AVERAGING EFFECT OF A RECEIVING APERTURE ON SPHERICAL WAVES REFLECTED FROM SPECULAR OBJECTS IN THE TURBULENT ATMOSPHERE

G.Ya. Patrushev and O.A. Rubtsova

Institute of Atmospheric Optics, Siberian Branch of the Russian Academy of Sciences, Tomsk Received June 3, 1993

Aperture averaging functions experimentally measured for spherical waves reflected from a specular disc and a two-dimensional matrix of corner-cube reflectors are analyzed for strong intensity fluctuations. In the case of the specularly reflecting disc the values of the aperture averaging function for small receiving aperture are less than that for forward propagation and in comparison with the values calculated by the asymptotic theory of saturated fluctuations taking into account the inner scale of turbulence. When the size of the receiving aperture considerably exceeds the radius of spatial coherence of fluctuations, the averaging function G(R) is close to the dependence $G(R) \sim R^{-2}$ that corresponds to the summation of the uncorrelated wave intensity fluctuations over the receiving aperture. In the case of reflection from the matrix of corner-cube reflectors the function G(R) has two scales. Fast decrease of the intensity fluctuations occurs on the aperture whose size is equal to the diffraction size of the corner-cube. Slow decrease of the fluctuations for large receiving apertures (saturation of averaging function) is caused by fluctuations of the flux across the aperture of the corner-cube matrix.

Averaging effect of a receiving aperture on the magnitude and spatial spectrum of the light flux fluctuations was considered in Ref. 1 for the forward propagation of a plane wave. The results of experimental investigations carried out with the help of a ground-based laser source were reported in Ref. 2, those with an astronomical source in the photon counting mode were reported in Ref. 3. Theoretical and experimental works carried out before 1975 and devoted to this problem were generalized in Ref. 4.

A brief review of theoretical and experimental works, published mainly in the USA and devoted to the forward propagation of radiation, is presented in Ref. 5*, in which approximate expressions for the averaging function G(R) were derived with allowance for the inner scale of turbulence l_0 . Asymptotic dependence $G(R) \sim R^{-7/3}$ pointed out earlier in Ref. 1 and found for the large size 2R of the receiving apertures in comparison with the radius of spatial correlation of fluctuations was refined with allowance for l_0 for weak and saturated intensity fluctuations. This behavior of the averaging function G(R) was not experimentally observed.

The measurement error of such devices as range finders, laser radar systems, and so on depends on the magnitude and spectrum of fluctuations of the received flux of reflected radiation.

In this connection Refs. 6-10 should be mentioned in which the averaging function^{7,8,10} and temporal spectra of the flux fluctuations^{7,9} of a reflected laser radiation were considered. The results reported in Ref. 10 substantiated conclusions of Ref. 7 in which the data

of Ref. 6 on the reduction of averaging effect of the aperture in the case of reflection of a plane wave from large specular disc were considered to be erroneous. Saturation of the averaging effect of the receiving aperture on the intensity fluctuations of a spherical wave reflected from specular point was confirmed experimentally for the weak intensity fluctuations.⁸

The magnitude and temporal spectrum of the fluctuations of light flux reflected from the matrix of spaceborne corner-cube reflectors^{11,12} and from an individual corner-cube⁹ placed at the end of a ground path were studied both theoretically and experimentally. The expression for probability density of the intensity fluctuations derived in Ref. 11 yields the value of the variance of the intensity fluctuations for wave reflected by a spatial matrix of corner-cube reflectors which disagrees with the experimental results.¹³ In the experimental study of the temporal flux spectra, the limited frequency range of a recording system¹² had an effect.

We studied the averaging effect of an annular aperture on the variance of fluctuations of the light flux reflected from a specular disc with diameters up to 500 mm and from a close–packed matrix of 12–prism high–quality corner–cube reflectors with a total light diameter of 12.5 cm. In the experiment we used a quasispherical wave and devices and technique described in Refs. 14 and 15 in ample detail. In our case the channel for measurement of instantaneous intensity of the reflected radiation was incorporated into the system described in Ref. 14. The structural characteristic of the refractive index C_n^2 and the inner scale of the turbulence l_0 were determined by the optical method for weak intensity fluctuations on an auxiliary 200 m path. The inner scale of turbulence l_0 was measured through the temporal spectrum of the intensity fluctuations.¹⁶

^{*}Reference 5 became available to us only after the present paper had been submitted for publication. Thereafter we revised our paper.

