ANALYTIC PREDICTIONS OF UPLINK THERMAL BLOOMING STREHL RATIOS

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We give an overview of functional reconstruction theory for predicting whole beam Strehl ratios in real time as applied to thermal blooming. This technique is based on our exact analytic solution to the problem of the interaction of thermal blooming and turbulence.[1] *We begin by writing down the reconstruction formula that relates a finite sized beam Strehl ratio to a sum involving the Strehl ratio of an infinite beam or periodic patch. We then define what a dynamically equivalent patch is and follow by describing the functional approach to scaling patch Strehl curves using a metric on the space of absorption profiles. We end by comparing results from our systems model based on functional reconstruction, AMPERES, which takes only seconds on any machine, with results from large nonlinear* 4–d *wave optics simulations.*

1. OVERVIEW OF FUNCTIONAL RECONSTRUCTION

Functional reconstruction involves three steps. The first step is to relate infinite beam (or periodic patch) structure functions[1] to the complex amplitude of a finite sized or whole beam. This provides a reconstruction formula that relates the subaperture or patch structure functions to the whole beam Strehl ratio. In this way, the computation of the Strehl ratio of the whole beam is reduced to a single sum of separate patch results. The next step is to match the dynamical characteristics of the patch to that point in the whole beam where the patch is placed. The final step is to apply functional calculus to the patch structure functions to scale a patch at one point in the beam to the appropriate patch at another point. In this way, relatively few patches are needed to reconstruct whole beam Strehl ratios under a wide variety of physical beam characteristics. The resulting numerical construction for predicting these whole beam Strehl ratios takes only seconds on any machine.

2. THE RECONSTRUCTION FORMULA

A reconstruction formula relates a finite sized or whole beam Strehl ratio to a sum involving the structure functions of an infinite beam, or equivalently, of a small periodic patch of the beam. The basic idea is to find a mathematical estimate of the complex amplitude of the whole beam in terms of infinite beam structure functions. Intuitively, one can see that a reconstruction is possible, because the dynamically relevant and dominant spatial scales, which are ultimately set by turbulence, are much smaller than the beam size. The Strehl ratio of a large beam then reduces to a local property of the beam. Mathematically, the existence of a reconstruction formula is guaranteed by Eq. (1) below because the complex amplitude is itself a local property of the beam.

Let $U(\mathbf{x}, z)$ be the complex amplitude of a whole (i.e., finite sized) beam at the point $\mathbf{x}$ and altitude z. The Strehl ratio, S, of a uniform flat–top beam at $z = L$ is

$$S = \left| \int d^2x\, U(\mathbf{x}, L) \right|^2 . \tag{1}$$

A mathematical estimate of the complex amplitude $U(\mathbf{x}, z)$, in terms of the patch or infinite beam structure functions is

$$\left| U(\mathbf{x}, z) \right| \sim S_p^{1/2} (\phi(\mathbf{x}, z)) , \tag{2}$$

where $\phi(x, z)$ is the total amount of heating that has occurred at $\mathbf{x}$ in the beam at altitude z and where S_p is the patch Strehl ratio evaluated at (nondimensional) time $\phi(\mathbf{x}, z)$ (Ref. 1). Using this estimate, the Strehl ratio of the whole beam in terms of the patch Strehl ratio is

$$S = \left| \int d^2x\, S_p^{1/2} (\phi(\mathbf{x}, L)) \right|^2 . \tag{3}$$

The generalization of the reconstruction formula to nonuniform beams is

$$S = \left| \frac{\int d^2x\, S_p^{1/2}(\phi(\mathbf{x}, L))\, I_0^{1/2}(\mathbf{x}, 0)}{\int d^2x\, I_0^{1/2}(\mathbf{x}, 0)} \right|^2 . \tag{4}$$

Therefore, to make Strehl predictions for a finite sized beam from patches or infinite beams, we need to know the patch Strehl curve, $S_p(\phi)$, as a function of time, ϕ, of the dynamically equivalent periodic patch.

Next we must define what a dynamically equivalent patch is.

3. DYNAMICALLY EQUIVALENT PATCHES

Consider a point $\mathbf{x}$ in the whole beam at $z = L$ (see Fig. 1). Looking back down the beam we can associate the following with each point $\mathbf{x}$ in the beam:

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a heating rate profile $= \dfrac{\partial}{\partial z}\dfrac{D}{Dt}\,\phi = \Gamma\, k\, \alpha(z)\, I_0(\mathbf{x}, z)$, (5)

a heating profile $= \dfrac{\partial f}{\partial z}$,

a heating rate $= \dfrac{d\phi}{dt}$,

and total heating $= \phi(\mathbf{x})$.

