ACTIVE STOKES POLARIMETRY UNDER CONDITIONS OF PARTIALLY–POLARIZED BACKGROUND ILLUMINATION

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In this paper we discuss an original technique for eliminating the contribution of a partially–polarized background component of an input optical field to the components of the Stokes vector in the study of the polarization characteristics of different natural objects. This technique is based not only on the polarization modulation of a light field in a recording channel but also on modulation of the intensity of a light field in a sounding channel of a polarimeter. This technique can be implemented in the optical scheme of any modulation polarimeter. It is shown that in so doing, no information about the absolute values of the initial phase and frequency of the amplitude modulation is needed.

In polarization measurements under natural conditions, a background component of optical field under investigation always acts at an input of a polarimeter in addition to an informative component. The former results in the significant distortion of the data on the polarization structure of the received optical field. Under laboratory condition this background component can be reduced to zero, but natural conditions this is practically impossible. The use of interference filters does not completely solve this problem. In this paper we discuss the active Stokes polarimetry in the sense that the polarization structure of optical field under investigation at the input of the polarimeter is the result of the transformation of conversion of sounding field in the process of its interaction with an object.

It is, obvious that, the informative and background components of input optical field are incoherent, and the resultant light action at the input of the Stokes polarimeter is described by the Stokes vector $S^{\rm res} = (S_0+S_0^{\rm bg}, S_1+S_1^{\rm bg}, S_2+S_2^{\rm bg}, S_3+S_3^{\rm bg})$, where (S_0, S_1, S_2, S_3) and $(S_0^{\rm bg}, S_1^{\rm bg}, S_2^{\rm bg}, S_3^{\rm bg})$ are the Stokes vectors of the informative and background components, respectively. Let us assume that the receiving channel of the Stokes polarimeter comprises a rotating phase plate, an immovable horizontally oriented analyzer, and a photodetector next to it. Such an optical scheme provides for a signal with the most simple spectrum containing four frequency components whose intensities are proportional to the corresponding components of the Stokes vector of the input field. The Stokes vector of optical field at the photodetector input can be found from the matrix equation[1]

$$S^{\rm pd} = [M_2]*[M_1]*S^{\rm res}, \qquad (1)$$

where M_1 and M_2 are the Müller matrices of the rotating phase plate and the analyzer, respectively.

As a result, the photodetector signal that represents the first component of the Stokes vector $S_0^{\rm pd}$ has the form

$$S_0^{\rm pd} = \frac{1}{2}(S_0 + S_0^{\rm bg}) + \frac{1}{4}(S_1 + S_1^{\rm bg})(1 + \cos\delta) + \frac{1}{4}(S_1 + S_1^{\rm bg})(1 - \cos\delta)\cos(4\Omega_2 t) + \frac{1}{4}(S_2 + S_2^{\rm bg})(1 - \cos\delta)\sin(4\Omega_2 t) - \frac{1}{2}(S_3 + S_3^{\rm bg})\sin\delta\sin(2\Omega_2 t), \qquad (2)$$

where δ is the phase shift due to the rotating phase plate and Ω_2 is the frequency of the polarization modulation.

Furthermore by taking the discrete Fourier transform of the intensities of the spectral components of signal (2), we obtain the explicit form of the components of the Stokes vector of the informative component of input field

$$S_0 = 2\left[I_0 - \frac{1}{4}(S_1 + S_1^{\rm bg})(1 + \cos\delta) \right] - S_0^{\rm bg},$$

$$S_1 = [4\, I_{4c} / (1 - \cos\delta)] - S_1^{\rm bg},$$

$$S_2 = [4\, I_{4s} / (1 - \cos\delta)] - S_2^{\rm bg};$$

$$S_3 = [2\, I_{2s} / \sin\delta] - S_3^{\rm bg}, \qquad (3)$$

where I_{4c}, I_{4s}, I_{2s}, and I_0 are the intensities of the corresponding spectral components of signal (2).

It follows from Eq.(3) that in order to calculate the components of the Stokes vector of the informative field, we must eliminate the quantities $S_0^{\rm bg}$, $S_1^{\rm bg}$, $S_2^{\rm bg}$, and $S_3^{\rm bg}$ from the system of equations (3). This can be done by additional intensity modulation of the sounding field. Actually, assuming that the intensity is modulated by the sinusoidal law and using matrix equation (1) for the signal at the photodetector, we obtain

$$S_{0\,\rm mod}^{\rm pd} = \frac{1}{2}\left[S_0^{\rm bg} + \frac{1}{2}S_1(1 + \cos\delta) + \frac{1}{2}S_1^{\rm bg}(1 - \cos\delta)\cos(4\Omega_2 t) + \frac{1}{2}S_2^{\rm bg}(1 - \cos\delta)\sin(4\Omega_2 t) - S_3^{\rm bg}\sin\delta\sin(2\Omega_2 t) \right] + \frac{1}{2}\left[S_0 + \frac{1}{2}S_1(1 + \cos\delta) + \right.$$

$$+\frac{1}{2}S_1\,(1-\cos\delta)\cos(4\Omega_2 t)+\frac{1}{2}S_2\,(1-\cos\delta)\sin(4\Omega_2 t)-$$

$$-S_3\sin\delta\sin(4\Omega_2 t)\Big]\,[1+\sin(\Omega_1 t)]\,, \qquad (4)$$

where Ω_1 is the frequency of modulation of the intensity of sounding optical field.