Figure 1 shows the experimentally measured averaging function G(R) of a quasispherical wave reflected from the specular disc with a diameter of $\simeq 500$ mm on the path of length $L \approx 1200-1250$ m for two runs of measurements. The outer diameter of the receiving aperture increased up to 60 mm for the data of the first run shown in Fig. 1 by dots, while the diameter of the inoperative inner circle was about 1.5 mm.



FIG. 1. A comparison of the averaging function $G(\mathbf{R})$ of spherical wave reflected from the specular disc with the averaging functions for weak (1) and strong (2) intensity fluctuations and with the experimental (small crosses) data from Ref. 5.

In the second run of measurements the results of which are shown by vertical bars whose lengths indicate the spread of the function G(R), the outer diameter of the receiving annular aperture increased up to 500 mm, and the inner diameter was equal to approximately 5 mm (see Ref. 14). For comparison, the averaging functions G(R) of the spherical wave used in the experiment performed in Ref. 5 on the forward propagation path with allowance for the inner scale of turbulence l_0 are also shown here by solid lines for the weak (1) and strong (2) fluctuations, and small crosses denote the data obtained in Ref. 5 (see Fig. 14 in Ref. 5) except for two experimental values obtained for small apertures when G(R) = 1. Ratio of the receiving aperture radius to the size of the first Fresnel zone on the path of length 2L with reflection is plotted on the abscissa below, while at the top of the figure — the ratio of the receiving aperture radius to the radius of coherence of spherical wave $\rho_0 = [0.545 C_n^2 K^2 L l_0^{-1/3}]^{1/2}$ (see Ref. 5), where K is the wave number of radiation.

As can be seen from the figure, for small apertures (up to 60 mm) our results are close to the data obtained in Ref. 5, but as a whole, they are slightly higher. This discrepancy is hardly caused by small difference between the turbulent conditions of propagation: our results were obtained for $\beta_0(L) = [1.21 \ C_n^2 \ K^{7/6} \ L^{11/6}]^{1/2} = 2-5$ and $l_0 = (5-6)$ mm; the measurement results reported in Ref. 5 and shown in the figure were obtained for $\beta_0(L) = 4.6-5$ and $l_0 = (5.98 \pm 0.35)$ mm. Apparently this is due to the fact that for a reflected quasispherical wave the radius of spatial correlation of the intensity fluctuations in the zone near the axis^{17,18} is larger than that for a forward propagation path, which is manifested for small apertures.

Our results indicate that within the range of size of the receiving apertures $0 < R < \sqrt{2\lambda L}$ the effect of averaging of the strong fluctuations is more pronounced in comparison with the case of weak fluctuations, but is still less than this is evident from the asymptotic theory.⁵ For the aperture size $R > \sqrt{2\lambda L}$ the averaging function is larger than this is evident from the asymptotic theory for the saturated fluctuations,⁵ and less in comparison with the case of weak fluctuations. For $R > 1.5 \sqrt{2\lambda L}$ the experimental results decrease close to the power-law dependence $G(R) \sim R^{-1}$, where μ is closer to 2 (curve 4) than to 2.33 (curve 3) predicted in Ref. 1 based on the energy conservation law for the spatial correlation function of the intensity fluctuations. In this case the last right point corresponding to a diameter of 500 mm is slightly above the dependence R^{-2} which corresponds to the summation of the independent random fluctuations over the receiving aperture.

Here in our opinion one should take into account the following fact. In our measurements we control the interception of the focused beam by the radiation reflector visually. However, even for complete interception of a focused beam by the reflector and receiving aperture with a diameter of 500 mm, the relative photocurrent fluctuations being equal to about 1% (0.01) were recorded on this path. These fluctuations cannot be completely attributed to a nonuniform sensitivity of a photocathode surface area of a receiving photomultiplier; more likely they are due to fluctuations in the optical thickness τ of the path which are associated with the motion of aerosol particles over the path. In our experiments the relative flux fluctuations $\beta_{\tilde{f}}^2$ were equal to 0.02–0.06 for 500 mm aperture and relative rms values of the intensity fluctuations $\beta_{\tilde{f}}^2 = 3.6-4$.

