ON THE POINT SPREAD FUNCTION OF SYSTEMS FOR OBSERVATIONS THROUGH ROUGH SEA SURFACE

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Physical models are proposed for the point spread function (PSF) of systems of vision through rough sea surface, which make it possible a simple interpretation of the effect of PSF shape distortion in the case of directional (solar) illumination.

The distribution of brightness over image of an underwater object, obtained through a rough sea surface after sufficient accumulation, is described using two types of transmission functions from the theory of optical systems, namely, the optical transmission function (OTF) and the point spread function (PSF). The effect of roughness on the visibility of underwater object was first analyzed in Ref. 1.

Further studies[2, 3] showed that, besides roughness, the conditions of illumination play an important role in forming OTF and PSF. For example, if the illumination is directional (e. g., solar), transmission functions of the sea surface are strongly distorted due to correlation of light entering and exiting the surface. These questions are considered in more detail in Refs. 4 and 5.

The analysis of OTF and PSF from Refs. 2–5 is sufficiently comprehensive and detailed. However, because of the formal presentation of basic expressions describing the process of the transfer of image of a point object through a rough sea surface, physical interpretation of the results of such an analysis becomes rather difficult. An attempt is undertaken in this paper to explain in context of approximate physical models the basic mechanisms of the PSF of the sea surface formation.

1. LINEAR MODEL

Within the small angle approximation[4] we write a general expression for a random value of the radiant flux in an element of the image

$$P(\mathbf{r}_r;\,\Omega_r) = \frac{B_s}{\pi\, m^2} \int\limits_{-\infty}^{\infty}\!\!\int R_0(\mathbf{r}_3)\, E_s(\mathbf{r}_3)\, E_r(\mathbf{r}_3)\, d\mathbf{r}_3, \qquad (1)$$

$$E_s(\cdot) = m^2 \int\limits_{-\infty}^{\infty} \dots \int D_s(\Omega_1 - \Omega_s)\, \delta\,[\Omega_1 - m\Omega_2 + A\, \mathbf{q}(\mathbf{r}_2)] \times$$
$$\times\, e_m(\mathbf{r}_3 - \mathbf{r}_2 - h\,\Omega_2)\, d\mathbf{r}_2\, d\Omega_{1,2}\,, \qquad (2)$$

$$E_r(\cdot) = m^2 \int\limits_{-\infty}^{\infty} \dots \int D_r(\mathbf{r}'_1 - \mathbf{r}_r;\, \Omega'_1 - \Omega_r)\, \delta\,(\mathbf{r}'_2 - \mathbf{r}'_1 - H\,\Omega'_1) \times$$
$$\times\, \delta\,[\Omega'_1 - m\,\Omega'_2 + A\,\mathbf{q}(\mathbf{r}'_2)]\, e_m\,(\mathbf{r}_3 - \mathbf{r}'_2 - h\,\Omega'_2)\, d\mathbf{r}'_{1,2}\, d\Omega'_{1,2}\,, \qquad (3)$$

where R_0 is the distribution of the reflection coefficient across the object at depth h; E_s and E_r are the functions describing the irradiance across the object produced by sources with directional patterns D_s and D_r ($D_s(0) = D_r(0) = 1$); B_s is the solar brightness; e_m is the scattering function of water describing the distribution of radiance from a point source in turbid medium at a distance h from that source; $\mathbf{q}$ is the vector-gradient of the rough sea surface; $\delta\,(\cdot)$ is the delta-function; $A = m - 1$, m is the refractive index of water; $\mathbf{r}_i$ are the coordinates of points in planes z_i (see Fig. 1); Ω_i are the projections of unit vectors $\mathbf{\Omega}_i^0$ onto the plane z_i; $\mathbf{r}_r$ is the coordinate of the central point of aperture of the detector of an observational system; Ω_s and Ω_r are the projections of unit vectors co-directed with axes of the directional patterns of a source and a receiver onto the plane z = const.

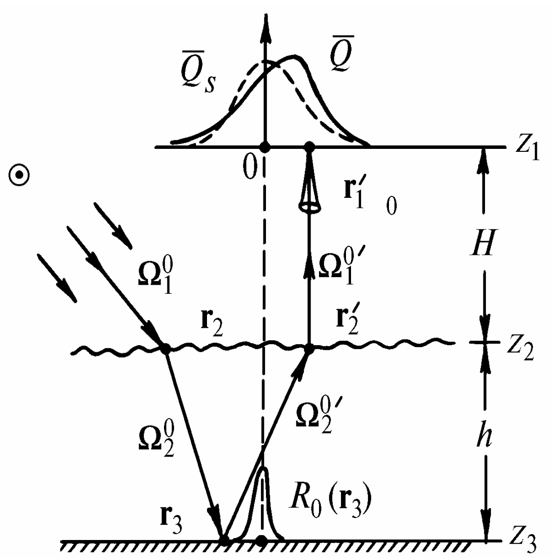

Fig. 1. Object illumination and observation geometry.

