INFLUENCE OF THE WAVE DIFFRACTION ON REFLECTOR EDGES ON AMPLIFICATION OF BACKSCATTERING IN A TURBULENT ATMOSPHERE

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It is shown in this paper that diffraction of an optical wave on the reflector edges can strongly influence on the distribution of the reflected wave intensity and the amplification of backscattering under conditions of weak turbulence. This study also revealed an oscillating dependence of the amplification factor on the Fresnel number of both a plane and a corner-cube specular reflectors. On the other hand, under conditions of strong turbulence in the atmosphere, the influence of an optical wave diffraction on the reflector edges becomes inessential.

Analysis of the efficiency of correction for atmospheric distortions by an adaptive optical system normally deals with the situation when predistortions introduced into a feedback are formed based on information extracted from reference waves.[1,2] The reference waves are, as a rule, produced by illuminating special reflectors (beacons) with a laser beam, so that in a number of cases the incident and reflected (reference) waves can propagate along one and the same path, passing, as a result, through the same inhomogeneities of a medium. Just this situation is observed in locating different objects in the atmosphere. It is well known[3,4] that in such cases some effects can occur due to correlation between the incident and reflected waves like an increase of the mean intensity of the reflected wave (amplification of the backscatter), an increase of fluctuations of its intensity and phase compared to those in the case of wave propagation through a double path along one direction, and so on. In this context it is important, for solving the problems in ranging and correcting atmospheric turbulence, to know how strong can be the amplification effects, if simultaneously accompanied by other phenomena, including such as the diffraction of a wave on the reflector edges.

In this paper we consider the influence of wave diffraction on the reflector edges on the backscattering amplification effect at different Fresnel numbers of a reflecting surface characterized by some effective radius. In Refs. 5 and 6 the authors considered only the case of reflectors with smooth edges, when the reflection coefficient varies across the reflector according to the Gauss law what sometimes gives results that poorly agree with experiment.[7] In Ref. 8 an account of diffraction effects was performed based on numerical simulations of wave propagation along the paths with reflection, but the results presented in this paper were calculated using the parameters that did not allow a comparison with the experimental data from Refs. 7 and 9 to be done, and physical conclusions to be drawn.

Let a spherical wave be incident on a reflector, as in the experiments described in Refs. 7 and 9. Then, in accordance with the representation of a wave field in a randomly inhomogeneous medium in the form of a path integral,[10,11] the complex amplitude of the reflected wave in the plane of a light source can be presented as follows:

$$U^R(x_0,\rho)=\left[i\,(x-x_0)\right]^2\int \mathrm{d}\rho'^2\mathrm{d}r^2\,V(\rho',\mathbf{r})\times$$

$$\times\exp\left\{i\,\frac{\kappa}{2(x-x_0)}\left[\rho'^2+(\mathbf{r}-\rho)^2\right]\right\}\lim_{N\to\infty}\left(\frac{\kappa}{2\pi i(x-x_0)}\right)^{2(N-1)}\times$$

$$\times\int \mathrm{d}a_1^2\ldots\mathrm{d}a_{N-1}^2\ldots\mathrm{d}b_1^2\ldots\mathrm{d}b_{N-1}^2\exp\left\{i\,\frac{\kappa}{2(x-x_0)}\sum_{l=1}^{N-1}\left(a_l^2+b_l^2\right)+\right.$$

$$+\,i\,\frac{\kappa}{2}\int\limits_{x_0}^{x}\mathrm{d}x'\left[\varepsilon_1\left(x',\frac{x'-x_0}{x-x_0}\rho'+\sum_{l=1}^{N-1}\nu_l(x')\,\mathbf{a}_l\right)+\right.$$

$$\left.\left.+\,\varepsilon_1\left(x',\left(1-\frac{x'-x_0}{x-x_0}\right)\rho+\frac{x'-x_0}{x-x_0}\mathbf{r}+\sum_{l=1}^{N-1}\nu_l(x')\,\mathbf{b}_l\right)\right]\right\},\quad(1)$$

