STABILITY ANALYSIS OF THE APERTURE SOUNDING ALGORITHM TAKING INTO ACCOUNT THE TRANSIENT PROCESSES EVOLVED IN ADAPTIVE SYSTEM

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The analysis of the stability and the speed of operation of the adaptive system of the aperture sounding designed for correction for the thermal blooming is presented based on the methods of numerical experiment. The model elaborated enables one to take into account the transient processes connected both with the establishing the temperature distribution in the channel of beam propagation and with natural vibrations of the reflecting surface of the corrector occurred under its deformation.

1. INTRODUCTION

One of the lines of evolution of mathematical modelling in adaptive optics is now bridging the gap between the numerical experiment conditions and the conditions of the laboratory and field experiments (unfortunately, up to present the hypothetic ones). The fullest progress in this area is presumably reached in constructing the model of interaction between the medium and radiation in which the following factors are taken into account: thermal blooming,[1] the stimulated convection,[2] the wind velocity pulsations along the propagation path,[3] and the random fluctuations of the refractive index.[4]

The calculation schemes of other elements of an adaptive system are also deeply developed, for example, the recording unit of the beam phase profile[5] with the wave–front corrector as the regulating unit. The mirror is most often used as a corrector. The limitations bringing by this unit were given in Refs. 6 and 7 by the set of the lowest–order Zernike polinomials. In Refs. 8 and 9 the model is described of a corrector presenting itself the elastic plate deformed by servo mechanisms, the number of which was changed from 4 to 30–40. The main drawback of the papers concerning the problems of the phase profile formation is that they were making use the statistical models, in other words, the vibrations of the reflecting surface resulting from the corrector deformation under the action of external forces were not taken into account.

In this paper, the further development of the model of the elastic adaptive mirror is presented. The calculation scheme used enables one to estimate the effect of transient processes conditioned by the natural vibrations of the reflecting surface onto the compensation stability of the thermal self–action. The control of the beam is treated relying on the algorithm of the aperture sounding. The main peculiarities of the algorithm are demonstrated by the example of the "ascent onto a hill" in the linear medium in which the control modification is proposed. Namely, to increase the stability the filtration of recording parameters is introduced. Further, using the filtration, the correction of non–stationary thermal self–action in the regular medium is realized.

2. MATHEMATICAL MODEL OF AN ADAPTIVE SYSTEM

Below the mathematical description of the adaptive system of the aperture sounding is presented which includes the dynamical model of the mirror. The base units are schematically depicted in Fig. 1.

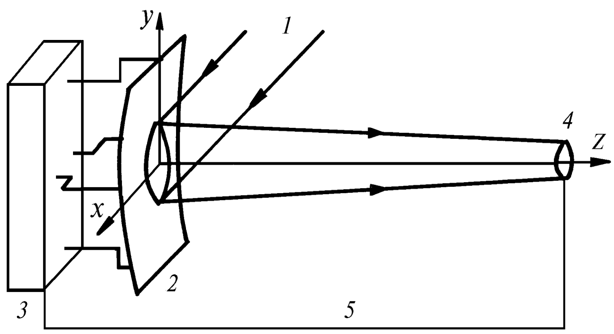

FIG. 1. Block diagram of main units of the adaptive system: 1) laser radiation source, 2) mirror, 3) control signal formation unit, 4) object, and 5) feedback loop.

In the model considered the mirror deformations $W(x, y, t)$ are described by the matrix equation:

$$[\mathrm{M}]\, \ddot{\mathbf{W}}(t) + [\mathrm{H}]\, \dot{\mathbf{W}}(t) + [\mathrm{K}]\, \mathbf{W}(t) = \mathbf{F}(t)\ , \qquad (1)$$

where [M] is the mass matrix, [H] is the vibration damping matrix, [K] is the rigity matrix, $\mathbf{F}$ is the vector of external forces applied to discrete points of the model, $\mathbf{F} = \{F_i\}$, $i = 1, 2, ..., N$, N is the number of servomechanisms, the displacement W is written in the vector form. Equation (1) is obtained from the general equation of deformations of a plate[10] using the finite element method.[11]

