NEW APPROACH TO MINIMIZATION OF NONLINEAR DIVERGENCE OF LASER BEAMS IN THE ATMOSPHERE

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We propose a new approach to solving the problem on optimizing angular divergence of radiation which is based on calculations of the effective angular beam divergence from the data on distributions of the beam intensity and phase over the beam cross section directly at the exit from a nonlinear medium (an atmospheric layer) avoiding the use of a Fourier transform. It is shown in the paper that such an approach to the selection of a quality criterion makes the optimization much easier. Calculations of the optimal initial profile of a laser beam wave front made for a defocusing medium with the Kerr–type nonlinearity well agree with the data obtained using other techniques.

As known, high–power radiation when passing through the atmosphere is subject to strong distortions.[1] It is also the case when an atmospheric layer mainly brings about phase distortions which result in an increased angular divergence of a laser beam. Since in the far diffraction zone the angular intensity distribution has a complex structure due to nonlinear aberrations,[2] its directional pattern is characterized by mean values: efficient angular divergence and position of the beam center of gravity in the far diffraction zone as well as by the higher–order moments.

For efficient transport of high–power optical radiation through the atmosphere, it is necessary to keep a narrow laser–beam divergence. To this end one must reduce or totally compensate for the wavefront distortions. To do this an optimal control of light beam characteristics is accomplished which allows one to attain the best beam quality in the far diffraction zone, in particular, minimum divergence or maximum intensity along the axis.[3–5]

In the aforementioned papers the field in the far diffraction zone was calculated using an integral Fourier transform for a complex amplitude of laser radiation at the exit from a nonlinear medium. Then based on an angular spectrum the authors calculated the chosen quality criterion whose extremum was found using the gradient method, a standard algorithm of coordinate discent, and the regularization method.[6] This approach resulted in a too complicated algorithm of optimizing an initial shape of the laser beam wave front.[7,8]

It is shown in this paper that the square mean angular divergence of a laser beam coincides with one of the integrals of the parabolic equation which describes light beam propagation behind a nonlinear layer of the atmosphere in free space. Because the integral of movement is kept constant its value, and hence nonlinear divergence can be calculated from the data on transverse distribution of the beam amplitude and phase directly at the exit from a nonlinear layer (atmospheric layer) without the use of complicated Fourier transform. An optimal wavefront shape is found, as before, using the gradient method of control.[7]

We consider the approach proposed, taking, as a case in point, the defocusing medium with Kerr nonlinearity. Let a controllable optical element that can modify in the initial wave front of the beam, e.g., a lens, be at the entrance to a nonlinear medium. Then the laser radiation with a Gaussian amplitude profile of an initial radius a_0 passing through a nonlinear layer enters a linear medium where, in the far diffraction zone, its directional pattern is formed (Fig. 1).

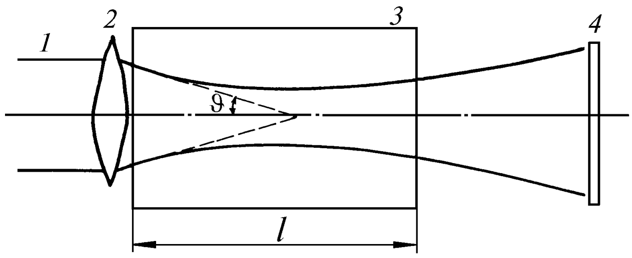

FIG. 1. Scheme of the problem under study: 1) laser beam; 2) controllable lens; 3) nonlinear medium; and 4) receiver in the far diffraction zone.

In quasioptical approximation the beam complex amplitude A in a nonlinear medium satisfies the following equation written for dimensionless variables:

$$2\,i\,\frac{\partial A}{\partial z} = \Delta_{\perp} A + \alpha\,|A|^2 A\,, \qquad 0 < z \le l \qquad (1)$$

with the boundary condition at $z = 0$

$$A(\,z = 0,\,x,\,y) = \exp\left(-\frac{x^2+y^2}{2}\right)\exp\left(i\,\frac{x^2+y^2}{2}\,\vartheta\right). \qquad (2)$$

Here x, y, and z are transverse and longitudinal coordinates; α is the parameter of nonlinearity; l is the nonlinear layer thickness; ϑ is the normalized initial focusing of the beam equal to the ratio of a diffraction length $l_d = k\,a_0^2$ to the focal length of the lens, F.