where $\kappa=2\pi/\lambda$ is the wave number; $x-x_0$ is the distance between the source in the plane $x'=x_0$ and a reflector (beacon) in the plane $x'=x$; $\rho=\{y,z\}$, ρ', $\mathbf{r}$, $\mathbf{a}$, $\mathbf{b}$ are two–dimensional vectors; $\varepsilon_1(x',\rho)$ is the fluctuating part of the dielectric constant of air;

$$\nu_l(x')=\frac{\sin\left(l\,\pi\,\dfrac{x'-x_0}{x-x_0}\right)}{\sqrt{2}\,N\sin(l\,\pi/2N)}\,.$$

The function

$$V(\rho',\mathbf{r})=A(\mathbf{r})\,\delta(\rho'\pm\mathbf{r})$$

characterizes the local reflection coefficient of the surface, $A(\mathbf{r})$ is the amplitude function, $\delta(\rho)$ is the Dirac delta function, the minus sign is for a plane mirror and the plus sign for a corner–cube retroreflector.

Assuming that integral of the field ε_1 in the exponent of Eq. (1), over the path is a normal random value and the field itself is locally homogeneous, isotropic, and satisfies the condition of delta–correlation[12] one obtains using Eq. (1) a formula for the mean intensity of the reflected spherical wave in the form of a path integral. Unfortunately, in the general case, it is impossible to analyze the resultant expression. To analyze the mean

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intensity of a reflected spherical wave in the limiting cases of weak ($\beta_0^2 < 1$) and strong ($\beta_0^2 \gg 1$) intensity fluctuations one could use the known approaches described in detail, for example, in Refs. 11 and 13. However, we shall use some approximation of Eq. (1) for a reflected field like a generalization of the Huygens–Kirchhoff method for the case of smoothly inhomogeneous media.[15] Grounds for some of approximations of this type are given in the monograph by Mironov.[14] The approximate expression, we use in this paper for complex amplitude of the wave field, provides absolute agreement of calculational results with those obtained using perturbation methods[4,13] in the region of weak fluctuations, assuming that similar series expansions over small parameter β_0^2 are used. At the same time this expression enables us to avoid limitations inherent in the perturbation methods when taking into account attenuation of the reflected wave intensity due to turbulence. In the case of strong fluctuations this approximation underestimates final results as compared with those obtained using strict approximate methods.[4,13] That means that coefficients of terms containing factor in the expansion $O(\beta_0^{-4/5})$ are underestimated.

Thus, for mean intensity of a spherical wave reflected from a round plane mirror and a round–shaped corner–cube retroreflector, we have, assuming $\beta_0^2 < 1$,

$$I_m^R(x_0, \rho) = \left(\frac{\Omega_r}{2}\right)^2 \int_0^1 \mathrm{d}r_1\, \mathrm{d}r_2 \int_0^1 \mathrm{d}x_1\, \mathrm{d}x_2 \cos\{\Omega_r(r_1 - r_2 - \rho\, p)\} \times$$

$$\times \exp\left\{7.02\ \beta_0^2 \int_0^1 \mathrm{d}\xi\ \mathrm{Re}\left\{[i\xi(1-\xi)]^{5/6}\ {}_1F_1\left(-5/6, 1; i\Omega_r \rho^2 \frac{1-\xi}{4\xi}\right)\right\} -\right.$$

$$- 1.175\Omega_r^{5/6}\, \beta_0^2 \int_0^1 \mathrm{d}\xi \left\{ \sum_{j=1,2} [\rho^2(1-\xi)^2 + \xi^2 q +\right.$$

$$\left.\left. + (-1)^j\, 2\xi(1-\xi)\rho\, p]^{5/6} + 2\xi^{5/6} q^{5/6} \right\}\right\}, \quad (2)$$

