

## ON A DIAL TECHNIQUE FOR TEMPERATURE PROFILING

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*A closed expression is derived for reconstructing the fluctuating temperature profiles in the low troposphere from the data obtained by a bifrequency DIAL technique based on absorption within an oxygen absorption band in the atmosphere. For a multipulse sounding regime it is recommended to use the stochastic approximation procedure proposed by Robbins and Monroe.*

The routine information about meteorological parameter profiles with the required spatial and temporal resolution is important in meteorology, ecology, and atmospheric physics. The temperature field  $T$  is stochastic in nature, and the vertical profiles  $T(h)$  are its spatial realizations at fixed instants of time. The DIAL technique of temperature sounding based on the temperature dependence of the population of rotational states of gas molecules, which was first proposed by Mason,<sup>1</sup> makes it possible to reconstruct these profiles. In so doing oxygen<sup>2</sup> or water vapor<sup>3</sup> can be used as an absorbing gas.

The bifrequency DIAL technique for measuring temperature and pressure<sup>2</sup> is based on the constant proportions of oxygen mixture in air depending on an altitude. The algorithm for estimating  $T(h)$  was derived in Ref. 2 from the relation between the oxygen absorption coefficient and the lidar signals and temperature by the iteration procedure for determining  $T(h)$ .

To improve and simplify the algorithm for processing of the lidar signals, the closed expression for estimating  $T(h)$  is derived in this paper, while for a multipulse sounding regime the stochastic approximation procedure proposed by Robbins and Monroe should be used.

As applied to the current photodetection regime, by writing the lidar equations for the sounding wavelengths  $\alpha_1$  and  $\alpha_0$ , then dividing the first equation into the second one, and differentiating the logarithm of the obtained ratio we can derive the formula<sup>4</sup> for estimating the oxygen absorption coefficient  $\hat{\alpha}_1(h)$  at the distance  $h$  and the wavelength  $\alpha_1$ . One of the indicated wavelengths ( $\alpha_1$ ) corresponds to the strong and another ( $\alpha_0$ ) to weak absorptions. As a result, using the finite difference formula for derivatives and neglecting the variations in the coefficients of backscattering and extinction caused by Rayleigh and aerosol scattering at the least resolved altitude range  $\Delta h$  we have the relation

$$\hat{\alpha}_1(h) = \frac{1}{2\Delta h} \ln \frac{\hat{P}(\lambda_1, h) \hat{P}(\lambda_0, h + \Delta h)}{\hat{P}(\lambda_1, h + \Delta h) \hat{P}(\lambda_0, h)}, \quad (1)$$

where  $\hat{P}(\lambda, h)$  is the estimate of the power  $P(\lambda, h)$  of the received signal. The power  $E_0$  of the sounding pulse fluctuates, and if its measurement in every pulse is possible then the received signals are routinely normalized. Then in Eq. (1) it is necessary to replace  $\hat{P}(\lambda, h)$  by  $\hat{P}_E(\lambda, h) = \hat{P}(\lambda, h)/E_0$ , where  $\hat{E}_0$  is the estimate of  $E_0$ .

In turn, one can express the absorption coefficient in terms of the absorption line parameters. The mass absorption coefficient of molecules can be represented in the form

$$K(\lambda, h) = S(\lambda, h) f(v - v_1), \quad (2)$$

where  $S(\lambda, h)$  is the intensity of the absorption line depending on the temperature  $T(h)$  and  $f(v - v_1)$  is the contour of the absorption line centered at  $v_1$  (see Ref. 4).

The bifrequency DIAL technique uses the change in the volume absorption coefficient of  $O_2$  at the maximum of the absorption line<sup>2,3</sup>

$$\hat{\alpha}_1(h) = q_0 (1 - q(h)) \rho(h) S(\lambda_1, h) f(0), \quad (3)$$

where  $q_0 = 0.2095$  and  $q(h)$  are the volume concentrations of oxygen in the dry atmosphere and of water vapor,  $\rho(h)$  is the profile of air density,  $f(0)$  is the value of  $f(v - v_1)$  at the center of absorption line. Taking into account only the broadening caused by collisions which predominates in the troposphere below 3 km,  $f(0) = 1/\pi\gamma_L$ , where  $\gamma_L(\lambda_1, h)$  is the half-width of the Lorentz contour depending on temperature and pressure.<sup>4</sup> In more general case of the Voigt profile let us use the analytical approximation<sup>2</sup>

$$f(0) = \frac{1}{3\gamma_L(\lambda_1, h)} \left( 1 - \frac{\exp(1)}{10 b_{DL}} \right), \quad (4)$$

which describes  $f(0)$  up to 2 km with an accuracy of 0.1%, and up to the tropopause boundary with an accuracy of 0.7%,  $b_{DL} = (\gamma_L/\gamma_0) \ln 2$ , where  $\gamma_D(\lambda_1, h)$  is the Doppler half-width.