In free space, behind the layer of the nonlinear medium, Eq. (1) takes the form

$$2\,i\,\partial A/\partial z = \Delta_{\perp} A\,, \qquad l < z \le z_0\,, \qquad (3)$$

where $z_0 \gg l_d$ which corresponds to the far diffraction zone.

The solution of this equation makes it possible to calculate radiation characteristics in the far diffraction zone, in particular, the effective angular divergence of the beam.

The gradient control of the beam wave front (initial focusing) is realized as

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$$\vartheta_{N+1} = \vartheta_N - \gamma \, \mathrm{grad}_{\vartheta_N} J \, , \qquad N = 0, 1, 2 \, , \tag{4}$$

where N is the number of the iteration; γ is the control coefficient; J is the quality criterion for radiation in the far diffraction zone equal the functional[7]:

$$J = \Theta^2 = \frac{\int\int_{-\infty}^{+\infty} (k_x^2 + k_y^2) \, | S(k_x, k_y, l)|^2 \, \mathrm{d}k_x \, \mathrm{d}k_y}{\int\int_{-\infty}^{+\infty} | S(k_x, k_y, l)|^2 \, \mathrm{d}k_x \, \mathrm{d}k_y} \, , \tag{5}$$

which has a meaning of the square mean angular width of the directional pattern of the laser beam passing through a l–thick nonlinear–medium layer. Here $S(k_x, k_y, l)$ is the Fourier image of a complex beam amplitude A taken in the cross section $z = l$:

$$S(k_x, k_y, l) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} A(x, y, l) \exp(- i \, k_x x - i \, k_y y) \, \mathrm{d}x \, \mathrm{d}y. \tag{6}$$

It is clear that the calculations of the functional (5) and the optimized focusing ϑ (4) are done for different cross sections in the medium, the control is made in the cross section $z = 0$, at the entrance to the medium, and the functional is taken in the far diffraction zone. Therefore to solve the problem formulated one must consider the systems of adjoint equations[7] both in a nonlinear layer and free space where a directional pattern is formed.

In this case, as noted above,[7] there can exist two types of problems. The first one is minimization of radiation angular divergence behind the layer of a nonlinear medium in the far diffraction zone in free space. Such problems partially arise in atmospheric optics when systems of space communication and measurements are used. The second type of problems is peculiar for laser technology: minimization of angular divergence in this case is realized in a focal plane of a lens placed at the exit from the nonlinear medium.

To solve such problems two systems of adjoint equations are used.[7] The first system consists of Eq. (1) which is solved on a segment of the nonlinear medium $0 < z \le l$ and the adjoint equation which is solved in the opposite direction. The second system incorporate Eq. (3) for a complex amplitude of the beam in free space which can be solved either on the segment $l < z \le z_0$ for the first type of problems or on the segment $l < z \le R_f$ (where R_f is the focal length of the lens placed at the exit from the nonlinear layer), as well as the related adjoint equations which are solved in the opposite direction.

It is easily seen that in such a method of calculating the quality criterion the calculational procedure is sufficiently complicated and entails lengthy computations.

At the same time it is possible to show that the quality criterion or the goal function of control (5) is the integral of movement of Eq. (1) and, hence, it can be calculated not only in the far zone field but also in any cross section, the exit from the nonlinear layer included. Actually, in the denominator of the functional (5) there is the total beam power which is kept constant in free space

$$P_0 = \int\int_{-\infty}^{+\infty} |S(k_x, k_y, l)|^2 \, \mathrm{d}k_x \, \mathrm{d}k_y = \mathrm{const} \, . \tag{7}$$

By substituting the expression for the spectrum (6) into the numerator of the functional (5), after integration by parts, we obtain

$$J = \Theta^2 = \frac{1}{P_0} \int\int_{-\infty}^{+\infty} \left(\left| \frac{\partial A(x, y, z)}{\partial x} \right|^2 + \left| \frac{\partial A(x, y, z)}{\partial y} \right|^2 \right) \mathrm{d}x \, \mathrm{d}y. \tag{8}$$

The value (8) is the movement integral of the parabolic equation (3) since it is easy to prove that $\partial J / \partial z = 0$, Ref. 9 (here z is the coordinate in free space at $\alpha = 0$)

Thus the square mean angular divergence of the beam is equal, accurate to the constant factor, to the movement integral (8) which is chosen as the quality criterion.