$$I_R^R(x_0, \rho) = \left(\frac{\Omega_r}{2}\right)^2 \int_0^1 \mathrm{d}r_1\, \mathrm{d}r_2 \int_0^1 \mathrm{d}x_1\, \mathrm{d}x_2 \times$$

$$\times \cos\left\{\Omega_r(r_1 - r_2 - \rho\, p) + 3.51\ \beta_0^2 \int_0^1 \mathrm{d}\xi \sum_{j=1,2} \mathrm{Im}\, R_j\right\} \times$$

$$\times \exp\left\{3.51 \sum_{j=1,2} \mathrm{Re} R_j - 2.35\, \beta_0^2\, \Omega_r^{5/6} \int_0^1 \mathrm{d}\xi\, \left\{\xi^{5/3} q^{5/6} + Q_j^{5/6}\right\}\right\}, \quad (3)$$

where $\beta_0^2 = 1.23\, C_n^2 \kappa^{7/6}(x - x_0)^{11/6}$, C_n^2 is the structure characteristic of the refractive index; $\Omega_r = \kappa a_r^2/(x - x_0)$ is the Fresnel number of a reflector with the radius a_r; $p = \sqrt{r_1}\cos 2\pi x_1 - \sqrt{r_2}\cos 2\pi x_2$, $Q_j = \rho^2(1-\xi)^2 + \xi^2 q + (-1)^j \times 2\xi(1-\xi)\rho p$, $q = r_1 + r_2 - 2\sqrt{r_1 r_2}\cos 2\pi(x_1 - x_2)$, $R_j = [(-1)^j i\xi \times (1-\xi)]^{5/6}\ {}_1F_1\left(5/6, 1; (-1)^j \Omega_r \left[\frac{\rho^2}{4}\frac{1-\xi}{\xi} + \rho\sqrt{r_j}\cos 2\pi x_j + \frac{\xi r_j}{1-\xi}\right]\right)$.

When the direct and counter waves propagate along uncorrelated paths, irregardless of the type of a reflector, we have

$$\langle I_{\mathrm{incoh}}^R(x_0, \rho)\rangle = \left(\frac{\Omega_r}{2}\right)^2 \int_0^1 \mathrm{d}r_1\, \mathrm{d}r_2 \int_0^1 \mathrm{d}x_1\, \mathrm{d}x_2 \times$$

$$\times \cos\{\Omega_r(r_1 - r_2 - \rho\, p)\} \exp\{-\ 0{,}88\, \beta_0^2\, \Omega_r^{5/6}\, q^{5/6}\}. \quad (4)$$

Note that formulas (2)–(4) are given in the form which is suitable for numerical integration. The latter procedure has been performed successively using Gauss formulas.[16] The calculations have been done only for the case of weak fluctuations ($\beta_0^2 < 1$).

Let us now introduce the factor

$$N(\rho) = \langle I^R(x_0, \rho)\rangle / \langle I_{\mathrm{incoh}}^R(x_0, \rho)\rangle - 1. \quad (5)$$

This factor allows one to make quantitative estimates of the influence of counter–wave correlation on the intensity distribution of a reflected wave by comparing with the case in which the incident and reflected waves propagate along uncorrelated paths.

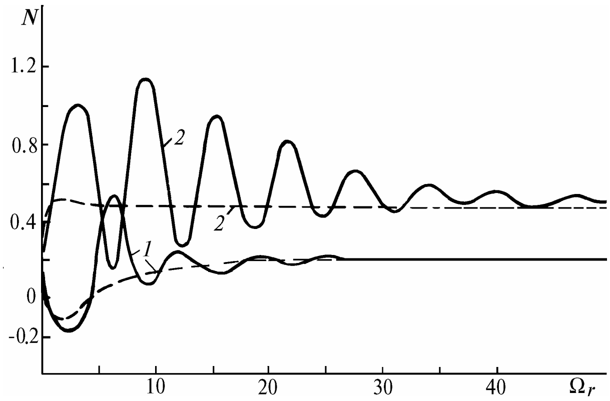

FIG. 1. The Ω_r–dependence of the amplification factor N: 1) plane mirror and 2) reflector. Solid line corresponds to rigorous calculations and dashed line is calculation on the basis of a Gaussian model of $A(\mathbf{r})$.