It should be noted that in the experiment described in Ref. 5 the authors found discrepancies between experimental and theoretical dependences of the same sign as we have for weak intensity fluctuations when the flux fluctuations were very small (see Figs. 10–12 in Ref. 5). As can be seen from our data, there is no clear distinction between two scales of the averaging function variations predicted by the asymptotic theory⁵: $G(R) \sim (R/\rho_0)^{-2}$ when $R \ge \rho_0$ and $(R/\rho_0)^{-7/3}$ when $R \gg \rho_0$. More likely and to some extent arbitrarily the first scale corresponds to the decrease of the function $G(R) \sim R^{-1/2}$ when $(R/\rho_0) \le 9\rho_0$, while the second is observed for $(R/\rho_0) > 20$ when $G(R) \sim R^{-2}$.

Figure 2 illustrates the experimental averaging function of a quasispherical wave reflected from a two-dimensional matrix of twelve corner-cube reflectors of high quality (deviation of dihedral angles at the apex of the individual corner-cube was no more than 2–5 sec of arc) with hexagonal arrangement of corner-cubes. The diameter of the individual corner-cube was ≈ 2.5 cm, the diameter of the whole matrix was close to 12.5 cm. The values of the parameter $\beta_0(L)$ were 2.5–3.1 except for the last point the position of which on the abscissa corresponds to the value $R/\sqrt{2\lambda L} = 6.2$ (the abscissa scale was changed after the point $R/\sqrt{2\lambda L} = 1$). The averaging function is shown by the solid line for the spherical wave on the forward propagation path under conditions of weak intensity fluctuations⁴ for $\beta_0(L) < 1$.

As can be seen from a comparison between the data obtained for the corner—cube reflectors (Fig. 2) and specular disc (Fig. 1), the effect of spatial averaging in the case of reflection from twelve corner—cubes with the same size of receiving aperture is several times stronger for small apertures $R < \sqrt{\lambda L}$ in spite of the fact that the diameter of the reflecting

mirror (≈ 500 mm) is vastly larger than the diameter of the matrix of the corner—cube reflectors (≈ 125 mm). For $R/\sqrt{2\lambda L} > 0.6$ the averaging function decreases very slowly. So for $R/\sqrt{2\lambda L} = 0.94$ the average value of the function G(R) = 0.09 and for $R/\sqrt{2\lambda L} = 6.2$ the average value G(R) = 0.07. It was derived from eight averaging functions G(R) measured for $\beta_0(L) = 1.9-5.2$. In Fig. 2 it corresponds to the last point with indicated spread of recorded values of the function G(R).



The above-indicated behavior of the averaging function becomes understandable if we take into account the fact that an image of the individual corner-cube with a diameter of 2.6 cm is approximately 5.2 cm in size on a receiving objective and it falls within the aperture with a diameter of 6 cm because the random jitters in the corner-cube image do not exceed two seconds of arc¹⁴ under these conditions. Therefore, the flux fluctuations across the receiving aperture with a diameter of 6 cm will be virtually identical to the flux fluctuations across the aperture of the spatial matrix of corner-cube reflectors. This is confirmed by a direct comparison between the fluctuations of the flux reflected from the matrix of corner-cube reflectors and from the specular disc with equivalent surface area when radiation is received by the aperture 500 mm in diameter. For $\beta_0(L) = 4.5$ the intensity fluctuations of radiation reflected from the cornercubes get closer to saturated fluctuations¹³ what provides an explanation for very slow decrease of the averaging function with an increase of the diameter of receiving aperture.

Stronger averaging effect of the receiving aperture in the case of reflection from the matrix of 12 corner–cubes in comparison with the specularly reflecting disc of larger diameter for small $R < \sqrt{2\lambda L}$ can be explained by the fact that the high–quality corner–cubes reflect a quasispherical wave practically in one and the same special area because the turbulent jitters of beams reflected from each corner–cube are highly correlated. This results in very spiky small–scale structure of radiation for strong fluctuations whose spatial correlation radius is much less than that for the case of reflection from the specular disc under the same conditions.