Here, D/Dt is the usual convective derivative based on the wind component transverse to the beam direction, $\kappa = 2\pi/\lambda$ is the beam wave number, Γ is a mixture of thermodynamics constants, and $\alpha(z)$ is the absorption profile.

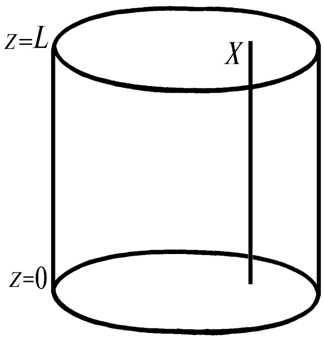

FIG. 1. With each column below the point **x** *in the beam we can associate a heating rate profile, a heating profile, a heating rate, and total heating. The dynamically equivalent patch has the same heating profile and heating rate.*

The dynamically equivalent patch or infinite beam appropriate to use at a point **x** in the whole beam is the one that matches the heating profile and total heating rate as seen at a point **x** looking down the beam. To arrive at the heating profile looking down from the point **x** in the whole beam, we must integrate the convective derivative. This integral over t at each altitude z in the whole beam case translates into a line integral in the transverse direction along the wind direction from the edge where the air entered the beam to the point **x**. Thus, depending upon the wind profile, the heating profile can vary from point to point in the whole beam. This means that potentially we would have needed a different patch Strehl curve at each point of the beam. So to make this reconstruction method practical, we will scale one patch Strehl curve generated with a particular heating profile and heating rate to another patch Strehl curve at a different heating profile and rate. In this way, we can reconstruct the entire beam with relatively few patch Strehl curves under a large variety of physical conditions.

4. SCALING HEATING PROFILES ON A PATCH

Infinite beams or patches with periodic boundary conditions have no mathematically relevant edges. Since there are no edges, all air spends the same amount of time in the beam. The moment of time $t = 0$ for a patch or infinite beam is defined by when the laser is turned on and not by when a parcel of air enters the beam as in the whole beam case. The patch heating rate profile does not involve the convective derivative, thus the heating profile on a patch is strictly proportional to the absorption profile:

$$\frac{\partial\phi}{\partial z} = \Gamma\, k\, \alpha(z) \left(\int_0^t d\,t'\, I(z, t') \right), \qquad (6)$$

Therefore, in order to scale the heating profile of a patch, we scale the absorption profile. Since the scaling parameter is a function, $\alpha(z)$, we must use the functional calculus to accomplish this. The heating rate on a patch may be scaled by adjusting the intensity of the patch.

5. FUNCTIONAL SCALING OF ABSORPTION PROFILES

The patch or infinite beam Strehl curve (i.e., the Strehl ratio as a function of time) is a functional of the absorption profile, $S_p = S_p[\alpha]$. The perturbative solution of the linear theory of thermal blooming of infinite beams for nonuniform atmospheres yields an analytical expression for the patch Strehl curve in the form of a functional Taylor expansion of $\ln(S_p[\alpha])$ about the profile $\alpha(z) = 0$ (i.e., no blooming). This result[1] can be written as

$$S_p(\alpha,\phi) = \exp\left(-\,0.093\,(N_T)^{-5/6} \sum_{nm} A_{nm}(\alpha, \phi) \right), \qquad (7)$$

where $N_T = r_0^2/\lambda L$ is the turbulence Fresnel number, and where $\phi = \Gamma\kappa t \int dz\alpha(z)I_0(z)$ is the number of radians of blooming – a nondimensional measure of time. The coefficients $A_{nm}[\alpha]$ are functionally proportional to α^{n+m} and depend on the atmospheric turbulence profile and the actuator spacing of the adaptive optics.

The explicit form for $S_p[\alpha]$, Eq. (7), can be used to scale patches with different heating profiles (i.e., absorption profiles). To do this, we use $S_p[\alpha]$ to define a metric on the space of all normalized absorption profiles. This metric tells us when two absorption profiles are close to each other in the sense that they produce the same patch Strehl curve. The functional scaling of absorption profiles is based on interpolating the "distance" between absorption profiles.

To construct the metric we first consider the space of all normalized absorption profiles (see Fig. 2). Above each point in this space of absorption profiles, we associate a "height" equal to $S_p[\alpha]$ (see Fig. 3). This produces a "surface" over the space of profiles. Now we look for the critical points of this surface, and in particular for the point, $\alpha_0(z)$, that maximizes the height and the curvature. We then use this artificial absorption profile as a metric on this space to do scaling with. Two points (i.e., two profiles) will be close to each other when they produce the same amount of blooming as measured by the Strehl ratio, or when they have the same overlap with $\alpha_0(z)$.

