STATISTICS OF THE SHEARING INTERFEROMETER SIGNAL WHEN RECORDING OF LASER RADIATION PASSED THROUGH THE ATMOSPHERIC TURBULENT LAYER

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Statistical characteristics of a signal of the shearing interferometer when recording of the laser radiation propagating through the turbulent atmosphere have been theoretically investigated. Analytic relationships for the statistical moments of the shearing interferometer signal under the various conditions for optical radiation propagation through the turbulent atmosphere are derived. New methods for processing of the shearing interferometer signal are proposed. These methods allow one to decrease distorting effect due to atmospheric turbulence when reconstructing the initial field distribution.

A problem on restoration of the image of an object of unknown form is investigated in two directions: study of mechanisms of an image formation of the object illuminated by incoherent radiation and analysis of peculiarities arising when restoring an image of coherently illuminated object. One of the specific methods for solving the problem on obtaining the information about the phase of wave front of coherent source is the use of such an instrument as the shearing interferometer.[1] This interferometer forms an interference pattern by mixing of a received optical wave with the same wave shifted by a certain short distance in the direction perpendicular to the wave propagation direction and passed through the optical wedge. Processing of the interference pattern allows us to obtain the information about the amplitude and phase of the optical wave front in the place of interferometer location. The initial characteristics of object illuminated by coherent radiation can be reconstructed using such information. However, the wave front distortions caused by fluctuations in amplitude and phase of the optical wave makes it difficult to reconstruct the initial field distribution.

To eliminate the distortion effect of random inhomogeneities of atmospheric turbulence, the series of short–exposure interferograms of transverse shear recorded in the time when medium is "frozen", i.e., during $\tilde{}10^{-3}$ s is processed. This paper presents the theoretical study of the statistical moments of the shearing interferometer signal when recording of laser radiation passed through the layer of atmospheric turbulence and the discussion of informative possibilities of the first two moments of measured value for reconstruction of the initial field distribution.

The coherent radiation with the wavelength λ passed through the path length x in the turbulent atmosphere is assumed to be received by the shearing interferometer. If the complex amplitude of the optical wave at the point of $\mathbf{r} = \{x, \rho\}$ is denoted as $U\{x, \rho\}$, then the shearing interferometer signal i will be proportional to[1]

$$\{U(x, \rho) + U(x, \rho + \Delta\rho) \exp(i\mathbf{a}\rho)\}\times$$

$$\times \{U^*(x, \rho) + U^*(x, \rho + \Delta\rho) \exp(-i\mathbf{a}\rho)\} =$$

$$= I(x, \rho) + I(x, \rho + \Delta\rho) + U(x, \rho)\, U^*(x, \rho + \Delta\rho) \times$$

$$\times \exp(-i\mathbf{a}\rho) + U^*(x, \rho)\, U(x, \rho + \Delta\rho) \exp(i\mathbf{a}\rho)\,,$$

where x is the longitudinal coordinate; ρ is the transverse coordinate; $\Delta\rho$ is the transverse spatial shear in the shearing interferometer; $\mathbf{a}\rho$ is the linear phase shift of the optical wave in the interferometer; $I\{x, \rho\} = U\{x, \rho\}\times U^*\{x, \rho\}$ is the intensity of optical field image at the point of $\mathbf{r}$; the asterisk denotes the complex conjugation.

Since the interferograms of transverse shear are recorded for the short exposures[1] ($\tilde{}10^{-3}$ s) every interferogram can be virtually considered as a random realization. Therefore, averaging over the series of the interferograms of short exposures allows us to perform averaging over an ensemble of realizations of the randomly inhomogeneous medium. Thus the mean value of the shearing interferogram signal is

$$\langle i\rangle = \langle I(x, \rho)\rangle + \langle I(x, \rho + \Delta\rho)\rangle + \Gamma_2(x, \rho, \rho + \Delta\rho) \times$$

$$\times \exp(-i\mathbf{a}\rho) + \Gamma_2(x, \rho + \Delta\rho, \rho) \exp(i\mathbf{a}\rho)\,, \quad (1)$$

where $\Gamma_2(x, \rho_1, \rho_2) = \langle U(x, \rho_1)U^*(x, \rho_2)\rangle$ is the second–order mutual coherence function for the optical field at the points of $\{x, \rho_1\}$ and $\{x, \rho_2\}$, and the angular brackets denote averaging over the ensemble of realizations.