The formation of the beam phase $\Phi(x, y, t)$ using the elastic mirror is performed as follows

$$\Phi(x, y, t) = 2k\, W(x, y, t)\ , \qquad (2)$$

where k is the wave number. The complex amplitude incident on the medium is defined as

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$$E(x, y, z = 0, t) = E_0 \exp(i\,\Phi(x, y, t)) . \qquad (3)$$

The propagation of the radiation under conditions of the thermal self–action is described by the system of equations

$$2ik\,\frac{\partial E}{\partial z} = \Delta_\perp E + 2\,\frac{k^2}{n_0}\left(\frac{\partial n}{\partial t}\right) TE\ ;$$
$$\frac{\partial T}{\partial t} + (\mathbf{V}_\perp\,\nabla_\perp)\,\mathrm{T} = \frac{\alpha I}{\rho C_p}\,, \qquad (4)$$

where $I = E\,E^*$, T is the temperature, $\mathbf{V}$ is the wind velocity vector and the other notations are commonly accepted. Nonlinear properties of the medium are characterized by a parameter

$$R = \frac{2\,k^2\,a_0^3\,I_0\alpha}{n_0\,\mathrm{r}\,C_\mathrm{p}V_0}\left(\frac{\partial n}{\partial t}\right) . \qquad (5)$$

The field on the object under focusing will be described using the criteria

$$J(x, y, z_0, t) = \frac{1}{P_0}\int\!\!\int \rho(x, y)\,I(x, y, z_0, t)\,\mathrm{d}x\,\mathrm{d}y\ , \qquad (6)$$

where ρ is the weighting function, P_0 is the total power of the beam. In the system under consideration, when the mirror is affected, the vibrations of its reflecting surface and, as a consequence, the oscillations of the field in the observation plane appear. The passage of the transient processes connected with the deformations of the mirror in the linear medium is illustrated in Fig. 2 *a* (the mirror is affected at the time $t = 0$). The oscillation damping is determined by the corrector parameters. The characteristic variations of the focusing criteria under conditions of the thermal self–action are presented in Fig. 2 *b*.

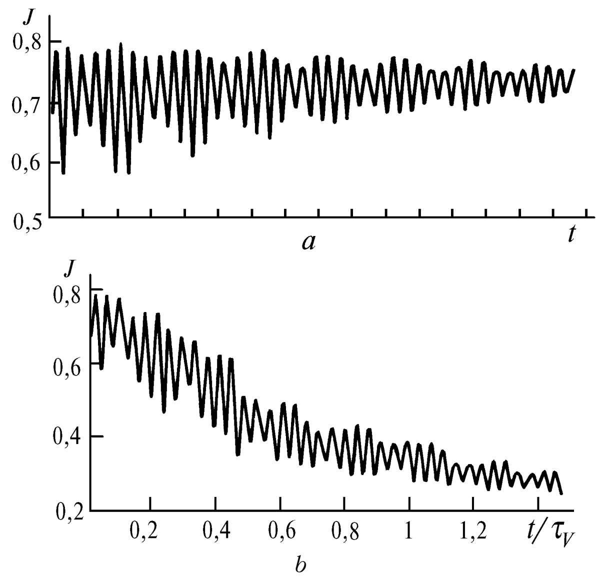

FIG. 2. Oscillations of the focusing criterium J in the observation plane under the action on the mirror. (a) linear medium and (b) nonlinear medium, $R = -20$.

The action on the mirror $\mathbf{F}(t)$ under control was determined on the basis of the algorithm of the aperture sounding

$$\mathbf{F}(\mathrm{t}) = \mathbf{F}(t - \tau_d) + \alpha\,(t - \tau_d)\ \mathrm{grad}\ J(x, y, z_0, t - \tau_d)\ , \qquad (7)$$

where α is the value of the gradient step, τ_a is the delay in forming of the control signal. The components of the vector grad J are the derivatives $\partial J/\partial F_i$ determined in the following way: under the action of the force ∂F_1 one of the servomechanisms of the mirror is displaced on a small amount, the corresponding change ∂J of the quality criteria J (probe variation) is recorded on the object under focusing. After calculations of all derivatives $\partial J/\partial F_i$, ($i = 1, \ldots, N$, N is the number of coordinates controlling the mirror) the simultaneous displacement is fulfilled of all the drives (the gradient step), whereupon the process of the probe variation is repeated.