Now let us formulate certain conditions which will make it possible to simplify expressions (1)–(3). Assume that

a) roughness is one-dimensional, $\mathbf{q}(\mathbf{r}) = q(x)$;

b) the angular size of the source is small, $D_s(\Omega) = \Delta_s \delta\,(\Omega - \Omega_s)$;

c) image is formed via spatial scanning of an object surface by a brightness meter aimed at nadir

$$D_r(\mathbf{r};\,\Omega) = \Sigma_r\, \Delta_r\, \delta\,(\mathbf{r} - \mathbf{r}_r)\, \delta\,(\Omega - \Omega_r)\,,$$

where $\Delta_{s(r)}$ is the solid angle of emission (detection); Σ_r is the area of the receiver aperture.

Under these conditions expression (1) takes the form

$$P(\mathbf{r}_r) = P_0 \int\limits_{-\infty}^{\infty}\!\!\int R_0(\mathbf{r}_3)\, Q(\mathbf{r}_r;\,\mathbf{r}_3)\, d\mathbf{r}_3\,, \qquad (4)$$

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where $P_0 = B_s \Delta_s \Sigma_r \Delta_r / \pi m^2$ is the radiant flux received from an object with uniform reflection coefficient through a smooth air-sea interface. Function Q describes an "instant" image of a point object and so may be (conditionally) called an "instant" PSF of the sea surface

$$Q(\cdot) = e_m[\mathbf{r}_3 - \mathbf{r}_r - a\mathbf{q}(\mathbf{r}_r)] \int\limits_{-\infty}^{\infty}\int e_m[\mathbf{r}_3 - h_0\mathbf{\Omega}_s - \mathbf{r}_2 - a\mathbf{q}(\mathbf{r}_2)]\mathrm{d}\mathbf{r}_2, \quad (5)$$

where $\mathbf{q}(\mathbf{r}_2) \equiv q(x_2)$, $a = A\, h_0$, $h_0 = h/m$.

To formulate the principal condition characterizing our model, i.e. the linear approximation, assume that the value of linear displacement $|a\mathbf{q}|$ of the light beam refracted at the rough air-sea interface in the plane z_3 is small compared to the characteristic width of the scattering function $e_m(\mathbf{r})$ of the water layer. In this case we may write

$$e_m(\mathbf{r} + a\,\mathbf{q}) \approx e_m(\mathbf{r}) + a\,\mathbf{q}\,\nabla\, e_m(\mathbf{r}). \quad (6)$$

In accordance with (6) expression (5) for a one-dimensional roughness will be reduced to

$$Q(\mathbf{r}_r; \mathbf{r}_3) = [e_m(\mathbf{r}_3 - \mathbf{r}_r) - a\, e'_x(\mathbf{r}_3 - \mathbf{r}_r)\, q(x_r)] \times$$
$$\times [1 - a \int\limits_{-\infty}^{\infty}\int e'_x(\mathbf{r}_3 - \mathbf{r}_2 - \Delta\rho)\, q(x_2)\, \mathrm{d}\mathbf{r}_2], \quad (7)$$

where $e'_x(\mathbf{r}) = \dfrac{\partial e_m}{\partial x}(\mathbf{r})$, $\Delta\rho = h_0\,\mathbf{\Omega}_s$.

By averaging expression (7) over realizations of slopes of a randomly rough sea surface, we obtain an expression for PSF of an averaged image:

$$\bar{Q}(\mathbf{r}; \Delta\rho) = e_m(\mathbf{r}) + a^2 e'_x(\mathbf{r}) \int\limits_{-\infty}^{\infty}\int e'_x(\mathbf{r} + \rho + \Delta\rho)\, M_q(\rho_x)\, \mathrm{d}\rho, \quad (8)$$

where
$\mathbf{r} = \mathbf{r}_r - \mathbf{r}_3$, $M_q(\rho) = \langle q(x)\, q(x + \rho) \rangle$;
M_q is the correlation function of slopes of the sea surface.

Expression (8) for PSF may be presented as a sum of the "un-correlated" $\bar{Q}_s$ and the "correlated" $\mathrm{D}\bar{Q}$ components

$$\bar{Q}(\mathbf{r}; \Delta\rho) = \bar{Q}_s(\mathbf{r}) + \Delta\bar{Q}(\mathbf{r}; \Delta\rho);$$
$$\bar{Q}_s(\mathbf{r}) = e_m(\mathbf{r}). \quad (9)$$

The component $\Delta\bar{Q}$ is presented as a product of two functions, one related to the detection of radiation coming from the object and the other one to irradiation of the object

$$\Delta\bar{Q}(\mathbf{r}; \Delta\rho) = a^2\, \psi_r(\mathbf{r})\, \psi_s(\mathbf{r} + \Delta\rho), \quad (10)$$

where

$$\psi_r(\cdot) = e'_x(\mathbf{r}); \quad \psi_s(\cdot) = \int\limits_{-\infty}^{\infty}\int e'_x(\mathbf{r} + \rho + \Delta\rho)\, M_q(\rho_x)\, \mathrm{d}\rho.$$

Note that it is the correlated component that controls the distortions of shape of the PSF in the case of a directionally illuminated object; if illumination is performed with a diffuse radiation, it is identically equal to zero.

