ON THE ACCURACY OF THE REFRACTION METHOD USED IN SPACE NAVIGATION

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Based on numerical calculations an explicit dependence of the perigee altitude of sighting line H_s *on the angle of total (astronomical) refraction* r *has been obtained. An accuracy in determining* H_s *has been analyzed for different seasons and regions of the Northern Hemisphere. It is shown that in order to reduce the effect of errors in measuring* r *as well as to minimize the effect of seasonal and regional peculiarities of the refractive index field on the accuracy of determining* H_s *it is advisable to carry out spaceborne measurements of the total refraction angle within the* 10.5–11.5 km *altitude range.*

An idea to use refraction of electromagnetic waves in space navigation has long been known (see, for example, Ref. 1). It relies on the dependence of the altitude of ray perigee or sighting line perigee on the angle of total refraction. In recent years this problem has received close study.[2,3] This problem was also investigated at the Institute of Atmospheric Optics of the Siberian Branch of the Russian Academy of Sciences. The results of these investigations are partially presented in this paper. To gain a better understanding of the subsequent presentation, we briefly recall the fundamental principles used in the development of specific techniques.

It is well known that the angle of total refraction r depends not only on the distribution of the refractive index $n(h)$ along the ray path but also on the altitude of ray perigee H_0. When a source and a receiver of radiation are located outside of the atmosphere (see Fig. 1), this function can be represented, for example, in the following form[4]:

$$r = 2\left\{ \int\limits_{H_0}^{H_{\text{eff}}} \frac{\mathrm{d}h}{(R_0+h)\sqrt{\left[\dfrac{(R_0+h)\,n(h)}{(R_0+H_0)\,n_0}\right]^2 - 1}} - \arccos\frac{(R_0+H_0)\,n_0}{R_0+H_{\text{eff}}} \right\}. \qquad (1)$$

It should be noted that the use of Eq. (1) in calculation of the angle of total (astronomical) refraction is more advisable in comparison with the conventional form because of algorithmic simplification and decrease of calculation time. In navigation calculations the altitude of sighting line H_s is used, which is related with the altitude of ray perigee H_0 by the formula[5]

$$H_s = (R_0 + H_0)\,n_0 - R_0\,. \qquad (2)$$

In Eqs. (1) and (2) (obtained under assumption of the spherically symmetrical atmosphere) R_0 is the Earth's radius, H_{eff} is the height of the atmosphere above which refraction can be neglected, n_0 is the refractive index at the point of the ray perigee at the altitude H_0, and h is the current altitude along the ray path.

The given formulas make it possible to find the altitudes of ray perigee H_0 and sighting line perigee H_s from spaceborne measurements of the angle of refraction. As an example, below we list some results of the numerical experiment on determination of the altitudes H_0 and H_s carried out for the typical conditions of the Northern Hemisphere.

r, sec of arc	3000	2000	1000	500	100	50
H_0, km	2.442	7.147	12.970	17.559	27.401	31.389
H_s, km	3.741	7.989	13.361	17.870	27.441	31.411

The altitude H_0 was found from Eq. (1) by the iterative method. To calculate H_0 with an error of 1 m 5–6 iterations are required given that the choice of initial approximation is correct.

However, for obtaining such an accuracy of this method it is necessary to get a real profile of the refractive index along the ray path at the instant of measurement of the refraction angles. Moreover, the calculations carried out in Ref. 4 showed that the needed accuracy of measurements could not be achieved yet. Another factor limiting potential accuracy of Eqs. (1) and (2), which is difficult to take into account, is the difference between the real Earth's shape and mathematical figure employed in calculations. And, finally, there is one more factor resulting in low efficiency of the exact formulas. This is the error in spaceborne measuring the refraction angle. Taking the preceding into consideration, we propose a simpler method for determination of the altitudes H_0 and H_s, which does not require routine data on the refractive index profile and large volume of calculations.