Figure 1 presents calculations of $N(0)$ for the case of weak intensity fluctuations ($\beta_0^2 = 0.5$) and for the detection being done at the source point. Dashed lines in this figure show the calculational results[5] obtained using a Gaussian model for the reflection coefficient distribution, i.e., $A(\mathbf{r}) = \exp\{-\,r^2/2a_r^2\}$. It is evident from this figure that in the region of values $\Omega_r \le 10^2$ the dependence of the amplification factor on the reflector Fresnel number calculated taking into account diffraction on the reflector edges is essentially different than that calculated neglecting the diffraction. This dependence is of oscillating character both in the case of a corner–cube retroreflector and a plane–mirror reflector. However, in the case of a corner–cube retroreflector the value $N(0)$ is positive though it has a large amplitude of oscillations while for a plane mirror in the region of $\Omega_r < 5$ it is negative what means that in this case a decrease of the mean intensity of a strictly backward reflected wave occurs compared to the case of uncorrelated direct and backward propagation paths. The Gaussian model also shows a decrease of the mean intensity,[5] but it does not describe an essential increase of the reflected wave intensity compared to I_{incoh} at $5 < \Omega_r < 10$. It is also can be seen from this figure that only at a sufficiently large value of Ω_r ($\Omega_r > 25$ for a plane mirror and $\Omega_r > 60$ for a corner–cube retroreflector) the influence of the diffraction on edges of a reflector on the mean intensity amplification is negligible.

It should be noted here that the effect of amplification of a wave reflected from an infinite plane mirror ($\Omega_r > 10^2$) at $\beta_0^2 < 1$ is much weaker than for a corner–cube retroreflector. The excess of the corresponding value I_{incoh} of the reflected wave intensity is only 25–35 percent what is well within the error of experimental measurements on field paths in the atmosphere.[17] It is most likely that just this circumstance explains the situation that the effect of amplification of mean intensity of a spherical wave reflected from a plane mirror[18] has been confirmed experimentally only recently and for β_0^2 exceeding unity ($\beta_0^2 \simeq 1.1$–7.6), though its existence has been shown, theoretically, long ago.[19]

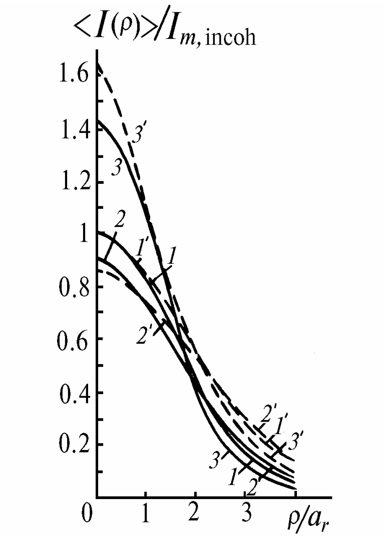

FIG. 2. Mean intensity distribution of the reflected spherical wave for $\Omega_r = 1$ and $\beta_0^2 = 0.5$: 1) forward and backward waves propagate along different paths, 2 and 3) forward and backward waves propagation along the same paths, 2′) plane mirror reflector, 3′) corner–cube retroreflector. Solid lines are rigorous calculations and dashed lines are calculations on the basis of the Gaussian model of $A(\mathbf{r})$.