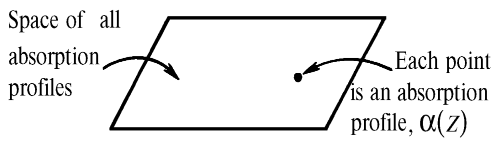

FIG. 2. The back drop upon which the functional scaling of patches occurs is the function space of all absorption profiles. Each point in this space is an absorption profile or function of z.

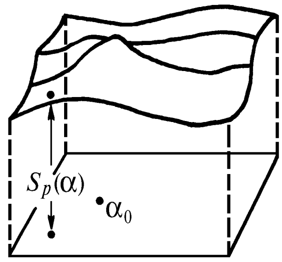

FIG. 3. Above each point in the space of absorption profiles we associate a "height" equal to the patch Strehl, $S_p[\alpha]$. This defines a "surface" above the space of profiles. α_0 is a critical point of that "surface".

In practice, our algorithm uses only the A_{11} term[1] of Eq. (7) to find $\alpha_0(z)$. Since this term is quadratic in α,

$$A_{11} = \int \mathrm{d}\, z' \mathrm{d}\, z\, \alpha(z')\, F(z', z)\, \alpha(z), \tag{8}$$

then $\alpha_0(z)$ is the eigenvector that produces the maximum eigenvalue of the operator $F(z', z)$. When there is significant wind shear, the maximum eigenvalue profile will depend on the heating rate.

An example of such an eigenvector absorption profile is given in Fig. 4. In this particular case, the atmospheric turbulence profile is Hufnagel–Valley 5/7, the background wind is Bufton, the heating rate is 60 radians/sec, and $L = 5$ km. A typical profile based on measurements is included for comparison.

6. FUNCTIONAL INTERPOLATION OF HEATING PROFILES USING α_0

Now consider two different patches, 1 and 2, with different absorption profiles, α_1 and α_2, i.e., with two different heating profiles. These patches could be two samples from different parts of the beam. Suppose that we have $S_p[\phi_1, \alpha_1]$, the patch Strehl curve for patch 1 and we would like to know $S_p[\phi_2, \alpha_2]$. Let us discuss how we scale the two patches using the metric $\alpha_0(z)$.

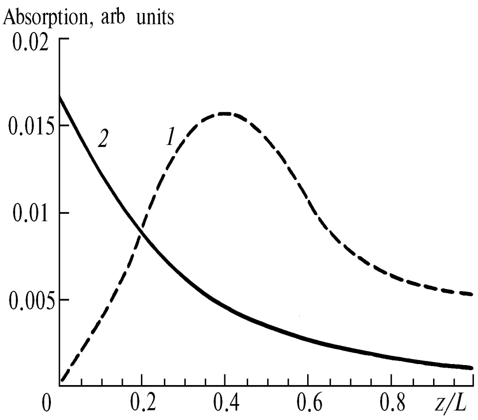

FIG. 4. An example of an eigenvector absorption profile, α_0, in comparison with an actual absorption profile.

The notation we use is

$$<(a \mid b)> = \int \mathrm{d}\, z\, a(z) b(z), \tag{9}$$

and for convenience assume, for the moment, that both profiles we are considering contain the same total amount of absorption,

$$<(\alpha_1 \mid 1)> = \int \mathrm{d}\, z\, \alpha_1(z) = <(\alpha_2 \mid 1)> = \int \mathrm{d}\, z\, \alpha_2(z)\,. \tag{10}$$

After an elapsed amount of time, t, the total amount of heating will be

$$\phi_1 = <\frac{\partial \phi_1}{\partial z} \mid 1> \text{ and } \phi_2 = <\frac{\partial \phi_2}{\partial z} \mid 1>\,, \tag{11}$$

$\phi_1 \neq \phi_2$ because the absorption is distributed differently in the two patches.

Since α_0 is the dominant contribution to the heating, ϕ_1 may be related to ϕ_2 by simply determining how much of an overlap there is numerically between α_0 and α_1 and how much overlap there is between α_0 and α_2. In formal terms, define an α_0 projection operator,

$$P = \frac{\mid \alpha_0 > < \alpha_0 \mid}{<\alpha_0 \mid \alpha_0>}\,, \tag{12}$$