Since, by definition of the second–order mutual coherence function

$$\Gamma_2(x, \rho, \rho + \Delta\rho) = \Gamma_2^*(x, \rho + \Delta\rho, \rho)\,,$$

relationship (1) can be written in the form

$$\langle i\rangle = \langle I(x, \rho)\rangle + \langle I(x, \rho + \Delta\rho)\rangle +$$

$$+ 2|\Gamma_2(x, \rho, \rho + \Delta\rho)| \cos\{\mathbf{a}\rho - \arg[\Gamma_2(x, \rho, \rho + \Delta\rho)]\}\,,$$

where $|\Gamma_2(x, \rho, \rho + \Delta\rho|$ and $\arg[\Gamma_2(x, \rho, \rho + \Delta\rho]$ are the modules and argument of the second–order mutual coherence function of the optical field, respectively. Thus

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the shear interferogram is the sinusoidal grating, the interference fringe curvature of which gives the information about the phase contribution into the mutual coherence function of optical radiation. The contrast of the interference pattern (ν) determined as a ratio of the difference between the maximum and minimum intensities to their sum is maximum when optical wave propagates through the homogeneous medium.

When the optical wave passes through the layer of randomly inhomogeneous turbulent atmosphere, the contrast of the mean interference pattern ν is lower than that in the homogeneous medium due to distorting effect caused by atmospheric turbulence. Therefore, it can be estimated by the following formula:

$$\nu = K(x, \rho, \rho + \Delta\rho)\, \gamma_2(x, \rho, \rho + \Delta\rho), \qquad (2)$$

where

$$\gamma_2(x, \rho, \rho+\Delta\rho) = |\Gamma_2(x, \rho, \rho+\Delta\rho)| / \sqrt{\langle I(x, \rho)\rangle \langle I(x, \rho+\Delta\rho)\rangle}$$

is the modules of the complex power of coherence (the value of modules of the second–order mutual coherence function of the optical field normalized to unity);

$$K(x, \rho, \rho+\Delta\rho) = 2\sqrt{\langle I(x, \rho)\rangle \langle I(x, \rho+\Delta\rho)\rangle} / [\langle I(x, \rho)\rangle + \langle I(x, \rho + \Delta\rho)\rangle]$$

is the factor which determines the effect of inhomogeneity in distribution of optical radiation intensity over its cross section on the contrast of the mean interference pattern.

To be specific, let us consider the model situation when optical radiation is assumed to be partially coherent beam with the initial amplitude U_0, initial radius a_0, wave–front curvature radius R_0, and initial coherence radius ρ_c. Then the quantities in Eq. (2) are, respectively, equal to[2]

$$\gamma_2(x, \rho, \rho + \Delta\rho) = \exp\{-[\Delta\rho/\rho_c(x)]^2\} \qquad (3)$$

and

$$K(x, \rho, \rho + \Delta\rho) = 2\exp\left\{\frac{(2\rho + \Delta\rho)\,\Delta\rho}{2a^2(x)}\right\} \Big/ \left[1 + \exp\left\{\frac{(2\rho + \Delta\rho)\,\Delta\rho}{2a^2(x)}\right\}\right], \qquad (4)$$

where

$$a(x) = a_0\{(1 - \mu)^2 + \Omega^{-2}(1 + \theta^2 + 4/3\,\Omega q)\}^{1/2}$$

is the current mean radius of a beam;

$$\rho_c(x) = \rho_0\Big\{[(1 - \mu)^2 + \Omega^{-2}(1 + \theta^2 + 4/3\,\Omega q)]/[1 - \mu + 1/3\,\mu^2 + \Omega^{-2}(1 + \theta^2 + 1/3\,\Omega q) + 1/4\,(\Omega q)^{-1}\theta]\Big\}^{-1/2}$$

is the current coherence radius of optical radiation; $\mu = x/R_0$ is the focusing parameter; $\Theta = a_0/\rho_c$ is the coefficient of the source coherence; $\Omega = ka_0^2/x$ is the Fresnel number of the radiating aperture; $q = x/(k\rho_0^2)$ is the parameter characterizing the turbulent propagation conditions along the path; $k = 2\pi/\lambda$ is the wave number; $\rho_0 = (1.45kC_n^2 x)^{-3/5}$ is the plane wave coherence radius; C_n^2 is the structural constant of the fluctuations in the atmospheric refractive index.

It is follows from Eqs. (3), (4), and (2) that if the linear scale of the interference pattern field $\sim a(x)$ is large as compared with the value of transverse shear in the interferometer $a(x) > \Delta\rho$ (usually this is always fulfilled) then

$$K(x, \rho, \rho + \Delta\rho) \simeq 1$$

and

$$\nu \simeq \exp\{-[\Delta\rho/\rho_c(x)]^2\}.$$

Thus, the contrast of the mean interference pattern will be satisfactory ($\nu \sim 1$) as long as the coherence radius of the recorded field is more than the value of transverse shear in the shearing interferometer

