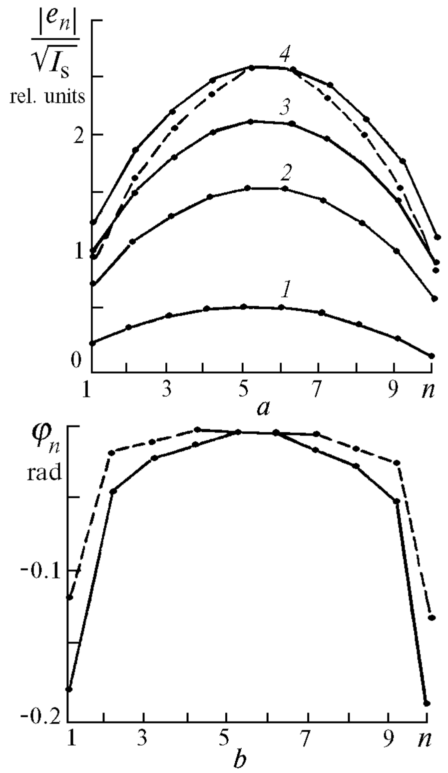

FIG. 3. Radiation profile of collective generation for $z_{co} = z_T/2$, $a/\sigma = 4$, *and* $G_{thr} = 0.067$: *a) distribution of the normalized modulus of the amplitude* $|e_n|/\sqrt{I_s}$ *of the waveguide modes over the channels of the array for cophased and antiphased generation: linear case is for* $G_0 = 0$ *(dashed line); nonlinear case is for* $G_0 = 0.1$ *(1),* 0.3 *(2),* 0.5 *(3), and* 0.7 *(4) and b) phase distribution* φ_n *of waveguide modes for cophased generation: linear case is for* $G_0 = 0$ *(dashed line) and nonlinear case is for* $G_0 = 0.7$ *(solid line).*

3. PROFILE OF THE OUTPUT RADIATION UNDER CONDITIONS OF GAIN SATURATION

The selective method of setting up of the stationary regime[16] is used to find the profile of the output radiation with allowance for the gain saturation in the modules. The main point of this method is that the iterative process for solving the nonlinear problem by the method of setting up of the stationary regime alternates with the analysis of the linear problem with "frozen" gain. To obtain the proper zeroth approximation for the iterative process, it is necessary to estimate the intensity of the chosen (cophased or antiphased) mode based on the condition of stationary generation. For Eq. (10) this condition has the form

$$|\gamma| = 1 .$$

Let us assume that under conditions of gain saturation the output radiation profile of the collective generation is not deformed. Then following Eq. (13), Eq. (10) can be written down in the form

$$\mathbf{E} = \gamma_0 \hat{U} \mathbf{E} .$$

Equation (9) allows us to obtain the value of u_{st} for stationary generation

$$\gamma_0 R u_{st} = 1.$$

The value of u_{st} thus derived determines the stationary intensity

$$I_{st} = \frac{G_0 + \ln|\gamma_0|}{(1 - |\gamma_0|)(R^2 + |\gamma_0|)} |\gamma_0|^2 . \qquad (16)$$

Such a value of the intensity is assumed to be attained in the channel with maximum amplitude e_n of the chosen collective mode.

The iterative procedure in the method of setting up of the stationary regime is expressed as

$$\mathbf{E}_{j+1} = \hat{U}(|\mathbf{E}'_j|^2)\,\mathbf{E}'_j ; \qquad \mathbf{E}'_j = \hat{M}\mathbf{E}_j , \qquad (17)$$

where j is the iteration number. In this case at each iteration Eq. (7) is solved for determining the matrix $\hat{U}$.

The profile vector $\mathbf{E}_J$ obtained in the last iteration is used for the solution of the linear eigenvalue problem with frozen gain

$$\gamma\mathbf{E} = \hat{U}(|\mathbf{E}_J|^2)\,\hat{M}\mathbf{E} . \qquad (18)$$

If one of the eigenvectors $\mathbf{E}^{(l)}$, $l = 1, ..., N$ of problem (18) coincides with $\mathbf{E}_J$, then the stationary solution has been found by iteration. The eigenvalue modulus $|\gamma^{(l)}|$ corresponding to this vector is equal to unity. If for the rest of the modes $|\gamma^{(\kappa)}| < 1$, $k \neq l$, then the solution is considered to be stable. The difference $|\gamma^{(l)}| - |\gamma^{(\kappa)}|$ determines the margin of stability for the obtained solution $\mathbf{E}^{(l)}$. When there are modes with $|\gamma^{(\kappa)}| > 1$, the solution is unstable. The situation can arise in which the moduli of eigenvalues of several modes with the highest Q–factors are close in values. This means that the regime of multimode generation is feasible with such parameters of the system.

In this case the total field saturating the gain must be considered as a superposition of fields of these modes.

If none of the eigenvectors $\mathbf{E}^{(l)}$ of problem (18) coincides with $\mathbf{E}_J$ found by the iteration process, it is necessary to return to the method of setting up of the stationary regime taking the eigenvector of problem (18) closest in value to $\mathbf{E}_J$ as the zeroth approximation.

The results of calculation of the nonlinear problem by the above–described procedure are shown in Fig. 3*a* in which the modulus of the field distribution over the channels is depicted for cophased and antiphased generation with different values of the gain G_0. Here the distance to the coupling mirror is $z_{co} = z_T/2$. Similar to the linear case, with this location of the coupling mirror these distributions coincide by modulus and differ only in relations between the phases of the fields in lasers.

