PROPAGATION OF AN AMPLITUDE–MODULATED WAVE THROUGH A MEDIUM WITH WEAKLY NONLINEAR DISPERSION

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Received July 29, 1993

The effect of weak nonlinearity in the dispersion of a medium on the complex form of simple signals is considered. Under such conditions the effect of spatially periodic transformation of AM to PM and back from PM to AM, so–called transmodulation, is distinctly pronounced. Some possible practical applications of this phenomenon are demonstrated.

Problems of the interaction of electromagnetic waves with the atmosphere are of permanent interest for investigators in the context of the prediction of propagation conditions for signals of various information–measuring systems[1] (for purposes of communication, detection and ranging, range finding, and so on) and of solving a wide class of problems of determining the parameters of a propagation medium[2] (for purposes of meteorology, ecology, geophysics, etc).

It is assumed *a priori* in these researches that the frequency dispersion is linear within the signal spectral width $\Delta\omega$ which is typically much less than the carrier frequency ω_0, that is, $\Delta\omega \ll \omega_0$.

However, this tentative assumption may violate under real conditions of measurements, for example, in remote sounding near selective absorption lines where the dispersion is not linear, or when broad band signals are used for which the dispersion is not linear over the bandwidth even if the spectrum of the signal is rather far from absorption lines.

In this connection it makes sense to estimate the effect of nonlinearity in the dispersion of a medium on a signal. This is the objective of the present paper.

To grasp the situation under above–indicated conditions, it will suffice to consider a quasimonochromatic plane wave train, that is, to represent a signal in the form

$$E_0(t) = A(t)\exp(i\,\omega_0\,t)\ , \qquad (1)$$

where $A(t)$ is a slowly varying function, to assume a non–absorptive medium, and to expand the phase of the signal in a power series retaining the first three terms

$$\varphi(\omega) = \varphi(\omega_0) + \varphi'(\omega_0)\,\Omega + (\varphi''(\omega_0)\,/\,2)\,\Omega^2 \ldots\ , \qquad (2)$$

where $\Omega = \omega - \omega_0$, and derivatives are taken with respect to ω.

In the linear theory wave field is represented in the form of a Fourier integral at the point of radiation emission and the inverse transform is taken at the point of signal reception.[3] The shape of signal transmitted through the medium assumes the form

$$E(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\!\!\int A(\tau)\exp[i\,(\omega_0\tau - \omega\,\tau + \omega\,t - \varphi(\omega))]\,\mathrm{d}\,\tau\,d\,\omega. \qquad (3)$$

It is evident that correct conclusion about the shape of the signal transmitted at a fixed distance through the medium can be made only if the functions $A(t)$ and $\varphi(\omega)$ are known.

In principle the problem can be completely solved for arbitrary form of the above–indicated functions by direct integration of expression (3) using modern computers to an accuracy being sufficient for practice. However, analytical solutions can be obtained only in relatively simple cases.

One case is the wave whose amplitude is modulated by harmonic signal with the frequency Ω and the degree of modulation m. This case is of great practical importance. At the point of radiation emission the amplitude modulated (AM) wave can be described by the following expression:

$$E_0(t) = (1 + m\cos\Omega\,t)\exp(i\,\omega_0\,t)\ . \qquad (4)$$

Substituting Eqs. (4) and (2) into Eq. (3) and omitting cumbersome intermediate manipulations, we finally derive for the field transmitted through the medium

$$E(t) = \left[1 + 2\,m\cos\frac{\varphi''(\omega_0)\,\Omega^2}{2}\cos(\Omega\,t - \varphi'(\omega_0)\,\Omega) + m^2\cos^2(\Omega\,t - \varphi'(\omega_0)\,\Omega)\right]^{0.5} \times$$

$$\times\exp i\left[\omega_0 t - \varphi(\omega_0) - \arctan\frac{m\sin\dfrac{\varphi''(\omega_0)\Omega^2}{2}\cos(\Omega t - \varphi'(\omega_0)\Omega)}{1 + m\cos\dfrac{\varphi''(\omega_0)\Omega^2}{2}\cos(\Omega t - \varphi'(\omega_0)\Omega)}\right]. \qquad (5)$$

A comparison of transmitted (4) and received (5) signals shows that even weakly nonlinear dispersion of the medium strongly affects the complex form of the signal.

Because of limitations on the length of the article, we omit the details of the computer analysis of propagation of AM wave and point out only qualitative features that are illustrated by expression (5).