3. REALIZATION OF THE "HILL CLIMBING" PROCEDURE IN THE LINEAR MEDIUM

If the aperture sounding is realized in the case of developing the transient processes in the system the determination of derivatives $\partial J/\partial F_i$, i.e., the isolation of the response of the recorded parameter on the probe action is sufficiently complicated. It is conditioned by the situation in which it is often impossible to determine the cause of variation of the focusing criteria, either by the probe action on the mirror or by the field oscillations in the process of stabilization. Therefore the most simple way of realization of the aperture sounding is the control according to the sustained field when the system is operating according to the scheme "recording J – probe action – "awaiting period," during which the transient processes conditioned by the action are finished – recording the increment ΔJ – gradient step – recording J – the probe action ... ".

The speed of operation of the system in this case is limited by the method of realization of searching for the extremum of the goal function, namely, by the presence of awaiting periods at each gradient step. It is possible to increase the control speed of operation due to reducing these periods, however, because the separating out the probe actions is performed when the field is sustained, it is necessary to gain a decrease of the duration of the transient processes, in other words, it is necessary to perform the optimization of the mirror construction with the aim to increase the coefficient of the vibration damping.

The "hill climbing" over the sustained parameters in the linear medium is illustrated in Fig. 3. The mirror hinged at the edges and having one servomechanism at the center was used as a corrector. It can be seen from Fig. 3 that the stable increase of the light field concentration is recorded at the object under focusing as a result of the control. At the same time if the hill climbing is performed on the basis of recording the increment ΔJ before ending the transient processes then the evolution of non–stabilities was observed in the system of the aperture sounding in numerical experiments.

The problem of separation of the response of the system on the action is illustrated in Fig. 4 *a* as well, where the variation of the criteria J under displacement of the servomechanism at the time t_0 is shown (the displacement amplitude corresponds to the gradient step) and at the time t_1 (the amplitude corresponds to the probe variation). It can be seen from the figure that in the example considered it is impossible to determine the increment ΔJ. The decrease of the oscillation amplitude of the criteria J in the observation plane without damping of the mirror is reached when the filtration of the recorded parameters is introduced. The signal corresponding to Fig. 4 *a* after passing through the filter is shown in Fig. 4 *b*. To separate the increment of the criteria should present no problems in this case.

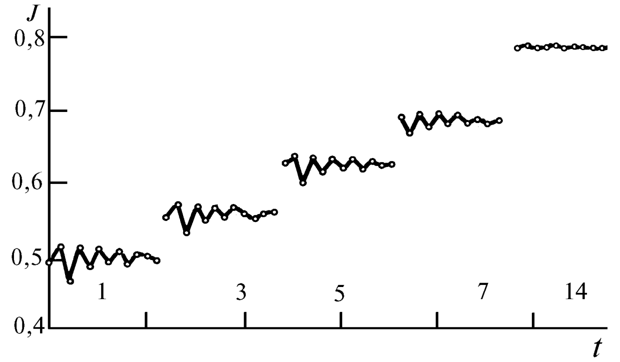

FIG. 3. The hill climbing over the sustained parameters of the light field. Linear medium. The numbers of iteration steps are given.

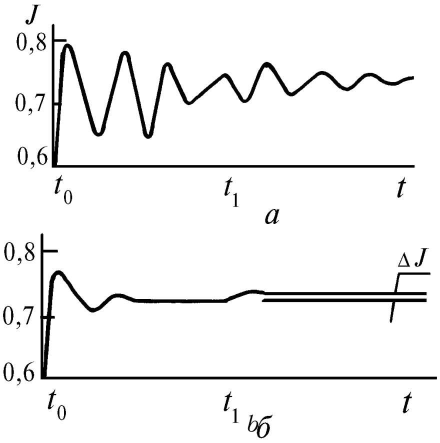

FIG. 4. Illustration to the problem of separation of the response to the probe action. (a) signal before filtration and (b) signal after filtration.