It can be seen that with increase of the gain G_0 the field amplitude increase in the channels slows down. The gain saturation manifests itself most strongly in the central channels of the array resulting in flattening of the collective radiation profile. At the same time, with increase of G_0 the phase shift of the field at the edges of the array increases (Fig. 3*b*).

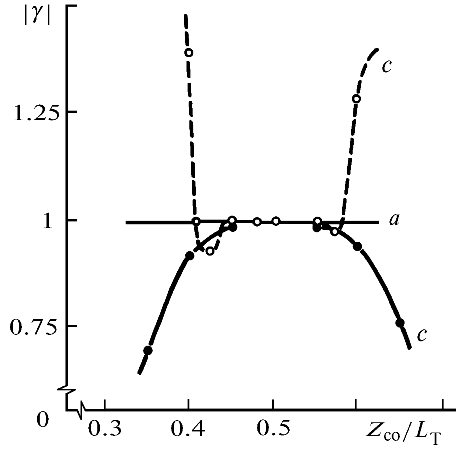

FIG. 4. Modulus of eigenvalues of cophased (c) and antiphased (a) collective modes vs the distance to the coupling mirror. Linear case ($G_0 = 0$): $a/\sigma = 4$ (solid curve) and $a/\sigma = 8$ (dashed curve). Nonlinear case: $G_0 = 0.7$ and $a/\sigma = 4$ (filled circles); $G_0 = 2$ and $a/\sigma = 8$ (empty circles).

Depicted in Fig. 4 is the change in the relation between the losses of cophased and antiphased modes due to variation of the distance to the coupling mirror. The results were obtained by choosing the antiphased mode as the zeroth approximation for the iterative process of setting up of the stationary regime. It can be seen that $|\gamma_a|$ for the antiphased mode is equal to unity for any z_{co}. For $a/\sigma = 4$ for the cophased mode $|\gamma_c| \le 1$ and this mode attenuates everywhere except a small vicinity of $z_{co} = z_T/2$, where a two–mode regime of generation is feasible. For $a/\sigma = 8$ the variation in $|\gamma_c|$ is of complicated nature. Flattening of the radiation profile of collective generation does not affect the relation between the losses of cophased and antiphased modes.

4. DIRECTIONAL PATTERN

It is of interest to analyze the angular divergence of radiation in the regime of collective generation of the diffraction–coupled lasers.

In the regime of antiphased generation the directional pattern has two side lobes at the angles $\theta = \pm 2\lambda/a$. In the regime of cophased generation there is one central and two side lobes at the angles $\theta = \pm\lambda/a$. The amplitude of the side lobes depends on the ratio a/σ, and for $a/\sigma = 4$ it is about 30% of the amplitude of the central lobe.

In the regime of cophased generation the beamwidth $\Delta\theta$ of the central lobe is determined as half the angular distance between the neighboring zeros $J(\theta)$ and for $N = 10$ and $a/\sigma = 4$ is equal to (Fig. 5)

$$\Delta\theta = 0.09\theta_d ,$$

where $\theta_d = \lambda/\pi\sigma$ is the angular divergence of radiation of an individual module. As a/σ increases, the relative beamwidth $\Delta\theta$ decreases.

To analyze the effect of the radiation profile in the regime of collective generation on the directional pattern, we show the zeroth maximum of field distribution in the far diffraction zone in the case of superposition of the radiation of cophased sources with equal amplitudes in Fig. 5. It can be seen that the decrease of the amplitude and the phase shift in the radiation profile of collective generation results in some broadening (by 20%) of the central lobe. As the analysis shows, the central lobe broadening is associated with the decrease of the radiation amplitude of the side lasers and in fact is independent of the phase shifts at the edges of the array. In this case flattening of the amplitude distribution has practically no effect on the radiation directional pattern.

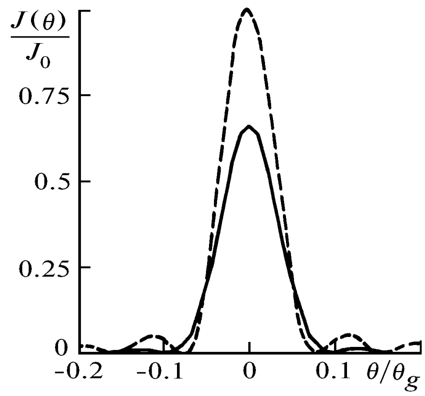

FIG. 5. Directional pattern of radiation for $N = 10$, $a/\sigma = 4$, and $G_0 = 0.7$: cophased collective generation (solid curve) and cophased sources of equal amplitude (dashed curve).

CONCLUSION

1. In the laser array consisting of the finite number of modules the diffraction losses at the edges of the array result in the decrease of the field amplitude at the periphery and the appearance of the phase shift between the peripheral and central modules ($\varphi \sim 7°$, $N = 10$, and $a/\sigma = 4$).

2. The gain saturation leads to flattening of the radiation profile and the phase shift increase at the periphery of the array ($\varphi \sim 10°$, $N = 10$, and $a/\sigma = 4$).

3. The field amplitude decrease at the edges of the array increases the beamwidth of the central lobe as compared to the lobe for cophased sources of equal intensity. The phase shifts as well as flattening of amplitude distribution under conditions of gain saturation weakly affect the angular distribution of radiation in the far diffraction zone.

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