# CALMAN–BUCY FILTRATION IN LIDAR SENSING OF TEMPERATURE BY THE MASON TECHNIQUE

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*Feasibility of application of the optimal Markovian filtration to trifrequency laser sounding of the atmospheric temperature by the Mason technique for a stochastic model of temperature fluctuations smoothed by a laser sounding pulse has been demonstrated. An algorithm of Calman–Bucy filtration has been synthesized that allows one to obtain an optimal estimate of a fluctuating temperature profile and its variance simultaneously with the maximum likelihood estimates of the aerosol and molecular scattering coefficients as well as of the density of a sounded gas.*

## 1. INTRODUCTION

The principle of laser sounding of temperature proposed by J.B. Mason[1] allows one to determine the population of molecular rotational states and to retrieve the atmospheric temperature from the data on differential absorption. Practically, this technique has been realized in the following variants: bifrequency[2] that imposes less stringent requirements upon emitting system and trifrequency[3] that allows one to decrease the number of *a priori* assumptions and parameters.

The spatially resolved trifrequency technique that involves sounding at the wavelengths $\lambda_1$ and $\lambda_2$ lying in the centers of the two absorption lines and at the wavelength $\lambda_0$ selected close to $\lambda_1$ and $\lambda_2$ but different from any of them, is based on the comparison between the estimates of the same concentration of a gas sounded by the differential absorption (DA) technique. The formula for the temperature estimate was derived in Ref. 1 disregarding absorption at $\lambda_0$. This disadvantage resulting in the systematic error in estimating the temperature by the trifrequency Mason technique was eliminated in Ref. 4. However, no one paper took into account the stochastic structure of the spatial temperature profiles $T(z)$ and the estimates obtained were not analyzed against the criterion of optimization.

Efficiency of laser sounding of the temperature by any technique is limited by fluctuations of the measurable profile and shot fluctuations in signal and noise. To increase the efficiency of sounding, it is necessary to increase the energy potential of a lidar and to optimize the algorithm for received signal processing. Feasibility of application of the optimal Markovian filtration of the vertical temperature fluctuations smoothed by a lidar pulse to bifrequency sounding at the wavelengths $\lambda_1$ and $\lambda_0$ lying at the line centers and off the line of oxygen absorption was demonstrated in Ref. 5.

The problem of synthesis of statistically optimal algorithms for processing of the DA lidar signals is further considered in this paper as applied to trifrequency sounding of the temperature. In particular, feasibility of application of the Calman–Bucy filter is shown and the efficiency of sounding is analyzed by the numerical modeling method for such processing of signals received mainly in the photon counting mode.

## 2. PHYSICAL PREMISES

Let us consider a ground–based monostatic lidar that emits the pulses with normalized power function $f(z)$ at the wavelengths $\lambda_0$, $\lambda_1$, and $\lambda_2$ lying at the centers $\lambda_1$ and $\lambda_2$ of the oxygen ($O_2$) or water vapor ($H_2O$) molecular absorption lines and off the absorption lines at $\lambda_0$ and sounds the atmosphere within the altitude range $[z_0, z_{max}]$. The power $P_{s\,i}(z)$ of the signal component at the input of a detector received from the distance $z$ at $\lambda_1$ is determined by the lidar equation[6]

$$P_{s\,i}(z) = \chi_1 E_0 S_a z^2 \beta_i(z) \frac{c}{2} Y^2_{a\,i}(0, z)\, Y^2_{R\,i}(0, z)\, J_i(z), \qquad (1)$$

where it is assumed, according to Ref. 5, that smoothing over the running interval $[z - L, z]$ essentially affects only the spatial realizations of the transmission function $\tilde{Y}_{g\,i}(0,z)$ at $\lambda_i$, caused by absorption[6] of $O_2$ or $H_2O$, and the vertical profiles of the absorption characteristics and thermodynamic parameters of the atmosphere associated with them. In formula (1), $\chi_1$ is the total coefficient of losses in the receiving and transmitting optics, $E_0$ is the energy of the emitted pulse, $S_a$ is the effective area of the receiving aperture, $\beta_i(z)$ is the profile of the aerosol and molecular backscattering coefficient, $Y_{a\,i}$ and $Y_{R\,i}$ are the transmission functions caused by aerosol and molecular scattering that during one sounding act are considered to be deterministic but unknown functions of altitude, $c$ is the light velocity, $\tau = 2\,z/c$, $L = c\tau_p/2$, $\tau_p$ is the effective duration of the emitted pulse,

$$J_i(z) = \frac{2}{c} \int\limits_0^z dz'\, f\left[\frac{2(z - z')}{c}\right] \tilde{Y}^2_{g\,i}(0, z'), \qquad (2)$$

and $z = 0, 1, 2$. The tilde denotes natural profiles.