$$\Delta\rho < \rho_c(x). \qquad (5)$$

When condition (5) holds the recorded shear interferograms are either unified field of continuous interference fringes or a few sufficiently large well–correlated speckles (the linear scale of the speckle is $l \sim \rho_c(x)$). In this case the averaged interferogram (since $\nu \sim 1$) allows us to reconstruct the information about the phase contribution into the mutual coherence function of optical radiation with the sufficient accuracy. Therefore, in processing of the individual interferograms, it is easy to find the continuations of the interference fringes in the neighbouring speckles, i.e., as if to reconstruct the whole shear interferogram. When the inverse relation is fulfilled

$$\Delta\rho \geq \rho_c(x). \qquad (6)$$

then the number of the speckles in the interferogram increases to a great extent, the interference fringes undergo strong displacements in discontinuities, and, consequently, the contrast of the averaged interferogram approaches to zero. That means the extremely high error in the phase determination. This situation is analogous, in its physical essence, to the case considered in Ref. 3 when the image of objects illuminated by incoherent radiation is recorded under the very long exposures.

Based on this analogy we can assume that the measurement of higher statistical moments of the recorded value appears to be fruitful for reconstructing the coherent (as well as incoherent[4,5]) images. It is known that in observation through the turbulent atmosphere one can obtain higher resolution in the case of processing of the great number of short–exposure images of incoherent source by Labeyrie[4] and Knox–Thompson[5] techniques based on the measurement of variance and correlation function of intensity fluctuation of optical image, respectively, than when recording of the averaged image. In this connection, it is proposed to record any second moment of the shearing interferometer signal, for example, in the simplest variant, to measure the interferometer signal variance, which is determined by the following expression:

$$\langle i^2\rangle = \langle I^2(x, \rho)\rangle + \langle I^2(x, \rho+\Delta\rho)\rangle + 4\langle I(x, \rho)\, I(x, \rho + \Delta\rho)\rangle +$$

$$+ 4|\Gamma_4(x, \rho, \rho, \rho + \Delta\rho, \rho)| \times$$
$$\times \cos(\mathbf{a}\rho - \arg[\Gamma_4(x, \rho, \rho, \rho + \Delta\rho, \rho)]) +$$
$$+ 4|\Gamma_4(x, \rho + \Delta\rho, \rho + \Delta\rho, \rho, \rho + \Delta\rho)| \times$$
$$\times \cos(\mathbf{a}\rho - \arg[\Gamma_4(x, \rho + \Delta\rho, \rho + \Delta\rho, \rho, \rho + \Delta\rho)]) +$$
$$+ 2|\Gamma_4(x, \rho, \rho + \Delta\rho, \rho, \rho + \Delta\rho)| \times$$
$$\times \cos(\mathbf{a}\rho - \arg[\Gamma_4(x, \rho, \rho + \Delta\rho, \rho, \rho + \Delta\rho)]) , \quad (7)$$

where $|\Gamma_4(x, \rho_1, \rho_2, \rho_3, \rho_4)|$ and $\arg(\Gamma_4(x, \rho_1, \rho_2, \rho_3, \rho_4))$ are the modules and argument of the fourth–order coherence function of the optical radiation field,[6–8] respectively;

$$\Gamma_4(x, \rho_1, \rho_2, \rho_3, \rho_4) = \langle U(x, \rho_1)\, U^*(x, \rho_2)\, U(x, \rho_3)\, U^*(x, \rho_4)\rangle,$$
$$\langle I^2(x, \rho)\rangle = \Gamma_4(x, \rho, \rho, \rho, \rho),$$
$$\langle I(x, \rho)\, I(x, \rho + \Delta\rho)\rangle = \Gamma_4(x, \rho, \rho + \Delta\rho, \rho, \rho + \Delta\rho) .$$

For the area of strong fluctuations in the intensity of optical radiation propagating through the turbulent atmosphere when the coherence radius of the plane wave is smaller than the radius of the first Fresnel zone, i.e.,

$$\rho_0 < \sqrt{x/k} ,$$

the fourth–order coherence functions for the field $U(x, \rho)$ included in the oscillating part of the variance of the shearing interferometer signal have the same characteristic scale over the difference coordinate which is proportional to $\rho_c(x)$. A value of this scale coincides with the transverse size of the speckle. When condition (5) holds the coherence functions (which are the coefficients of the first and second harmonics) are approximately expressed by the second–order coherence functions:

$$\Gamma_4(x, \rho, \rho, \rho + \Delta\rho, \rho) \approx 2 \langle I(x, \rho)\rangle \Gamma_2^*(x, \rho, \rho + \Delta\rho) ,$$
$$\Gamma_4(x, \rho+\Delta\rho, \rho+\Delta\rho, \rho, \rho+\Delta\rho) \approx 2 \langle I(x, \rho+\Delta\rho)\rangle\, \Gamma_2(x, \rho, \rho+\Delta\rho),$$
$$\Gamma_4(x, \rho, \rho + \Delta\rho, \rho, \rho + \Delta\rho) \approx 2\, \Gamma_2^2(x, \rho, \rho + \Delta\rho) . \quad (8)$$