Consider now a mathematical algorithm for calculating the optimal initial focusing in this case. The equation adjoint with Eq. (1) has the form

$$- 2 \, i \, \partial\psi / \partial z = \Delta_\perp \psi + 2\alpha \, |A|^2 \, \psi - \alpha \, A^{*2} \, \psi^* \, . \tag{9}$$

After variations of the functional (8) have been calculated we obtain the following boundary condition for it;

$$\psi(x, y, l) = - [\partial^2 A^*(x, y, l) / \partial x^2] - [\partial^2 A^*(x, y, l) / \partial y^2]. \tag{10}$$

The functional gradient J is calculated, as earlier,[7] by formula

$$\frac{\delta J}{\delta \vartheta}(z = 0) = \mathrm{Re} \left[\, i \int\int_{-\infty}^{+\infty} \psi(x, y, 0) \,)(x^2 + y^2) \exp\left(- \frac{x^2 + y^2}{2} \right) \times \right.$$
$$\left. \times \exp\left(i \, \frac{(x^2 + y^2) \, \vartheta}{2} \right) \mathrm{d}x \, \mathrm{d}y \right] \tag{11}$$

The optimal initial focusing is found using the gradient method (4).

The numerical calculations of the optimal initial focusing for different values of radiation and medium parameters were made using the foregoing scheme. Depicted in Figs. 2 and 3 are the plots obtained in such calculations for the following values of the parameters of the beam and the medium: $\alpha = - 10$, $l = 0.25$. As seen from Fig. 2, the use of both the control algorithm with the Fourier transform[7] and the method under study provide an optimal regime at about equal number of iterations which is also determined by the initial value of the focusing parameter chosen. The plots in Fig. 3 enable one to follow the process of minimization of the goal function of control during iteration. The results obtained using the two different methods are in good agreement as it is the case in Fig. 2.

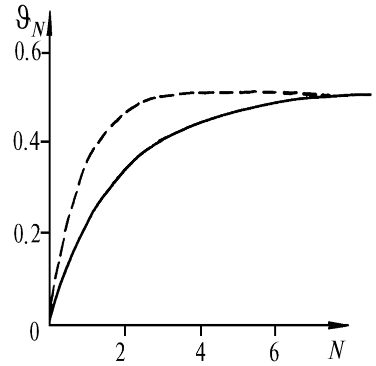

FIG. 2. A plot of the value of initial beam focusing vs a number of iterations for $\alpha = - 10$, $l = 0.25$, and $\vartheta_0 = 0$. Solid curve illustrates calculations using the method described; dashed curve the calculations using the Fourier transform technique.

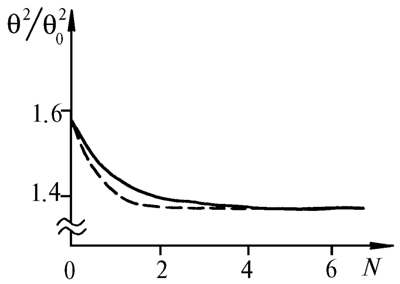

FIG. 3. A plot of the normalized quality criterion vs a number of iterations for $\alpha = -10$, $l = 0.25$, and $\vartheta_0 = 0$. Solid curve is for calculations made using the method described; dashed curve is for calculations by Fourier transform (Q_0^2 is the angular divergence in a linear medium).