The mass absorption coefficient of molecules at the frequency $\nu$ centered at $\nu_i$ has the form[6]:

 

$$\tilde{K}_g(z;\, v - v_i) = \tilde{S}_g(z;\, \lambda_i)\, \tilde{f}_g(z;\, \lambda - \lambda_i)\,, \qquad (3)$$

where $\tilde{S}_g(z;\, \lambda_i)$ is the intensity of the absorption line at the wavelength $\lambda_i = c/\nu_i$, depending on $\tilde{T}(z)$; $f_g(z;\, \nu - \nu_i)$ is the Voigt profile that describes the absorption line shape in the general case with allowance for collisional and Doppler broadening.

Following Ref. 5, let us represent the random quantities in the form $\tilde{T}(z) = \overline{T}(z) + \Delta\tilde{T}(z)$, where $\overline{T}(z)$ is the *a priori* mean profile of $\tilde{T}(z)$ with sufficient statistics averaged over an ensemble of temperature fluctuations.

For arbitrary profiles $\tilde{T}(z)$ and $\tilde{P}(z)$ (temperature and pressure, respectively), $\tilde{S}_g(z,\, \lambda_i)$ and $f_g(z;\, \nu - \nu_i)$ have the form[6]

$$\tilde{S}_g(z;\, \lambda_i) = S_g(\overline{T}(z);\lambda_i) \left\{\frac{\overline{T}(z)}{\tilde{T}(z)}\right\}^{3/2} \exp\left\{-\frac{hc}{k} E''_i \left(\frac{1}{\tilde{T}(z)} - \frac{1}{\overline{T}(z)}\right)\right\}, \qquad (4)$$

where $S_g(\overline{T}(z);\, \lambda_i)$ is the line intensity at the temperature $\overline{T}(z)$; $h$ and $k$ are the Planck and Boltzmann constants, respectively; $E''_i$ is the energy of the lower level of the transition to $\lambda_i$;

$$\tilde{f}_g(z;\, \nu - \nu_i) = \frac{\tilde{f}'\,\tilde{a}}{\pi} \int\limits_{-\infty}^{\infty} \mathrm{d}y\, \frac{\exp(-y^2)}{\tilde{a}^2 + (\tilde{\mu} - y)^2}\,, \qquad (5)$$

where $\tilde{f}' = \dfrac{\sqrt{\ln(2/\pi)}}{\tilde{\gamma}_\mathrm{D}}$, $\tilde{a} = \sqrt{\ln 2}\, \dfrac{\tilde{\gamma}_\mathrm{L}}{\tilde{\gamma}_\mathrm{D}}$, and $\tilde{\mu} = \sqrt{\ln 2}\, \dfrac{(\nu - \nu_i)}{\tilde{\gamma}_\mathrm{D}}$,

$$\tilde{\gamma}_\mathrm{L}(\lambda_i) = \gamma_\mathrm{L}(\lambda_i;\, \tilde{T}(z),\, \tilde{P}(z)) = \gamma_\mathrm{L}(\lambda_i;\, \overline{T}(z)) \left[\frac{\tilde{T}(z)}{\overline{T}(z)}\right]^m \frac{\tilde{P}(z)}{\overline{P}(z)}, \qquad (6)$$

$$\tilde{\gamma}_\mathrm{D}(\lambda_i) = \gamma_\mathrm{D}(\lambda_i;\, \tilde{T}(z)) = \gamma_\mathrm{D}(\lambda_i;\, \tilde{T}(z)) = \gamma_\mathrm{D}(\lambda_i;\, \overline{T}(z))\sqrt{\tilde{T}(z)/\overline{T}(z)} \qquad (7)$$

are the Lorentz and Doppler half–widths of the absorption lines at $\tilde{T}(z)$, $\tilde{P}(z)$; $\gamma_\mathrm{L}(\lambda_i,\, \overline{T}(z),\, \overline{P}(z))$, $\gamma_\mathrm{D}(\lambda_i,\, \overline{T}(z))$ are the same half–widths but for $\overline{T}(z)$, $\overline{P}(z)$, $m = 1/2$ in the Doppler pressure–broadening regime and depends on the transition in this regime and thus in general case.