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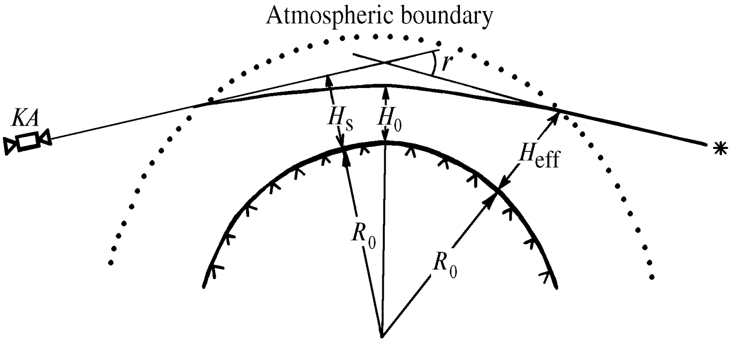

FIG. 1. Scheme of radiation propagation through the Earth's atmosphere in spaceborne measuring the total (astronomical) refraction angle r. Here H_s is the altitude of sighting line perigee, H_0 is the altitude of ray perigee, and H_{eff} is the height of the atmosphere.

For this purpose we calculated the angles of total (astronomical) refraction for H_0 up to 50 km with a step of 1 km using formula (1) for various models of vertical profile of the refractive index. The refractive index was calculated from the Owens formulas[6] for the wavelength $\lambda = 0.5$ μm and five models of vertical profiles of temperature, pressure, and humidity of air. These models were developed from the data of balloon and satellite measurements performed in 1961–1977 in three latitude belts of the Northern Hemisphere: polar (summer and winter) extending from 60 to 90°, middle (summer and winter) from 30 to 60°, and tropical from 0 to 30° (see Ref. 7). Moreover, based on these data we obtained the average model of $n(h)$ distribution for the entire Northern Hemisphere. The standard deviations of the refractive index were also calculated for all models. An integration in Eq. (1) was carried out to $H_{eff} = 100$ km (see Ref. 8). To take into account the Earth's asphericity upon integrating, we used a mean curvature radius of normal cross section of the Earth's ellipsoid instead of R_0 (see Ref. 8). The value of H_s was calculated from Eq. (2) for each H_0.

The values of H_s and refraction angles r obtained in such a manner in a wide interval of the altitudes H_0 were tabulated and used in searching for an explicit dependence of H_s on r. As a result, a simple but sufficiently exact formula was obtained

$$H_s = b_0 + b_1 \ln r + b_2 (\ln r)^2 . \quad (3)$$

Similar dependence was found for the attitude H_0 as well

$$H_0 = a_0 + a_1 \ln r + a_2 (\ln r)^2 . \quad (4)$$

In these formulas the refraction angle is in sec of arc, while H_0 and H_s are in km. The coefficients a_i and b_i ($i = 0, 1, 2$) were calculated for all models by the method of least squares in different altitude ranges $\Delta H_0 = H_0 \ldots 50$ km, where H_0 changed from 0 to 25 km with a step of 1 km. The calculations showed that the rms error in approximating H_s and H_0 by formulas (3) and (4) decreased with increasing H_0. Minimum rms errors in Eqs. (3) and (4) were obtained for $H_0 \geq 13$ km for "cold" models (polar models and mid–latitude model in winter). As to "warm" models (tropical model and mid–latitude model in summer), this threshold altitude was about 17 km. The coefficients a_i and b_i change as functions of the altitude H_0 and an employed model. Some results of these calculations for the typical conditions of the Northern Hemisphere are listed in Table I.

TABLE I. Typical values of the coefficients a_i and b_i in formulas (3) and (4) for indicated ranges of altitudes $\Delta H_0 = H_0 \ldots 50$ km *and their rms errors σ_H by the example of average–annual model of the Northern Hemisphere.*

ΔH_0, km	a_0	a_1	a_2	σ_H, km	b_0	b_1	b_2	σ_H, km
5...50	43.675	−1.5382	−0.42666	0.30	46.773	−2.7601	−0.3053	0.25
10...50	51.666	−4.4801	−0.16257	0.12	53.464	−5.2240	−08413	0.10
15...50	56.700	−6.4090	0.01884	0.06	57.676	−6.8382	0.06774	0.05
20...50	59.670	−7.5928	0.13534	0.02	60.170	−7.8307	0.16522	0.01