For $\Omega_1 \gg 4\Omega_2$ the frequency spectrum of the photodetector signal is divided into two ranges: the low–frequency range caused by the modulation of polarization, and the high–frequency range caused by the intensity modulation. In this case the partially–polarized background component of the input field makes no contribution to the high–freguency photodetector signal. For instrumental separation of the frequency ranges $4\Omega_2$ and $(\Omega_1 - 4\Omega_2)$ must be separated fully by an order of magnitude. It is well known[2] that a rate of decay of the amplitude-frequency characteristics (AFC) of 80 dB per octave can be obtained by means of iterative passive filtration with 80–90 dB noise suppression outside the transmission band of a filter. When the informative and background components at the input of the polarimeter are equal to each other, the background contribution to the relative error in measuring the components of the Stokes vector of the informative field does not exceed 0.01%.

By calculating the intensities of the high–frequency signal components given by Eq. (4), we obtain

$$S_0 = I_0 - [4\, I_1\,(1+\cos\delta)\,/\,(1-\cos\delta)]\,,$$

$$S_1 = 8\, I_1\,/\,(1-\cos\delta)\,,$$

$$S_2 = 8\, I_2\,/\,(1-\cos\delta)\,,$$

$$S_3 = -\,4\, I_3\,/\,\sin\delta\,, \qquad (5)$$

where I_0 is the intensity of component of the signal (4) at the frequency Ω_1; I_1, I_2, and I_3 are the signal intensities at the frequencies $\sin(\Omega_1 - 4\Omega_2)$, $\cos(\Omega_1 - 4\Omega_2)$, and $\cos(\Omega_1 - 4\Omega_2)$, respectively. The summed combination frequencies of signal (4) can also be used.

Of fundamental importance here is the fact that for measuring the components of the Stokes vector, in addition to the data on the absolute value of the polarization modulation frequency and its initial phase, the information is needed about the absolute value and the initial phase of the frequency of the intensity modulation and about the phase difference between the signals at these frequencies. In practice this means that the instrumental synchronization of the measuring process with the frequency of the intensity modulation as well as of these frequencies is needed. This can be avoided if we take the Fourier transform of the square of the signal intensity rather than of the signal given by Eq. (4) in the high–frequency range of the spectrum.

$$(S^{\rm pd}_{0\,{\rm hf}(4)})^2 = \frac{1}{2}\Big[\,S_0^2 + \frac{1}{2}S_1^2\,(1+\cos\delta)^2 + \frac{1}{2}S_1^2\,(1-$$

$$-\cos\delta)^2\cos^2(4\Omega_2\ t) + \frac{1}{2}S_2^2\,(1-\cos\delta)^2\sin^2(4\Omega_2\, t) +$$

$$+S_3^2\sin 2\delta\sin^2(2\Omega_2\, t) + S_0\,S_1\,(1+\cos\delta) + S_0\,S_1\,(1-$$

$$-\cos\delta)\cos(4\Omega_2\, t) + S_0\,S_2\,(1-\cos\delta)\sin(4\Omega_2\, t) -$$

$$-\,2S_0\,S_3\sin\delta\sin(2\Omega_2\, t) + \frac{1}{2}S_1^2\,(1-\cos\delta)\cos(4\Omega_2\, t) +$$

$$+\frac{1}{2}S_1\,S_2\,(1-\cos 2\delta)\sin(4\Omega_2\, t) - S_1\,S_3\,(1+$$

$$+\cos^2\delta)\sin\delta\sin(2\Omega_2\, t) + \frac{1}{2}S_1\,S_2\,(1-$$

$$-\cos\delta)^2\cos(4\Omega_2\, t)\sin(4\Omega_2\, t) - S_1\,S_3\,(1-$$

$$-\cos\delta)\cos(4\Omega_2 t)\sin(2\Omega_2\, t)\sin\delta - S_2\,S_3\,(1-$$

$$-\cos\delta)\sin(4\Omega_2\, t)\sin(2\Omega_2\, t)\,{\rm sind}\,\Big]\sin^2(\Omega_1\, t)\,. \qquad (6)$$