Since the condition $\sigma[\tilde{T}(z)] \ll \overline{T}(z)$ is satisfied in the atmosphere for the root–mean–square deviation $\sigma[\tilde{T}(z)]$ of the temperature, we can linearize the mass coefficient $\tilde{K}_g(z;\, \lambda_i)$ at the center of the absorption line at $\tilde{T}(z)$ and $\tilde{P}(z)$ for $\Delta\tilde{T}(z)$ in the form

$$\tilde{K}_g(z;\, \lambda_i) = \overline{K}_g(z;\, \lambda_i) + \left.\frac{\mathrm{d}\tilde{K}_g(z;\, \lambda_i)}{\mathrm{d}\tilde{T}}\right|_{\tilde{T} = \overline{T}} \Delta\tilde{T}(z)\,, \qquad (8)$$

where

$$\overline{K}_g(z;\, \lambda_i) = K_g(z;\, \lambda_i,\, \overline{T}(z),\, \overline{P}(z))\,,$$

$$\frac{\mathrm{d}\tilde{K}_g(z;\, \lambda_i)}{\mathrm{d}\tilde{T}(z)} = \left[\frac{h\,c}{k} E''_i - \frac{3}{2} - G_i(z)\right] \frac{\overline{K}_g(z;\, \lambda_i)}{\overline{T}(z)}\,, \qquad (9)$$

$$G_i(z) = 1 + 2\, \tilde{a}_i^2 - \tilde{a}_i \exp(-\tilde{a}_i^2) \Big/ \int\limits_{\tilde{a}_i}^{\infty} \exp(-y^2)\, \mathrm{d}y \qquad (10)$$

describes the line shape depending on the altitude, affected by collisional ($\tilde{a}_i = \infty$, $G(z) = 0$) and Doppler ($\tilde{a}_i = 0$, $G(z) = 1$) broadening.

Let us expand $\tilde{Y}^2_{g\,i}$ in the Taylor series around the profile $\Delta\tilde{T}(z)$ in the vicinity of the smoothed altitude realization

$$\Delta T(z) = \frac{2}{c} \int\limits_0^z f[2(z - z')/c]\, \Delta\tilde{T}(z')\, \mathrm{d}z'\,. \qquad (11)$$

In the linear approximation we can write for the functional

$$\tilde{Y}^2_{g\,i}(0,\, z) \simeq \overline{Y}^2_{g\,i}(0,\, z) \left\{\mathrm{e}^{-z\,\Delta s_i} + \int\limits_0^z dz' \mathrm{e}^{-z\,\Delta s_i}(\Delta\tilde{\gamma}_i - \Delta\gamma_i)\right\}, \qquad (12)$$

where $\Delta\gamma_i(z) = \rho(z)\, \overline{K}_g(z;\, \lambda_i)\, B_i(z)\, \dfrac{\Delta T(z)}{\overline{T}(z)}$ and $\Delta\tau_i(0,\, z) = \displaystyle\int_0^z \mathrm{d}z' \Delta\gamma_i(z)$ are the smoothed fluctuations of the absorption coefficient and optical depth due to absorption, respectively; $\Delta\tilde{g}_i$ and $\Delta\tilde{\tau}$ are the same at $\Delta\tilde{T}(z)$; $B_i(z) = \dfrac{h\,c}{k} E''_i - \dfrac{3}{2} - G_i(z)$.

After integration of Eq.(2), taking into account that $\Delta\tilde{\gamma}_i$ is close to $\Delta\gamma_i$ for the functional $J_i(z)$, we obtain

$$J_i(z) \simeq \overline{Y}^2_{g\,i}(0,\, z) \exp\left\{-2 \int\limits_0^z dz'\, \rho(z')\, \overline{K}_g(z';\, \lambda_i)\, B_i(z')\, \frac{\Delta T(z')}{\overline{T}(z')}\right\}, \qquad (13)$$

where $\overline{Y}^2_{g\,i}(0,\, z)$ is the transmission function due to absorption by $O_2$ and $H_2O$ at $\overline{T}(z)$ and $\overline{P}(z)$.