Then,

$$\phi_1 = \frac{\partial \phi_1}{\partial z} \mid 1 \propto <\alpha_1 \mid 1> \xrightarrow{P} \frac{<\alpha_1 \mid \alpha_0><\alpha_0 \mid 1>}{<\alpha_0 \mid \alpha_0>}, \tag{13}$$

and,

$$\phi_2 = \frac{\partial \phi_2}{\partial z} \mid 1 \propto <\alpha_2 \mid 1> \xrightarrow{P} \frac{<\alpha_2 \mid \alpha_0><\alpha_0 \mid 1>}{<\alpha_0 \mid \alpha_0>}, \tag{14}$$

hence,

$$\frac{\phi_1}{\phi_2} \approx \frac{<\alpha_1 \mid \alpha_0>}{<\alpha_2 \mid \alpha_0>}\,, \tag{15}$$

If the total absorption is not the same in the two patches, then

$$\frac{\phi_1}{\phi_2} \approx \frac{<\alpha_1 \mid \alpha_0>}{<\alpha_2 \mid \alpha_0>} \frac{<\alpha_2 \mid 1>}{<\alpha_1 \mid 1>}\,. \tag{16}$$

Thus, if we know $S_p[\alpha_1]$ and are given ϕ_2 and α_2 but not $S_p[\alpha_2]$, then we substitute $S_p[\tilde{\phi}_1, \alpha_1]$ for $S_p[\phi_2, \alpha_2]$ where $\tilde{\phi}_1$ is computed from ϕ_2 via Eq. (16). In this way, a limited number of patch Strehl curves may be used to construct a whole beam under a large variety of wind/slew/absorption conditions.

7. AMPERES

We have developed a systems model, AMPERES, implementing the functional reconstruction theory just described. This systems model employs just a few patch Strehl ratios to reconstruct the Strehl for a finite beam under a large variety of physical conditions. It runs on any machine in seconds and predicts the Strehl ratio of a finite sized beam at a wind clearing time. The systems model has been implemented at four different wavelengths: 0.41 μm, 1.06 μm, 1.3 μm, and

3.8 μm. So far, AMPERES is designed for unidirectional background winds.

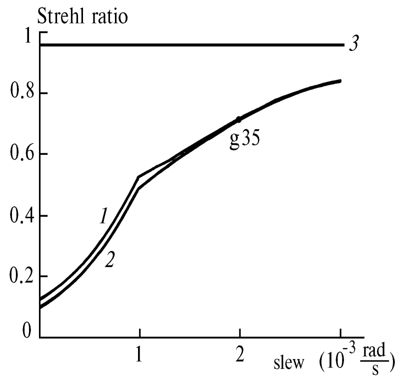

FIG. 5. Comparison of Strehl ratio predictions at 2.5 km *at a wind clearing time for a* 2.5 mW, 1.4 m *uniform flat–top beam. The background turbulence profile (3) is SLCday, the absorption is nonuniform, and the background wind is uniform. All AMPERES points (1) are based on just* 2 *patches. Each PHOTON point (2) represents a large nonlinear* 4–d *wave optics simulation. g35 is a PHOTON run designation.*

In Fig. 5. we compare Strehl predictions at a wind clearing time from AMPERES with results from NERA's nonlinear 4–d wave optics code PHOTON. All AMPERES points are based on just 2 patches. Each PHOTON point represents a large and long numerical simulation. In this comparison, the background wind is uniform. In Fig. 6 a similar comparison is made for a truncated Gaussian beam propagated to 5 km. However, in this set the background wind is Bufton which contains a large amount of wind shear above 2.5 km.

Functional reconstruction offers an economical and fast prediction of whole beam Strehls whether for routine calculations, increased precision or pre–experiment calibration or guidance. Reconstruction accuracy improves as the beam diameter increases, thus functional reconstruction is the economical approach to large beam diameter predictions where numerical simulations are impractical or inaccurate due to numerical resolution requirements.

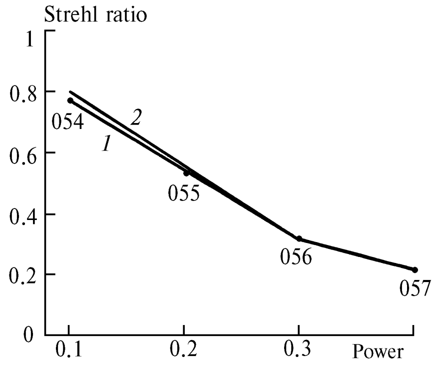

FIG. 6. Comparison of Strehl ratio predictions at 5.0 km *at a wind clearing time for a* 1.5 m *truncated Gaussian beam. In this case, the background wind is Bufton which contains a large amount of wind shear above* 2.5 km. *o54–o57 are PHOTON run designations. 1) AMPERES and 2) PHOTON runs.*

REFERENCES

1. S. Enguehard and B. Hatfield, J. Opt. Soc. Am. A. **8**, 637 (1991).