It follows from equation (5) that both the amplitude and phase of the AM signal of the above–considered form involve the harmonic functions of time mutually displaced in phase by $\pi/2$. This means that at arbitrary point of reception of the AM wave one can simultaneously observe both amplitude and phase modulation. The modulation type transforms periodically in space from AM to PM and back, that is, the signal propagation is accompanied by transmodulation phenomenon, as pointed out in Ref. 4, but has not yet been investigated in detail.

0235-6880/94/03 227-02 $02.00

If the condition $\varphi''(\omega)\,\Omega^2 \simeq \pi$ is valid on the path from a transmitter to a receiver, this leads to the formation of blind zones during reception of the AM signal or to spectral distortion of the AM signal at the frequences at which this condition is valid.

Treating the frequency as the derivative of the signal phase with respect to time, deviation of the carrier frequency of the received signal should be mentioned. It may result in the tuning instability in the receiving channel especially for systems with automatic phase tuning.

In addition it follows from Eq. (5) that when operating through the real atmosphere the conditions $\varphi''(\omega)\,\Omega^2 \ll \pi$ and $m < 1$ are most often realized. Under these conditions Eq. (5) can be reduced to

$$E(t) = [1 + m\cos(\Omega\, t - \varphi'(\omega_0)\,\Omega)] \times$$

$$\times \exp i\,[(\omega t - \varphi(\omega_0) - m\,(\varphi''(\omega_0)\,\Omega^2 / 2)\cos(\Omega\, t - \varphi'(\omega_0)\,\Omega)]. \quad (6)$$

Differentiating the third term of the exponent with respect to time, we obtain the equation

$$\Delta\,\omega = (m\,\varphi''(\omega_0)\,\Omega^3 / 2)\,\sin(\Omega\, t - \varphi'(\omega_0)\,\Omega)] =$$

$$= \Delta\,\omega_{\text{A}}\,\sin(\Omega\, t - \varphi'(\omega_0)\,\Omega)\,, \quad (7)$$

where $\Delta\omega_{\text{A}}$ is the measurable amplitude of deviation of the carrier frequency at the point of reception. This quantity enables one to reconstruct the parameters of the medium.

For instance, we dwell on the use of Eq. (7) for calculation of the concentration of the component of homogeneous dispersive medium when operating near the absorption line of this component.

Taking into account the well–known expression for the phase of the signal

$$\varphi(\omega) = (\omega / c)\, n(\omega)\, L\,, \quad (8)$$

where n is the refractive index of the medium, and L is the length of the signal propagation path, differentiating Eq. (8) two times, and substituting the obtained expression into Eq. (7), we derive for the carrier frequency deviation

$$\Delta\,\omega_{\text{A}} = \frac{m\,\Omega^3\, L\, d\, n_{\text{gr}}}{2\,c\,d\,\omega}\,, \quad (9)$$

where n_{gr} is the group refractive index of the medium. Assuming that the spectral line has the Lorentz shape with the central frequency ω_i, rate of decay g_i, and oscillator strength A_i and using successfully the Kramers–Kronig and Rayleigh relations, we obtain

$$n_{\text{gr}}(\omega) = N\,A_i\,\omega_i\,\omega\,(\omega_i^2 + \omega^2)\,\frac{(\omega_i^2 - \omega^2)^2 - g_i^2\,\omega_i^2\,\omega^2}{[(\omega_i^2 - \omega^2)^2 + g_i^2\,\omega_i^2\,\omega^2]^2}\,. \quad (10)$$

Transformation to new coordinates affixed to the line center $\Delta = \omega - \omega_i$ in Eq. (10) and subsequent differentiation yield for Eq. (9)

$$\Delta\,\omega_{\text{A}} = \frac{m\,\Omega^3\,L\,N\,A_i}{2\,c}\;\frac{-\,64\,\omega_i^3\,\Delta^3 + 80\,g_i^2\,\omega_i^{10}\,\Delta}{(4\,\omega_i^2\,\Delta^2 + g_i^2\,\omega_i^4\,\Delta)^3}\,. \quad (11)$$

Solving Eq. (11) for N at maximum amplitude of deviation for $\Delta = 0.21\,g_i\,\omega_i$, we obtain the sought–after concentration

$$N = \frac{\Delta\omega_{\text{A}}\,c\,g_i^3\,\omega_i}{8.1\,m\,\Omega^3\,A_i\,L}\,.$$

Thus it has been shown in the paper that the propagation of amplitude–modulated wave through a medium with even weakly nonlinear dispersion is accompanied by significant changes of the complex signal form that must be taken into account when operating through this medium. Moreover, a possible application of these changes for determining some parameters of the propagation medium has been demonstrated as well.

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