### 3. MODEL OF SIGNAL AND NOISE

Let $L \gg \tilde{z}_\mathrm{kT}$, where $\tilde{z}_\mathrm{kT}$ is the vertical correlation length of the unsmoothed fluctuations $\Delta\tilde{T}(z)$. Then, following Ref. 5, let us take the approximation of the normalized fluctuations $\eta_1(\tau) = \Delta T(c\tau/2)/\sigma_\mathrm{T}(z)$ in the form of a Gaussian Markovian process. Full statistical description of the temperature fluctuations and absorption characteristics connected with them gives a four–dimensional vector–process $\eta = \{\eta_j\}^T$, where $j = 1, 2, 3, 4$, whose components satisfy the system of stochastic differential equations (SDE) of the form

$$\dot{\eta}_1 = -\alpha\,\eta_1 + w_1(t)\,,$$
$$\dot{\eta}_{2+i} = c\,\rho_g(z)\,\overline{\sigma}_{ki}\,\eta_1(\tau)\,, \quad (14)$$

where $w_1(\tau)$ is the white Gaussian noise: $\langle w_1(\tau)\rangle = 0$, $\langle w_1(\tau)\,w_1(\tau')\rangle = 2\alpha\delta(\tau - \tau')$, $\alpha = 1/\tau_p$, and

$$\overline{\sigma}^{\,2}_{k\,i} = \overline{K}_g(z;\lambda_i)\,B_i^2(z)\,\sigma_T^2 \,/\, \overline{T}^{\,2}(z) \quad (15)$$

is the variance of the mass absorption coefficient $K_g(z;\lambda_i)$ of the sounded gas.

Then in the main photon counting regimes we have for the given realizations $\eta(\tau)$ at the output of optical detection channels at the wavelengths $i = 0, 1, 2$ :

$$\mathbf{y}(\tau) = \mathbf{s}(\tau;\,\mathbf{V},\,\boldsymbol{\eta}) + \mathbf{n}(\tau)\,, \quad (16)$$

where, according to Eqs. (1) and (13),

$$\mathbf{s}(\tau;\,\boldsymbol{\eta},\,\mathbf{V}) = \overline{\mathbf{s}}(\tau;\,\mathbf{V})\,\xi(\tau;\,\boldsymbol{\eta})$$

is the vector of the ensemble averaged shot fluctuations of the signal components $s_i(\tau,\,\mathbf{V},\,\boldsymbol{\eta})$ of the photoelectric current in the form of digital values or the number of photoelectrons accumulated in the gate interval $\Delta\tau_c \ll \tau_p$,

$$\mathbf{V} = \{\mathbf{u},\,\rho\} = \{\mathbf{u}_i,\,\rho\}\,,\ \mathbf{u}_i = \{\beta_i,\,\overline{Y}_i\}\,,\ \xi(\tau;\,\boldsymbol{\eta}) = \{\xi_i(\tau;\,\boldsymbol{\eta})\}$$

is the three–dimensional vector, whose components, according to Eq. (13), are

$$\xi_i(\tau;\,\boldsymbol{\eta}) = \exp\{-\,2\,\eta_{2i}\}\,, \quad (17)$$

where $\eta_{2i} = \Delta\tau_{g\,i}(0,\,z)$ are the optical depth fluctuations caused by absorption of the sounded gas at $\lambda_i$ ,

$$\overline{Y}_i = \exp\left\{-\int\limits_0^z \mathrm{d}z'\,s_i(z')\right\}, \qquad \beta_i = g_\pi\,\sigma_i\,. \quad (18)$$