The analysis of Eq.(6) shows that the values S_0, S_1, S_2, and S_3, can be found from the expression for the intensities of the fundamental, second, and fourth harmonics of the polarization modulation frequency, i.e., from the system of equations

$$I_0 = S_0^2 + \frac{1}{4}S_1^2\,(1+\cos\delta)^2 + \frac{1}{8}S_1^2\,(1-\cos\delta)^2 +$$

$$+\frac{1}{8}S_2^2\,(1-\cos\delta)^2 + \frac{1}{2}S_3^2\sin 2\delta + S_0\,S_1\,(1-\cos\delta)\,,$$

$$I_{4s} = S_0\,S_2\,(1-\cos\delta) + \frac{1}{2}S_1\,S_2\,(1-\cos 2\delta)\,, \qquad (7)$$

$$I_{2s} = -\,2\,S_0\,S_3\sin\delta - S_1\,S_3\,(1+\cos\delta)\sin\delta -$$

$$-\frac{1}{2}S_1\,S_3\,(1-\cos\delta)\sin\delta\,,$$

$$I_{4c} = -\frac{1}{2}S_3^2\sin\delta + S_0\,S_1\,(1-\cos\delta) + \frac{1}{2}S_1^2\,(1-\cos 2\delta)^2\,.$$

It can also be shown that in general the Jacobian of the system of equations (7) is nonzero.

Thus the intensity of the sounding field can be additionally modulated to eliminate the effect of the partially–polarized background component on the results of measuring the Stokes vectors of the informative field. In so doing we don't need any information about the absolute value and the initial phase of the intensity modulation frequency. Moreover, our analysis shows that this frequency may arbitrarily vary with time, but these variations must be slower than the quadrupolar polarization modulation frequency. This allows one to use electromechanical modulators.

Let us consider the implementation of the technique of double modulation in the above described polarimeter with the use of mechanical chopper for the intensity modulation. After the chopper the sounding field takes the form of a train of pulses whose shapes are nearly rectangular, i. e., the pulse modulation of the intensity is carried out. Then using the representation of the pulse train in the form of a Fourier series and Eq. (1) for the photodetector signal on account of Eq.(2) we found

$$S^{\rm pd}_{0\,{\rm imp}} = S^{\rm pd}_{0\,(2)}\left[\,1 + \frac{2}{\pi}\sum\frac{1}{(2\kappa+1)}\sin\left((2\kappa+1)\Omega_1\right)\right]. \qquad (8)$$

The signal given by Eq. (8) is analogous to the signal given by Eq. (4) in the structure of the frequency spectrum. If we neglect all the terms of the Fourier series except the first one, a situation occurs that completely coincides with the aforementioned one for the sinusoidal intensity modulation by means of, for example, electro–optical modulator. Knowing the frequency Ω_1 and its initial phase as well as the initial phase of the difference frequency Ω_1-Ω_2, we can directly use Eq.(5) for the calculation of the components of the Stokes vector of the informative component. Moreover, taking the Fourier transform of the square signal in the high–frequency range of the signal spectrum given by Eq. (8) in a way similar to that used for Eq. (6) in order to avoid the additional synchronization with the frequency Ω_1, it is easy to verify that the components of the Stokes vector of the informative field can be found from the system of equations (7). Only the common numerical factor on the right side will change. The value of this factor is equal to a sum of a numerical series that appears in taking the square of the Fourier representation of the pulse train (the second parenthesis in Eq. (4)). Thus the problem of measuring the components of the Stokes vector under conditions of the partially–polarized background component of the optical field at the input of the polarimeter can be successfully solved by means of the pulse modulation of the intensity of sounding field.

We have implemented the proposed technique (with sinusoidal modulation of the intensity by means of an electro–optical modulator fabricated from a $LiNbO_3$ crystal) in the Stokes polarimeter operating at two wavelenghts λ = 0.63 and 1.15 μm. An LG – 126 laser was used as a radiation source, and a Glan prism was used as an analyzer. The transition to the far–IR range will require a replacement of the polarization units and photodetector. The technique based on the use of a mechanical chopper was implemented in a Stokes polarimeter operating in the spectral range centered at 10.6 μm. Measurements of the polarization characteristics of various types of smokes were carried out with this device under real conditions in the daytime. The experiment has shown that polarization measurements in this optical frequency range with keeping the constant component of the photodetector signal is problematic. This is connected with the presence of the background component as well as with the fact that, as is known, the photoresistors cooled by nitrogen are used for radiation recording in this spectral range. Their resistance significantly varies with time in an arbitrary way as a result of nonuniform cooling. This gives rise to the irregular variations of the constant component of the recorded signal. It seems impossible to develop a compensating or tracking scheme for recording the constant component due to the significant spread of the photoresistor parameters. In this case the error in measuring the components of the Stokes vector with tenfold averaging in the best case is about 15-20%. The double modulation provided the decrease of the error of one measurement down to 2-3%. Numerical simulation of the operation of the Stokes polarimeter with double modulation of field made it possible to reveal that the measurement error in this case is connected with the instability of the initial phase of the rotating plate (mechanically rotated plate fabricated from a CdS crystal was used) in the receiving channel of the polarimeter rather than with the instability of pulse modulation of the optical field intensity.

REFERENCES

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