The decrease of the number of corner-cubes in matrix causes reduction of the averaging effect for the same diameter of the receiving aperture, all other factors being the same, due to the increasing flux fluctuations across the reflector aperture. To a lesser degree these flux fluctuations engender the increase of the receiving intensity fluctuations. It can be seen from the measurements of the function G(R) for three closely–spaced corner–cubes (their total diameter was about 5.5 cm) shown in Fig. 2 above the data obtained for twelve corner–cubes with the same receiving aperture $R/\sqrt{2\lambda L} = 6.2$ (500 mm). For these data the mean value of the averaging function G(R) = 0.12 for 6 measurements (i = 1, 2, ..., 6); the relative root–mean–square values of the flux fluctuations $\beta_f = 0.69$; and, β_{fi}^2 varies within the range 0.663–0.774. Analogously, for the intensity fluctuations $\beta_I = 1.98$ and $\beta_{Ii} = 1.83-2.16$; in this case the values of the parameter $\beta_0(L) = 3-5.5$.

The authors would like to express their deep gratitude to O.A. Pelymskii, A.I. Petrov, and A.P. Rostov for their support of performed investigations.

REFERENCES

1. V.I. Tatarskii, *Wave Propagation in the Turbulent Atmosphere* (Nauka, Moscow, 1967), 548 pp.

2. G.E. Houstad, J.W. Strohben, R.U. Berger, and J.M. Henegam, J. Opt. Soc. Amer. **64**, No. 2, 161 (1974).

3. R.S. Iyer and J.L. Bufton, Opt. Commun. 22, No. 3, 377 (1974).

4. A.S. Gurvich, A.I. Kon, V.L. Mironov, and S.S. Khmelevtsov, *Laser Radiation in the Turbulent Atmosphere* (Nauka, Moscow, 1976), 277 pp.

5. J.H. Churnside, Appl. Optics 30, No. 15, 1982 (1991).

6. V.P. Lukin and V.V. Pokasov, in: *Abstracts of Reports at the First All-Union Conference on Atmospheric Optics*, Tomsk (1976), Vol. 4, p. 134.

7. E.A. Monastyrnyi, G.Ya. Patrushev, A.I. Petrov, and V.V. Pokasov, Kvant. Elektron. 7, No. 7, 1580–1582 (1980).

8. M.S. Belenkii, A.A. Makarov, V.L. Mironov, and V.V. Pokasov, Izv. Vyssh. Uchebn. Zaved. SSSR, Radiofizika **21**, No. 2, 299–301 (1978).

9. V.I. Grigor'evskii, A.I. Lomakin, and S.V. Tarakanov, Izv. Vyssh. Uchebn. Zaved. SSSR, Radiofizika **29**, No. 3, 364–366 (1986).

10. Z. Song, X. Feng, S. Han, Yu. Lin, and Q. Ding, Chines Physics Laser 16, No. 6, 442 (1988).

11. J.L. Bufton, R.S. Iyer, and L.S. Taylor, Appl. Optics 16, No. 9, 2408 (1977).

12. J.L. Bufton, Appl. Optics 16, No. 9, 2654 (1977).

13. G.Ya. Patrushev, A.I. Petrov, and O.A. Rubtsova, Atm. Opt. **2**, No. 3, 221–226 (1989).

14. G.Ya. Patrushev, O.A. Pelymskii, and A.I. Petrov, Izv. Vyssh. Uchebn. Zaved. SSSR, Radiofizika **32**, No. 6, 673–679 (1989).

15. G.Ya. Patrushev, A.I. Petrov, and V.V. Pokasov, Izv. Vyssh. Uchebn. Zaved. SSSR, Radiofizika **26**, No. 7, 823–831 (1983).

16. E.A. Monastyrnyi, G.Ya. Patrushev, A.I. Petrov, and V.V. Pokasov, Author's Certificate, No. 913794 (1981).

17. V.E. Zuev, V.A. Banakh, and V.V. Pokasov, *Optics of the Turbulent Atmosphere* (Gidrometeoizdat, Leningrad, 1988), 271 pp.

18. A.P. Ivanov, G.Ya. Patrushev, and A.P. Rostov, Atmos. Oceanic Optics **6**, No. 5, 313–315 (1993).