Using the Bunyakowskii–Schwarz inequality and linerizing Eq.(17) for $\Delta\tau_{g\,i}(0,z)$, one can show that[5] $\sqrt{\overline{\Delta\tau^2_{g\,i}(0,\,z)}} \ll 1$ and the approximation holds

$$\xi_i(\tau;\,\boldsymbol{\eta}) = 1 - 2\,\eta_{2+i}\,. \quad (19)$$

In the current regime of operation under condition $\Pi_i\,\tau_p \gg 1$, where $\Pi_i$ is the bandwidth of postdetector filter of the first channel, $\mathbf{n}(\tau) = \{n_i(\tau)\}$, and $n_i(\tau)$ is the white Gaussian noise with spectral power density

$$N_{0i} = 2\,e\left[\overline{s}_i(\tau) + \chi_{qe}\,P_{bg\,i} + \overline{s}_{di}\right], \quad (20)$$

where $P_{bg\,i}$ and $s_{di}$ are the background power at the input and the dark current of the $i$th photodetector channel, respectively; $\chi_{qe}$ is the quantum efficiency of the photodetector.

## 4. FILTRATION EQUATION

Let us process the data sample given by Eq. (16): $\mathbf{y}(\tau) = \{y_i/\tau\}^T$, to provide the optimal estimate of the realization $\boldsymbol{\eta}(\tau)$ in the sense of maximum *a posteriori* probability density (APD). In accordance with the aforementioned considerations of the component $\mathbf{V}$, it is necessary to estimate the unknown profiles of $\mathbf{u}_i$ and $\rho$ simultaneously. Following Ref. 7, let us resolve the *a priori* uncertainly about $\mathbf{u}_i$ and $\rho$ with the help of the variant of maximum likelihood (ML).

Since the sounding wavelengths are close, we may ignore the spectral dependence of the aerosol and molecular scattering coefficients. Thus, the components of $\mathbf{u}_i$ at all wavelengths are equal, so it would suffice to obtain the ML estimate of the scattering coefficient $\sigma_0(z)$ at $\lambda_0$. The other characteristics of aerosol and molecular scattering we may obtain using Eq. (18) and the data of measurements of the meteorological parameters.

According to Ref. 8, the ML estimate $\hat{\mathbf{u}}_0$ of the components of unknown vector $\mathbf{u}_0$ with additive Gaussian noise is determined from the relation

$$y_0(\tau) = s_0(\tau;\,\boldsymbol{\eta}^*,\,\hat{\mathbf{u}}_0,\,\hat{\mathrm{r}}_g)\,, \quad (21)$$

where the ML estimate $\hat{\mathrm{r}}_g(z)$ of the density $\rho_g(z)$ is obtained using the data sample at $\lambda_2$

$$y_2(\tau) = s_2(\tau;\,\boldsymbol{\eta}^*,\,\hat{\mathbf{u}}_0,\,\hat{\mathrm{r}}_g)\,. \quad (22)$$

Since the pairs $\lambda_0$, $\lambda_1$ and $\lambda_1$, $\lambda_2$ are close, the estimates $\hat{\mathbf{u}}_0$ and $\hat{\mathrm{r}}_g$ obtained at $\lambda_0$ and $\lambda_2$ can be used for data processing at $\lambda_1$.

As a result, the statement of the problem of synthesis of the statistically optimal algorithm for signal processing at $\lambda_1$ to obtain the estimate $\boldsymbol{\eta}^*$ state vector $\boldsymbol{\eta}$ is described by the expression

$$y_1(\tau) = \overline{s}_1(\tau;\,\hat{\mathbf{u}}_0)\,\exp\left\{-\,2\int\limits_0^z \mathrm{d}z'\,\hat{\rho}(z')\,\times\right.$$
$$\left.\times\left[\overline{K}_g(z';\,\lambda_1)\,B_1(z') - \overline{K}_g(z';\,\lambda_0)\,B_0(z')\right]\eta_1\right\} + \eta_1(\tau), \quad (23)$$

where $y_1(\sigma)$ are the data sample $\hat{\mathbf{u}}_0$ and $\hat{\mathrm{r}}_g$ are the running ML estimates of $\mathbf{u}_0$ and $\rho_g$ that are obtained simultaneously with $\boldsymbol{\eta}^*$ when Eq. (23) is completed by Eqs. (21) and (22). The *a priori* information on the statistical structure of the state vector $\boldsymbol{\eta}$ components is embedded in the system of differential equations (15). It is necessary to find the algorithm for optimal processing of the received signals in the since of the maximum APD at $\lambda_1$ corresponding to the transition with the energy of the lower level $E''_1 \gg E''_0$ and $E''_1 \gg E''_2$.