If we replace the functions entering into expression (10) by their expressions we have

$$e_m(\mathbf{r}) = \frac{1}{g}\exp\left(-\frac{\pi r^2}{g}\right), \quad M_q(\rho) = \sigma_q^2 \exp\left(-\frac{\pi r^2}{S_q}\right),$$

where g is the characteristic area of the scattering function of the medium at the depth h; σ_q^2 is the variance of wave slopes; $S_q = \pi r_q^2$; ρ_q is the radius of correlation of slopes at the interface; then, after simple transformations we arrive at an analytic formula for the "correlated" component of PSF

$$\Delta\bar{Q}(\xi; \delta) = \frac{2\Sigma_q}{g^2}\frac{\sqrt{\alpha_g}}{(1 + \alpha_g)^{3/2}}\,\xi(\xi + \delta)\exp\left(-\xi^2 - \frac{(\xi + \delta)^2}{1 + \alpha_g}\right), \quad (11)$$

where
$\xi = \sqrt{\pi/g}\; x$; $\quad \delta = \sqrt{\pi/g}\; \Delta\rho$; $\quad \alpha_g = S_q/g$; $\quad \Sigma_q = 2\pi\, a^2\, s_q^2$;
and, Σ_q is the characteristic area of a speck on water surface as observed from the depth h.

The parameter $\Delta\rho$ plays an important role in the above derived formulas, since it characterizes the angular "discrepancy" between the directions of irradiation and observation of an object.

Figure 2 illustrates the dependence of the "addition" $\Delta\bar{Q}$ to PSF $\bar{Q}_s$ on the coordinate $x = x_d - x_3$. It can be seen from Fig. 2 that the function $\Delta\bar{Q}(x)$ is, generally, non-monotonic and sign-alternating. At $\Delta\rho = 0$ (i. e. for coaxial irradiation) the function $\Delta\bar{Q}$ is symmetric with respect to $x = 0$; for $\Delta\rho \to \infty$ it vanishes, and in this case $\bar{Q}(r) = \bar{Q}_s(r)$.

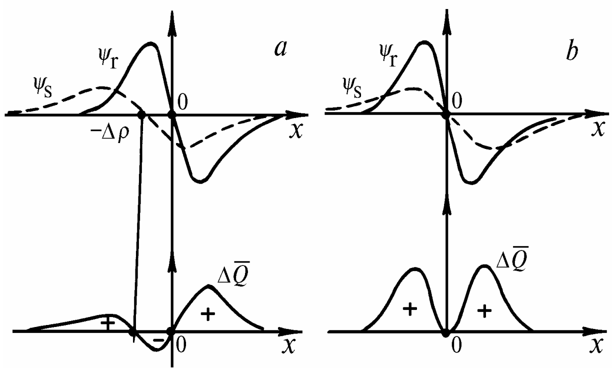

Fig. 2. "Correlated" component of a PSF at $\Delta\rho > 0$ (a) and $\Delta\rho = 0$ (b).

Peculiarities of the PSF, revealed using this model, have been described in Ref. 5, where a strict numerical analysis of the process of formation of this function is done. This fact prompts one to assume that the exact and the approximate models are qualitatively similar. In what follows we propose a physical explanation of the features of formation of PSF for the case of directional irradiation, so based on the approximate linear model.

Assume, first of all, that the interface between the two media is smooth. Apparently, the object is then irradiated uniformy. The image of the object (its PSF) is described by the function of scattering of the water medium, $e_m(\mathbf{r})$. Now assume that the interface is randomly rough. In this case oscillations of the illuminating and the sighting beams below the interface occur nonsynchronously and the averaged image of the object is described by a convolution of scattering functions of the medium and the interface.

Since we assume that the interface takes part in scattering to much lower extent than the water medium (the linear approximation), the PSF will still be described by the function $e_m(\mathbf{r})$.

Now assume that within a narrow vicinity of the point of entry of the beam into water $\mathbf{r}_r$, the slopes of the rough interface strongly correlate. It this case oscillations of the irradiating and sighting beams, passing through the correlated area, will occur synchronously (in a correlated mode). The result may be seen in Fig. 3, depending on the position of the point of observation $\mathbf{r}_r$, with respect to the coordinate of the object $\mathbf{r}_3$. For example, a clockwise variation of tilt of the interface at the point $\mathbf{r}_r$ results in a change of direction of the sighting beam *1* and of irradiating beam *2* (Fig. 3 *a*). Scanning of the object by the directional patterns of scattering which correspond to beams *1* and *2* results in a modulation of E_s and E_r. