DIFFRACTION ERRORS AND SEGMENTED OPTICS

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The present approach to segmented mirror control used for phase compensation is to measure phase tilts with a wavefront sensor, apply these tilts to the segments, and then adjust the pistons to minimize the edge mismatch by bringing adjacent segment edge midpoints together. Matching the heights of adjacent segment edge midpoints leads to a linear system of equations for the segment pistons U_i*, in terms of the segment tilts* $\mathbf{t}_i$*, which looks like a discretized Poisson equation,* $\nabla^2 U = -\nabla\mathbf{t}$*. We demonstrate that the discrete linear system is ill–posed in 2 dimensions. Moreover, diffractive effects that arise from the segmentation must be accounted for in the wavefront control.*

1. INTRODUCTION AND SUMMARY

The main purpose of a large segmented primary mirror in a ground–based laser power beaming system operating at near infrared wavelengths is to provide atmospheric compensation. Since this primary mirror is segmented diffractive effects that arise from this segmentation must be accounted for in the wavefront reconstruction.

Wavefront fitting error results from the inability of the adaptive optics to exactly reproduce the beacon phase front and to impart the conjugate on the high power laser beam. Ideally, the conjugate of the beacon phase is applied to the uplink beam by the active mirror. And, even if the adaptive optics system were able to exactly measure and apply the conjugate to the high power beam, the distortion due to atmospheric turbulence would not be completely cancelled. This is due to the fact that the phase compensation is only perfect in the geometric or ray optics limit. Finally, there are diffractive scattering effects that arise due to the fact that the segmented primary mirror is not continuous. Neighboring segments with different tilts necessarily will have edges and corners that "stick up" or down out of the surface. The diffractive effects from this have been totally ignored until now. Edge mismatch between neighboring segments will impart discontinuities on the outgoing high power beam. We are the first to show that the minimization of the edge mismatch by matching the centers of adjacent segment edges, upon which the Poisson–solving control algorithms are based, is ill–posed. There are not enough segment degrees of freedom to satisfy the constraints. The difference between the total number of constraints and the total number of degrees of freedom gets larger as the segmented mirror gets larger (as the square of the number of segments).

In Sec. 2, we begin by showing the 2–D problem for a segmented surface is ill–posed leading to a bumpy basin of approximate solutions (i.e., any straight–forward linear representation is singular). We also discuss physically correct systems performance criteria. In Sec. 3, we describe diffraction due to edge mismatch.

2. THE PISTONED *n*–SEGMENTED SURFACE IS AN ILL–POSED PROBLEM

For the 1–D case, setting tilts and iterating pistons converges to a well–defined solution. Let U_i be the height of the piston of the i th segment and t_i be the tilt of the i th segment (see Fig. 1).

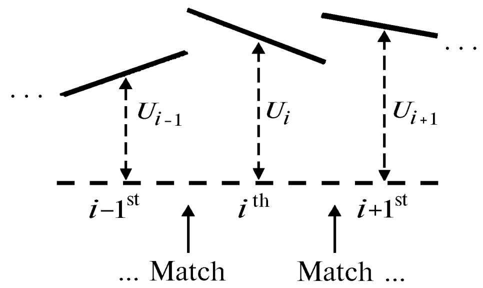

FIG. 1.

The height of the left edge of the i th segment is $U_i - \frac{1}{2} t_i$. The height of the right edge of the $(i - 1)$ segment is $U_{i-1} + \frac{1}{2} t_{i-1}$. Matching the edges we obtain $U_i - \frac{1}{2} t_i = U_{i-1} + \frac{1}{2} t_{i-1}$. Now consider the height of the right edge of the i th segment which is $U_i + \frac{1}{2} t_i$ and the height of the left edge of the $(i + 1)$ segment, $U_{i+1} - \frac{1}{2} t_{i+1}$. Again matching we get $U_i + \frac{1}{2} t_i = U_{i+1} - \frac{1}{2} t_{i+1}$. Combining the two matching equations $U_{i-1} + U_{i+1} - 2U_i = \frac{1}{2}(t_{i+1} - t_{i-1})$ which is the continuum limit of the 1 – D Poisson equation:

$$\frac{d^2}{dx^2} U = \frac{d}{dx} t$$

and both of which have well defined solutions. Since $t = -\mathrm{d}\phi/\mathrm{d}x$, we arrive at $U = -\phi$.

We can now show that in the 2–D case, the discretized problem is ill–posed. Let us begin by defining the system. We must first define a criterion for "edge matching". If two neighboring segments have different tilt components along the edge, then the edges cannot be aligned. At best, we can only match one point along the edge.

For simplicity, let us consider square segments. The analysis is identical for hexagonal segments. Let U_{ij} equal the height or piston setting of the (i th, j th) segment and $\mathbf{t}_{ij} = (t_{ij}^{(1)}, t_{ij}^{(2)})$ equal the tilt vector of the (i th, j th) segment. Match heights of adjacent segments at mid–points of neighboring edges (i.e., at the × in the following figure):

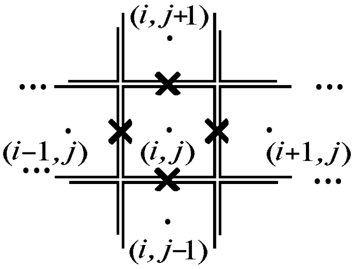

FIG. 2.

If we now match the right–hand edge of segment (i, j) we obtain:

$$U_{ij} + \frac{1}{2}\,t_{ij}^{(1)} = U_{i+1j} - \frac{1}{2}\,t_{i+1j}^{(1)}\ ;$$

matching the upper edge of segment (i, j):

$$U_{ij} + \frac{1}{2}\,t_{ij}^{(2)} = U_{ij+1} - \frac{1}{2}\,t_{ij+1}^{(2)}\ ;$$

matching the left–hand edge of segment (i, j):

$$U_{ij} - \frac{1}{2}\,t_{ij}^{(1)} = U_{i-1j} + \frac{1}{2}\,t_{i-1j}^{(1)}\ ;$$

and finally matching the lower edge segment (i, j):

$$U_{ij} - \frac{1}{2}\,t_{ij}^{(2)} = U_{ij-1} + \frac{1}{2}\,t_{ij-1}^{(2)}\ .$$

If we now combine all four matching equations and move the U's to one side, t's to other,

$$U_{i+1j} + U_{ij+1} + U_{i-1j} + U_{ij-1} - 4U_{ij} =$$

$$= \frac{1}{2}\left(t_{i+1j}^{(1)} - t_{i-1j}^{(1)} + t_{ij+1}^{(2)} - t_{ij-1}^{(2)}\right)$$

which looks like the discretized version of Poisson equation

$$\nabla^2 U(x,\, y) = \nabla\mathbf{t}\ .$$

But Poisson equation is not continuum limit of the discretized system of matching equations. The reason for this is that the discretized system is ill–posed and has no exact solution. There are more edge matching equations than there are piston degrees of freedom. In other words, the "best" fit with the "minimum" edge mismatch is not in the configuration of state space or the n–segmented membrane.

Here is a simple example to illustrate that the 2–D problem is ill–posed. Consider a segmented mirror made of just four square segments:

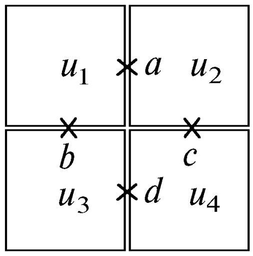

FIG. 3.