The control over non–sustained field with the filtration is depicted in Fig. 5. The damping of the mirror vibrations is practically absent. The maximal frequency of probe variations which was succeeded to be obtained in numerical experiments is approximately four times less than the frequency of the natural vibrations of the corrector. At the further decrease of the delay in forming of control actions the divergence of the algorithm of the aperture sounding is observed again. As a whole, according to the results of realization of the hill climbing in the linear medium, the conclusion can be arrived that the introduction of the signal filtration enables one to realize the control over non–sustained parameters of the light field and to increase essentially the speed of operation of the aperture sounding. In doing so it is unnecessary to suppress the natural vibrations of the mirror.

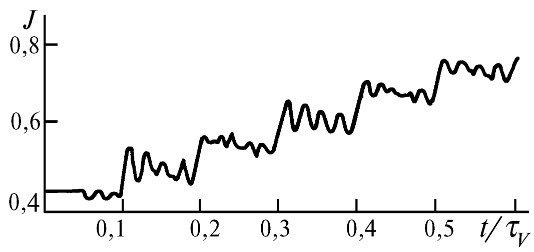

FIG. 5. Control over non–sustained parameters in the linear medium. The perpendicular lines refer to the times when the displacement of the servomechanism takes place.

4. CORRECTION FOR THE THERMAL BLOOMING

When the laser radiation propagates through the nonlinear atmosphere the transient processes evolve in the adaptive system under consideration connected with the heating of the medium by the beam. The similar processes evolve after switching up the laser source and also after any variation of the phase profile.[12] The control under such conditions was considered in Refs. 12 and 13, where, among other things, it was shown that to realize the aperture sounding the delay in forming of control signals τ_d should be much smaller than the characteristic time of variation of the thermal lens: $\tau_V = a_0/V$, a_0 is the initial radius of the beam, V is the wind velocity.

It was conceived under the realization of the control in the nonlinear medium with taking into account the natural vibrations of the corrector that these vibrations are sufficiently high–frequency ones and the thermal lens is practically unchanged during some periods. The time of frozen nonlinearity was of 0.12 τ_V (four periods of the mirror vibrations). To reduce the time of calculations the control was considered only over one coordinate. Despite of the fact that the efficiency of correction making use the simplest mirror is considerably lower the similar approach allows one to make the analysis of main peculiarities of the control process.

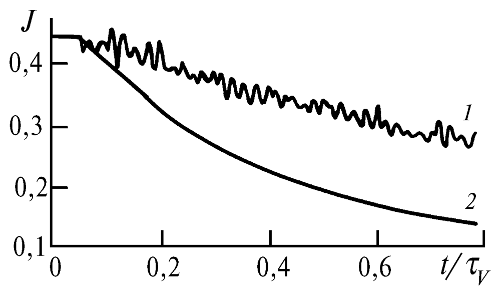

FIG. 6. Control over one coordinate under conditions of the thermal blooming. $|R| = 40$. *Curve 1 refers to the result of correction and curve 2 refers to the case without control.*

The results of the hill climbing in the nonlinear medium are presented in Fig. 6, the correction being made over non–sustained parameters with the use of filtration. Also depicted are the variations of the criteria J for the beam with the plane phase profile. It can be seen that the essential increase of the light field concentration in the observation plane is recorded as a result of the control.

5. CONCLUSION

The results obtained allow one to conclude that despite of the evolution of the transient processes connected with the natural vibrations of the corrector the adaptive control of the beam is possible to be realized both in linear and in nonlinear medium on the basis of the algorithm of the aperture sounding. In doing so the increase of the speed of operation of the correction is obtained due to filtration of the recorded parameters.

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