Let us write down the system of equations of the quasioptimal Calmam–Bucy filtration[5] using the Gaussian approximation of the APD of the state vector $\boldsymbol{\eta}$ with the conditional mean $\boldsymbol{\eta}^*$ and correlation matrix $R = \langle(\boldsymbol{\eta} - \boldsymbol{\eta}^*)\times$ $\times\,(\boldsymbol{\eta} - \boldsymbol{\eta}^*)^T\rangle$

$$\dot{\boldsymbol{\eta}}^* = A(z)\,\boldsymbol{\eta}^* + \frac{2}{N_{01}}\,R\,\mathbf{C}\left[y_1(\tau) - \hat{s}_1(\tau)\,\hat{\mathbf{u}}_0\,\xi_1(\tau;\,\boldsymbol{\eta}^*)\right], \quad (24)$$

$$\dot{R} = A\,R + R\,A^{\mathrm{T}} + b - \frac{\hat{s}_1^2(\tau)}{N_{01}}\,R\,F_2\,R\,, \quad (25)$$

where

$$A = \begin{pmatrix} -\alpha & 0 \\ c\,\hat{\rho}_g(\bar{\sigma}_{C1} - \bar{\sigma}_{C0})/2 & 0 \end{pmatrix},$$

$F_2$ is the matrix of order (2×2) with nonzero component $F_{22} = -4$.

The optimal processing is the simultaneous solution of the system of Eqs. (24) and (25), as the sample data on $y_1(\tau)$ become available, with Eqs. (21) and (22) for the *a priori* profiles $\bar{T}(z)$, $\sigma_T(z)$, and $L$ with preset initial conditions, by the appropriate finite–difference technique on a computer.

The recurrent finite–difference solution of this system of equations gives the optimal estimate $\eta_1^*$ and thereby the estimate of the temperature profile T($z$)

$$T^*(z) = \bar{T}(z) + \sigma_T(z)\,\mathrm{h}_1^*(c\,\tau/2),$$

as well as the estimate $R_{11}(\tau)$ of the variance of realization $\eta_1^*(\tau)$ and thereby of the variance $D[T^*(z)] = \sigma_T^2(z)R_{11}(\tau)$ of the profile $T^*(z)$.

## 5. FILTRATION EFFICIENCY

Let us characterize the efficiency of filtration by the dependence of the variance of the temperature estimate on the sounding altitude in different spectral ranges in the absorption lines of $O_2$ and $H_2O$.

According to Eq. (25), the elements of the correlation matrix satisfy the following system of differential equations

$$\begin{cases} \dot{R}_{11} = -2\,\alpha\,R_{11} + 2\,\alpha - 4\,\dfrac{\hat{\bar{s}}_1^2(\tau)}{N_{01}} R_{12}^2(\tau), \\ \dot{R}_{12} = -2\alpha R_{12} + c\,\hat{\rho}_g(\bar{\sigma}_{C1} - \bar{\sigma}_{C0})/2R_{11} - 4\dfrac{\hat{\bar{s}}_1^2(\tau)}{N_{01}} R_{12}R_{22}, \\ \dot{R}_{22} = c\hat{\rho}_g(\bar{\sigma}_{C1} - \bar{\sigma}_{C0})/2R_{12} - 4\dfrac{\hat{\bar{s}}_1^2(\tau)}{N_{01}} R_{22}^2(\tau), \end{cases} \quad (26)$$

where the independent variable $\tau = 2z/c$ is related to the altitude $z$, so the profiles of the relative variance $R_{11}(2z/c)$ and $R_{22}(2z/c)$ characterize the filtration efficiency and its dynamics as function of altitude.

For comprehensive study of the dependencies $R_{11}(z)$, $R_{12}(z)$, and $R_{22}(z)$ it is necessary to model the profiles $s_1(z)$, $\rho_g(z)$, $\sigma_C(z;\lambda_i)$, etc. considering all the factors that affect sounding of temperature by the Mason technique and to integrate Eq. (26).