By substituting Eq. (8) into Eq. (7), we can show that for $\Delta\rho \ll \rho_c(x)$ (when the shear in the shearing interferometer is smaller than the linear size of speckle) the extracting of the first harmonic in the expression for $\langle i^2\rangle$ allows us to obtain the information about the phase contribution into the second–order mutual coherence function of the optical radiation field in the same volume approximately as the measurements of ν from $\langle i^2\rangle$. Moreover, it is necessary to note that the amplitude of the first harmonic $\langle i^2\rangle$ decreases slower ($\sim\exp\{-1/2\, [\Delta\rho/\rho_c(x)]^2\}$) with decrease of $\rho_c(x)$ than the contrast of the mean interference pattern. When condition (6) is satisfied, the variance of the shearing interferometer signal becomes the same not informative as the mean interferogram. In a word, transfer to the measurement of the second simplest moment as the variance does not allow us to remove restriction (6) or to solve the problem of "sewing" the broken interference fringes. An advantage of this transfer is in the following fact. When reconstructing the initial field distribution from the measurement of $\langle i^2\rangle$ the distortions in the signal amplitude are proportional to $\exp\{-1/2\, [\Delta\rho/\rho_c(x)]^2\}$ rather than $\exp\{-[\Delta\rho/\rho_c(x)]^2\}$ as for the measurement of the contrast ν, i.e., the accuracy of the initial field restoration can be increased.

As for the spatial correlation function of the fluctuation in the shearing interferometer signal $\langle i(x, \rho_1)\, i(x, \rho_2)\rangle$ by analyzing asymptotically analogously to Refs. 6–8, it can be shown that for $\Delta\rho < \rho_c(x)$ the oscillating contribution to the function $\langle i(x, \rho_1)\, i(x, \rho_2)\rangle$ is proportional to $\langle I(x, \rho_1)\, I(x, \rho_2)\rangle$, where $I(x, \rho_1)\, I(x, \rho_2)\rangle = \Gamma_4(x, \rho_1, \rho_1, \rho_2, \rho_2)$ is the spatial correlation function of intensity fluctuations of the optical wave incident on the interferometer transverse shear. From here one can make a conclusion about two–scale character of the function $\langle i(x, \rho_1)\, i(x, \rho_2)\rangle$ in the area of strong fluctuations in optical radiation. The first scale is determined by the size of regions with high correlation of the fluctuations in the optical field intensity, and therefore, of the interferograms of transverse shear $\sim\rho_c(x)$. The second characteristic scale $r_0 = x/(k\rho_c(x))$ is proportional to the spatial size of the area of weak correlation of the fluctuations in identity of the optical radiation field (since $r_0 \gg \rho_c(x)$ this area includes a great number of the single speckles weakly correlated between each other). Therefore, a measurement of the spatial correlation function i can allows us to reconstruct the shear interferogram not only inside the speckles but to determine the correlation of the interference fringes in the various speckles. As a result, the size of reconstructed areas can be expanded from $\sim \rho_c(x)$ up to $\sim r_0$ ($r_0 \gg \rho_c(x)$), making the statistical "sewing" of the interference fringes, which are in the neighborings speckles. In the case of $\Delta\rho \geq \rho_c(x)$ correlation $\langle i(x, \rho_1)\, i(x, \rho_2)\rangle$ has the only scale proportional to $\rho_c(x)$ and, consequently, a measurement of the spatial correlation function does not give an advantage as compared with the recording $\langle i^2\rangle$ or $\langle i\rangle$.

Thus, as a result of the analysis carried out, two following conclusions arise: 1) to reconstruct the initial distribution of the coherent source field (or an object illuminated by the coherent radiation) from the interferograms of transverse shear when recording of any moments of the shearing interferometer signal, it is necessary for condition (5) to be satisfied, and 2) the second moment of the signal of interferometer of transverse shear allows us to obtain the information about the phase contribution into the second–order spatial coherence function in the greater space area and with the larger accuracy than the first one.

In spite of the fact that the Gaussian profile of optical radiation field distribution is considered in this paper, an analogous character in the behavior of statistical moments of recorded quantity can be apparently expected and for more realistic profiles also (for example, for an object of the arbitrary form with the sharp edges). In conclusion it should be noted that the proposed processing techniques of the series of short–exposure interferograms of transverse shear (recorded during, the time when the turbulence is "frozen") based on the measurement of variance or spatial correlation function of the shearing interferometer signal allows us, in principle, to reconstruct an image of the prolonged objects of unknown form being illuminated by the coherent radiation in a wider region of change in the conditions of propagation through the randomly inhomogeneous medium than the techniques based on the contrast recording of the mean interference pattern of the transverse shear.

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