Let us write down

$$T(h) = T_m(h) + \Delta T(h),$$

where  $T(h)$  is the statistically provided and *a priori* known model profile,  $\Delta T(h)$  is the deviation of the reconstructed vertical dependence  $T(h)$  including two components

$$\Delta T(h) = \langle \Delta T(h) \rangle + \tilde{\Delta T}(h),$$

where  $\langle \Delta T(h) \rangle$  is the mesoscale average under the given conditions (inversion, stable or unstable stratification, etc.) and  $\tilde{\Delta T}(h)$  is the fluctuation.

Let  $K_m(\lambda_1, h)$  be the mass absorption coefficient for the *a priori* prescribed model profiles  $T_m(h)$  and  $p_m(h)$ ,

$K(\lambda_1, h)$  is the same but for  $T(h)$  and  $p(h)$ ;  $p(h)$  and  $p_m(h)$  are the vertical dependences of pressure. Thus one can show that

$$K(\lambda_1, h) = K_m(\lambda_1, h) \left[ \frac{T_m(h)}{T(h)} \right]^{0.5} \frac{p_m(h)}{p(h)} \exp \left\{ 1.439 E_1'' \left( \frac{1}{T_m} - \frac{1}{T} \right) \right\}, \tag{5}$$

where  $E_1''$  is the rotational energy of the lower transition level under the standard conditions  $p_0$  and  $T_0$ .

Since in the low troposphere  $|T_m(h) - T(h)| \ll T_m(h)$ , we can restrict ourselves to only the linear terms of the expansion in  $\Delta T/T(h)$  in Eq. (5), and taking into account Eqs. (3) and (4) derive the closed formula for the temperature estimate

$$\hat{T}(h) = T_m(h) \left\{ 1 + \left( \frac{1.439 E_1''}{T_m(h)} - \frac{3}{2} \right)^{-1} \ln \frac{\hat{\alpha}_l(h)}{q_0(1-q(h)) \rho(h) K_m(\lambda_1, h)} \right\} \tag{6}$$

which makes it possible to determine it without using the iteration procedure.<sup>2</sup>

For a multipulse sounding regime it is necessary to use the values of  $\hat{P}$  averaged over the observation time interval  $t_{\text{obs}}$  of the temporal resolution instead of  $\hat{P}$  (or  $\hat{P}_E$ ) in the right side of Eq. (1). Let us consider the case in which there are no data on *a priori* distribution of these values. Then it is expedient to apply for its estimating the nonparametric procedure of stochastic approximation proposed by Robbins and Monroe of the form<sup>5</sup>

$$\hat{P}_l(h, t_{k+1}) = \hat{P}_l(h, t_k) + a_k \{ P_l(h, t_{k+1}) - \hat{P}_l(h, t_k) \}. \tag{7}$$

Such a procedure can solve the problem of determination of the value of  $\bar{P}_l(h)$  being constant but unknown at  $t_{\text{obs}}$ , where  $l = 0, 1$ . The selected values  $P_l(h, t_k)$  correspond to

the altitude  $h$  and the instant of time  $t_\kappa = \kappa \Delta t$  from the observation time interval  $[0, t_{\text{obs}}]$ , where  $\Delta t = t_{\text{obs}}/N$ , where  $N$  is the number of sounding events,  $\kappa = 1, \dots, N$ . The subsequence of the weight coefficient  $a_\kappa > 0$  satisfies

the conditions:  $\sum_{\kappa=1}^{\infty} a_\kappa = \infty$ ,  $\sum_{\kappa=1}^{\infty} a_\kappa^2 < \infty$ . If we choose

$a_\kappa = 1/\kappa$  then the Robbins and Monroe procedure becomes identical to the procedure for the recurrent calculation of the maximum likelihood estimate. The errors in determining  $\bar{P}_l(h)$  are asymptotically normal

with the variances<sup>5</sup>  $D[\hat{P}_l(h, t_N)] = D[P_l]/N$ .

As a result we have a convenient algorithm for estimating  $\alpha_l(h)$  and thus  $T(h)$  by Eq. (6) in both monopulse and multipulse sounding regimes. When *a priori* distributions  $\hat{P}_l$  are known accurate to the parameters one can use one of the parametric methods of estimation instead of Eq. (7) in order to decrease the fluctuation error in the multipulse regime.

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