u_1 represents the piston for square *1*, u_2 represents the piston for square *2*, etc., and $\mathbf{t}_1$ represents the two–component tilt for square *1*, etc. We match the edges at mid–points a, b, c, and d. For a we have:

$$u_1 + \frac{1}{2}\,t_1^{(1)} = u_2 - \frac{1}{2}\,t_2^{(1)} \Rightarrow u_1 - u_2 = (-)\,\frac{1}{2}\,(t_1^{(1)} + t_2^{(1)})\ ;$$

and for b :

$$u_1 - \frac{1}{2}\,t_1^{(2)} = u_3 + \frac{1}{2}\,t_3^{(2)} \Rightarrow u_1 - u_3 = \frac{1}{2}\,(t_1^{(2)} + t_3^{(2)})\ ;$$

and for c :

$$u_2 - \frac{1}{2}\,t_2^{(2)} = u_4 + \frac{1}{2}\,t_4^{(2)} \Rightarrow u_2 - u_4 = \frac{1}{2}\,(t_2^{(2)} + t_4^{(2)})\ ;$$

and for d :

$$u_3 + \frac{1}{2}\,t_3^{(1)} = u_4 - \frac{1}{2}\,t_4^{(1)} \Rightarrow u_3 - u_4 = (-)\frac{1}{2}\,(t_4^{(1)} + t_3^{(1)})\ .$$

Rewriting the last four equations as the matrix equation $M\mathbf{u} = \mathbf{T}$,

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -\,(t_1^{(1)} + t_2^{(1)}) \\ (t_1^{(2)} + t_3^{(2)}) \\ (t_2^{(2)} + t_4^{(2)}) \\ -\,(t_4^{(1)} + t_3^{(1)}) \end{pmatrix}$$

and expanding the determinant of M in co–factors we find

$$\det \mathrm{M} = 1 \begin{vmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & -1 \end{vmatrix} - (-1) \begin{vmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} =$$

$$= -\,1 + 1 = 0\ .$$

Therefore, M is singular and cannot be inverted. The problem is ill–posed and an exact solution does not exist.

The midpoint edge matching condition led to a set of discretized equations whose continuum limit appeared to be the Poisson equation with the source term $\nabla\mathbf{t}$. Since $\mathbf{t}$ represents $-\nabla\phi$, we again arrive at $U = -\nabla\phi$ which is desirable. When the tilts of each segment are updated and the pistons have not yet been readjusted, the segmented membrane starts out at one point in the configuration space. The "Poisson solvers" then adjust the pistons to move the configuration of the segmented membrane towards the point in the configuration space that is a solution of Poisson equation. But if the "best" fit solution is not in the segmented membrane configuration space, then what is the Poisson iteration converting to?

There are many approximate solutions to the matching equations where the various neighboring segments do not align in the middle but nearby. Once the Poisson solver moves the segmented membrane to a configuration that is in the basin of approximate solutions, then additional iterations just move the configuration from one approximate solution to another. The Poisson solver keeps trying to find the solution, but since it isn't in the space, the Poisson iterations just bounce it around the group of approximate solutions.

3. POWER BEAMING AND MINIMIZING EDGE MISMATCH

Since we are interested in power beaming and not just imaging, the criterion for determining the "best" surface is the energy on target in a given radius of the beam. Since not all of the approximate solutions lead to excellent system behavior (energy on target), it is desirable to find additional constraints or characteristics of the "best" approximate solutions so as to dampen the fluctuations. The criterion for how good an approximate solution is should involve the error spectrum (due to diffraction) and not just the rms error.

Due to the diffracting propagation leg associated with beaming, the rms surface fit error is not a sensitive enough criterion of control loop system performance. As far as diffraction is concerned, the spectrum of the surface fit error is more relevant. The high spatial frequency phase disturbances will scatter energy out of the beam as it propagates via diffraction (independent of turbulence).[1,2,3]

To put this in proper perspective, consider this system with tilts directly measured from the wavefront sensor then applied to the segments and with tilts locked until the next wavefront sensor measurement update. The pistons can be adjusted to minimize edge mismatch. But implicit in this method is the assumption that segment tilt error is far more important than edge error. Edge mismatch imparts phase discontinuities onto the outgoing beam, and thereby introduces high spatial frequency phase disturbances. Segment tilt error imparts lower spatial frequency phase disturbances. The high spatial frequency disturbances will be uncorrectable due to bandwidth limitations in the hardware. The low frequency disturbances can be minimized since they are in the compensation band.

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