To use these formulas in practice it is necessary to bear in mind that for high altitudes of ray perigee the values of refraction angles can be comparable to the rms errors in their measurements. In its turn, this can result in large rms errors in determining H_s and H_0. The rms error $\sigma_H(r)$ caused by the rms error in measuring the refraction angles σ_r can be evaluated by the formula following from Eq. (3)

$$\sigma_H(r) = (b_1 + 2b_2 \ln r) \frac{s_r}{r} . \quad (5)$$

Its typical values for the rms error in measuring the refraction angles $\sigma_r = 10$ sec of arc are presented in Table II. The preceding can be confirmed by the tabulated data. For the given permissible rms error in determining the altitude of sighting line perigee formula (5) can be used to estimate the needed accuracy in spaceborne measuring the refraction angles as well as the upper boundary of the interval ΔH_0 in which measurements of the refraction angles are considered to be appropriate.

TABLE II. Altitude of the sighting line perigee H_s *and the rms error in its determination* $\sigma_H(r)$ *for indicated values of the refraction angle r measured with the rms error* σ_r = 10 sec of arc.

r, sec of arc	50	100	200	500	1000	2000
H_s, km	31.4	27.4	23.2	17.9	13.4	8.00
$\sigma_H(r)$, km	1.31	0.63	0.30	0.12	0.06	0.03

As noted in discussing the limitations of the method of determination H_s from the exact formulas, the main and practically uncorrectable source of errors is the spatiotemporal variability of the refractive index profile. This is especially true for Eqs. (3) and (4) whose coefficients a_i and b_i are determined by the seasonal regional atmospheric models. In order to estimate the systematic rms error $\sigma_H(n)$ caused by the seasonal and regional variability of the refractive index, the rms errors of refraction angles $\sigma_r(n)$ were calculated for each model. The values of $\sigma_r(n)$ were calculated by the formulas presented in Ref. 9 without regard for correlations. The value of $\sigma_H(n)$ was calculated from Eq. (5) in which $\sigma_r(n)$ was taken instead of σ_r for the corresponding model.

The calculations show that within the 10–30 km altitude range the value of $\sigma_H(n)$ ranges from 0.6 to 0.3 km slightly varying from model to model. As the altitude H_0 decreases, the value of $\sigma_H(n)$ increases markedly and reaches 1–2 km near the Earth's surface. It becomes possible to slightly decrease the variance of the ray altitude with due regard to the vertical correlations of the meteorological parameters.[2] The estimates carried out for three sites of the territory of the Commonwealth of Independent States showed that $\sigma_H(n)$ varied from 0.15 to 0.55 km within the 5–20 km altitude range. Further decrease of $\sigma_H(n)$ requires closer consideration of the regional and seasonal peculiarities of vertical structure of meteorological fields. This problem can be solved in different ways: from a simple averaging over some latitude belts[2,7] to the choice of the quasiuniform regions with allowance for their temporal stability for atmospheric processes of global and synoptic scales.[10] Thus, the authors of Ref. 2 propose to use 10 models to determine H_s. In Ref. 10 20 quasiuniform regions were indentified in winter season and 17 – in summer for the Northern Hemisphere. In addition, a monthly classification was performed for each region.

One can use one or other number of models depending on the performance characteristics of onboard computers and permissible error in the determination of H_s. The measurement error of the refraction angle contributing significantly to the total error in the determination of H_s is also of great importance. However, it is evident that closer consideration of the regional, synoptic, and seasonal peculiarities of the vertical distribution of the refractive index decreases to a greater extent the systematic error in determination of the ray perigee altitude ΔH_s.

In order to estimate the possible value of ΔH_s, we calculated the values of H_s^i for the models presented in Refs. 2 and 7 as well as the values of $\overline{H}_s$ for the entire Northern hemisphere using the average–annual profile of the refractive index. Calculations of H_s^i and $\overline{H}_s$ were carried out by Eqs. (1) and (2) for the refraction angles ranging from 50 to 4 000 sec of arc. The error in calculation of H_s by the iterative method was assumed equal to 1 m. Some results of these calculations are presented in Table III. As could be expected, the values of H_s^i calculated for one and the same refraction angle differ essentially for various models. In this case the differences in the values of H_s^i exhibit some regular trends. This is readily illustrated by Fig. 2 which shows the differences $\Delta H_s^i = H_s^i - \overline{H}_s$ within the investigated range of refraction angles.