Thus, the above calculations confirm the conclusion drawn in Ref. 7 that the Gaussian model for distribution of the reflection coefficient, which neglects the diffraction on the reflector edges, correctly describes the intensity distribution of a reflected wave at $\beta_0^2 < 1$ only in limiting cases of $\Omega_r \ll 1$ and $\Omega_r \gg 1$. Qualitatively it well agrees with the rigorous calculations in the region $10^{-1} \ll \Omega_r \leq 1$. A comparison of the model calculations with the rigorous ones for the distribution of a reflected wave intensity at $\Omega_r = 1$ and $\beta_0^2 = 0.5$ is shown in Fig. 2. One can see in this figure at least qualitative agreement in the magnitude and in the rates of the curves fall off down to zero level. In both cases the curves are normalized by the corresponding maximum values of the intensity I_m for uncorrelated direct and backward propagation paths.

Further increase of the parameter Ω_r into the region $1 \ll \Omega_r \leq 10^2$ in which, according to Fig. 1, the influence of the diffraction effects becomes essential, results not only in quantitative but also in qualitative differences in the reflected wave intensity distributions. Most strong these differences are at the 2π–fold Fresnel numbers of a reflector, that is, when an even number of Fresnel zones are within the reflector area. In this case we have, at the center of a diffraction pattern, the minimum of the intensity instead of its maximum.[20]

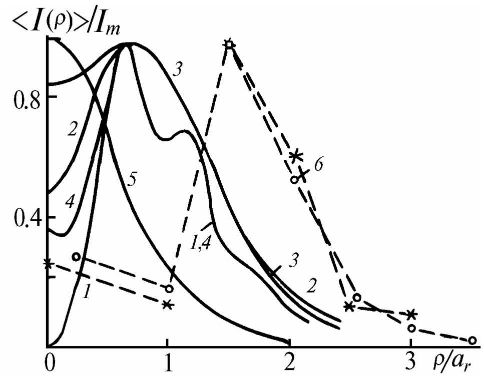

FIG. 3. Mean intensity distribution of a reflected spherical wave for $\Omega_r = 6.28$. 1) $\beta_0^2 = 0$, 2–6) 0.3 , 2–4) calculations by formulas (4), (2), and (3), respectively, 5) Gaussian model of the reflection coefficient, (3 is the reflection from a plane mirror and 4, 5, 6 – from a corner–cube refroreflector), and 6) experimental data from Ref. 7.

In Fig. 3 one can see the experimental and calculational data[7] that correspond to $\Omega_r = 6.28$. The curves presented in this figure are normalized by maximum values relevant to each of them. It can be seen from this figure that the diffraction pattern observed in a homogeneous medium ($\beta_0^2 = 0$) is washed out (see curve 1) in the presence of turbulence. In the case of reflection from a mirror the correlation of the counter waves results in a more strong washing out of the diffraction pattern while at reflection from a corner–cube retroreflector the influence of turbulence is much weaker because of a partial reversal of the wave front occurring in this case. The experimental and calculational data agree only qualitatively. One of possible causes of the difference between the experimental and calculated intensity distributions is, as it was noted in Ref. 7, deviation of actual corner–cubes from an ideal one what inevitably smears the diffraction pattern.

Analysis we have carried out above shows that under conditions of weak fluctuations ($\beta_0^2 < 1$) the diffraction on a reflector can strongly influence the distribution of intensity of a reflected wave and the backscattering amplification. All this should necessarily be taken into account when studying effects of ultra–high resolution of coherent images of objects in randomly inhomogeneous media that occur due to the backscattering amplification, as well as when analyzing efficiency of adaptive systems, in which a reflected radiation is used as a reference one.

In the case of large β_0^2 values the turbulent distortions of a reflected wave dominate compared to those due to the diffraction effects, so that the use of a Gaussian model does not introduce any noticeable error in the final results, thus being well justified. The same is also valid for reflectors with a diffusely scattering surface. In the latter case the diffraction effects are inessential at any intensity of the atmospheric turbulence along the propagation path.

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