We can analyze the dependence of $R_{11}(z)$ on the altitude if we replace $R_{12}(z)$ in the first equation of system (26) by its approximation

$$R_{12}(z) \simeq \hat{\rho}_g(\bar{\sigma}_{C1} - \bar{\sigma}_{C0})\,z_L\,R_{11}(z),$$

where

$$z_L = \begin{cases} z - z_0, & \text{for } z - z_0 \ll L, \\ L, & \text{for } z - z_0 \gg L. \end{cases} \quad (27)$$

In this case the equation for $R_{11}(z)$ can be integrated independently of the system of equations (26). In particular, we have

$$\frac{\mathrm{d}\dot{R}_{11}(z)}{\mathrm{d}z} = -\frac{2}{L}R_{11}(z) + \frac{2}{L} - \frac{2}{L}Q(z;\lambda_1)\,R_{11}^2(z), \quad (28)$$

where

$$Q(z;\lambda_1) = \frac{\hat{\bar{s}}_1^2\,\hat{\rho}_g^2\,(\bar{\sigma}_{C1} - \bar{\sigma}_{C0})\,z_L^2}{N_{01}\,\alpha}. \quad (29)$$

In analogy with Refs. 5 and 7, in which one–and two–channel filtration of signals of lidars harnessing scattering and DA was considered, let us call the parameter $Q(z;\lambda_1)$ the generalized signal–to–noise ratio. It is seen that $Q(z;\lambda_1)$ of the form given by Eq. (29) differs from the ratio $Q(z;\lambda_1)$ introduced in Refs. 5 and 7 by a larger number of parameters specifying $Q(z;\lambda_1)$. Taking into account that

$$\bar{\sigma}_{Ci} = \bar{K}_g(z;\lambda_i)\,B_i(E_i'', G_i(z))\,\mu_T(z),$$

where $\mu_T = \sigma_T/\bar{T}$, $\hat{\bar{s}}_1(l_0)$, $\hat{\rho}_g(\lambda_2;\bar{\sigma}_{C2} - \bar{\sigma}_{C0})$ we obtain that $Q$ is determined by the following parameters: the signal–to–noise ratio $\bar{s}_1^2/N_{01}\,\alpha$ due to elastic scattering, the variation coefficient $\mu_T$, and the nonlinear dependence of the quantities $\rho_g(z)$,

$\bar{K}_g(z,\lambda_1)\,B_1(E_1'', G_1) - \bar{K}_g(z,\lambda_0)\,B_0(E_0'', G_0)$,

$\bar{K}_g(z;\lambda_2)\,B_2(E_2'', G_2) - \bar{K}_g(z,\lambda_0)\,B_0(E_0'', G_0)$,

and $z_L$ determining the value of variance $D\left[\overline{\Delta\tau_{10}^2}\right] = (\bar{\sigma}_{C1} - \bar{\sigma}_{C0})^2\,\rho_g^2\,z_L^2$ of the differential optical depth $\Delta\tau_{10}(0, z_L)$.

Filtration is efficient only at altitudes at which $Q(z;\lambda_1) \gg 1$ (see Ref. 8). Let us determine the corresponding value of $R_{11}(z)$ taking $dR_{11}(z)/dz = 0$ in Eq. (28). Then the solution $\bar{R}_{11}(z)$ of the quadratic equation has the form

$$\bar{R}_{11}(z) = \frac{1}{2\,Q(z;\lambda_1)}\left\{\sqrt{1 + 4\,Q(z;\lambda_1)} - 1\right\}. \quad (30)$$

Using the derived quantities $Q(z;\lambda_1)$, we can determine from the Eq. (30) the profiles $\bar{R}_{11}(z)$ and the errors $\bar{s}(T^*) = \sigma_T\sqrt{\bar{R}_{11}(z)}$ in retrieving $T^*(z)$. It is seen that optimization of the data processing with the use of the Calman–Bucy filtration algorithm provides the efficient retrieval of the fluctuating temperature profiles, since it allows us to obtain the error in retrieving $\sigma[T^*(z)] < \sigma[T(z)]$.

## 6. CONCLUSION

The Calman–Bucy filtration algorithm has been synthesized that makes it possible to optimize the processing of the DA lidar signals in trifrequency sounding of the temperature by the Mason technique.

Simultaneously it is possible to determine the ML estimates of the aerosol and molecular scattering coefficients and the density of the sounded gas. It is shown that the efficiency of filtration of the spatial realizations of temperature depends on the generalized signal–to–noise ratio (introduced in the paper) that accumulates all the factors that show promise for this technique.


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