The above approach was also applied to calculations made in noniterative approximation. In this approximation the complex amplitude of a Gaussian beam is

$$A(x, y, z) = \frac{A_0}{f(z)} \exp\left(-\frac{x^2 + y^2}{a_0^2 f^2} - \frac{ikx^2}{2f}\frac{df}{dz} - \frac{iky^2}{2f}\frac{df}{dz} + i\varphi(z) \right), \quad (12)$$

where A_0 is the amplitude of the field at the beam axis; $\varphi(z)$ is the phase difference at the beam axis; $f(z)$ is the dimensionless beam width. Equation (1) is converted to the form

$$d^2f/dz^2 = (1 + \alpha_{NL}) / f^3 \quad (13)$$

with boundary conditions at $z = 0$

$$f(0) = 1\ ,\ df(0)/dz = -\vartheta\ , \quad (14)$$

where $\alpha_{NL} = \alpha/4$ is the nonlinearity parameter.

As shown in Ref. 4, the total angular divergence in the far zone (chosen, as in the previous consideration, as a quality criterion) can be calculated as a sum of nonlinear and diffraction limited divergences at the boundary of the medium at $z = l$:

$$J = \Theta^2 = \left(\frac{df}{dz}\right)^2\bigg|_{z=l} + \frac{1}{f^2}\bigg|_{z=l}\ . \quad (15)$$

Let us show that this value, accurate to the constant factor, corresponds to the movement integral I_3. Actually, by substituting Eq. (12) into Eq. (8) and making integration we obtain the expression (15) for the total angular divergence accurate to the constant factor.

To perform the algorithm of gradient control (4) we obtain the equation adjoint to Eq. (13). By varying Eq. (13) and the boundary conditions (14) over the variable ϑ we obtain

$$\frac{d^2(\delta f)}{\delta z^2} = (-3)\frac{1 + \alpha_{NL}}{f^4}\delta f\ ; \quad (16)$$

$$\delta f(0) = 0\ ,\ \frac{d\delta f(0)}{dz} = -\delta\vartheta\ . \quad (17)$$

The variation of the functional (15) is written as

$$\delta J(z = l) = (-2)\ \frac{\delta f}{f^3}\bigg|_{z=l} + 2\ \frac{df}{dz}\frac{d\delta f}{dz}\bigg|_{z=l}\ . \quad (18)$$

By multiplying Eq. (16) by ψ and integrating it over z from 0 to l and determining the boundary conditions for a new equation from a comparison with Eq. (18) we obtain the adjoint equation

$$\frac{d^2\psi}{dz^2} = (-3)\frac{1 + \alpha_{NL}}{f^4}\psi \quad (19)$$

with the boundary conditions

$$\psi(l) = 2\frac{df(l)}{dz}\ ,\ \frac{d\psi(l)}{dz} = \frac{2}{f^3(l)}\ . \quad (20)$$

Then the derivative of the functional for angular divergence of radiation is found from the boundary conditions for

$$\frac{\delta J}{\delta\vartheta}(z = l) = -\psi(0)\ . \quad (21)$$

Thus the optimal initial focusing ϑ is found using the expression (4) and solving Eqs. (13) and (19) with proper boundary conditions by numerical methods and calculating the gradient of the functional (21).

As known equation (13) has an analytical solution, and the dependence J on ϑ can be represented in an explicit form.[4] However, it was shown, that calculation of the gradient leads to the fifth–power equation with respect to ϑ. Therefore the numerical methods are also needed for its solution. As in the previous case, the optimal initial focusing for different values of the nonlinear layer thickness l and the nonlinear parameter α was calculated using the above calculational scheme. The results obtained coincide with the values ϑ_{opt} obtained previously from numerical solution of the equation for the optimal initial focusing.[4]

Thus, it is clear that the proposed approach treated in quasioptical and nonaberrational approximations provides a simpler mathematical algorithm of the initial focusing optimization compared to the one from Ref. 7. As a result combersome calculations of the radiation quality criterion in the far zone and solving the direct and adjoint equations for the beam complex amplitude in free space are avoided. The method reduces the bulk of numerical calculations by a factor of two thus increasing the response of the optical control system and simplifying the problem of optimization.

The higher efficiency of the system operation on minimizing nonlinear beam divergence can be attained using an aberration lens (mirror) which creates an optimal wave front of a more complex shape. Relevant algorithm does not substantially differ from the one described in this paper.

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