TABLE III. Altitude of sighting line perigee H_s (km) *for various models of the atmosphere. 1) Tropical model, 2 and 3) mid–latitude model in winter and summer, 4 and 5) polar model in winter and summer, and 6) Northern Hemisphere as a whole.*

r, sec of ars	Models					
	1	2	3	4	5	6
3500	2.44	2.73	3.57	3.22	4.12	3.32
3000	3.74	3.96	4.59	4.39	5.27	4.43
2000	7.29	7.60	8.12	8.13	8.69	7.99
1000	13.84	13.83	13.26	13.21	12.89	13.36
500	18.69	18.15	17.64	17.46	17.11	17.87
50	31.54	31.90	31.29	32.01	31.01	31.41

As can be seen from this figure, the differences in the values of H_s^i for various models are considerable and exhibit regular trend except for narrow range of the refraction angles stretching approximately from 1200 to 1350 sec of arc. In this interval, which corresponds to the 10.5–11.5 km altitude range, the values of ΔH_s^i are minimum and practically independent of the employed atmospheric model. This conclusion is also confirmed when we use atmospheric models developed in Ref. 2 in more detail. Moreover, as statistics claims, the systematic error can be neglected if its value does not exceed 1/5 of the total random error.[11] Since, as has already been noted above, the random error in determining H_s caused only by intraseasonal and intraregional variability of the atmosphere reaches ~ 0.5 km, the values of ΔH_s^i can be neglected.

The results make it possible to simplify substantially the technique for determining the altitude of sighting line perigee H_s. Virtually, one can measure the refraction angles within a narrow angular range and H_s can be calculated from simple formula (3) for the minimum number of models or only for one model of the entire Northern Hemisphere. In addition, as follows from Eq. (5) and Table II, the refraction measurements in the indicated range of altitudes substantially decrease the contribution of measurement errors in the total error in determining H_s.

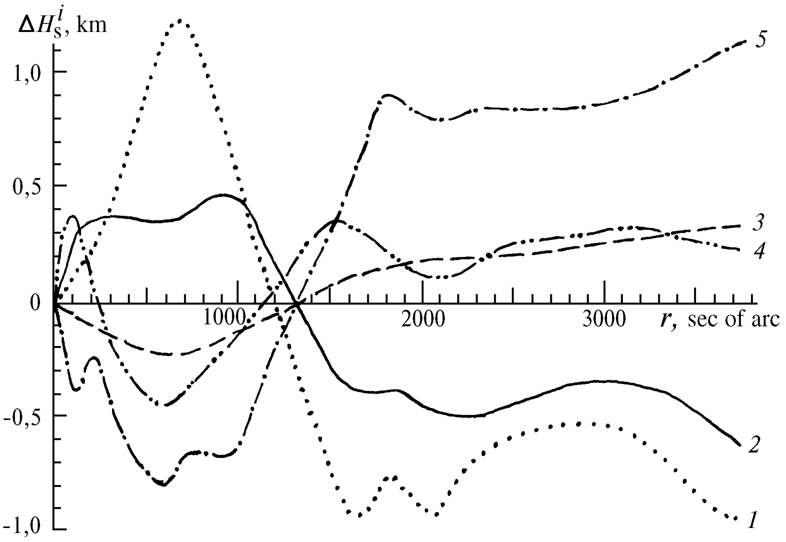

FIG. 2. Systematic error in determining the altitude of sighting line perigee ΔH_s^i as a function of measured astronomical refraction angle r for various models of the atmosphere. 1) Tropical model, 2 and 3) mid–latitude model in summer and winter, and 4 and 5) polar model in summer and winter.

In conclusion it should be noted that no consideration has been given to the effect of horizontal nonuniformity of the refractive index field on the accuracy of the determination of H_s in this paper. This is due to the lack of the reliable data on the profiles of horizontal gradients of the refractive index both for the territory of the Northern Hemisphere and for the individual latitude belts.

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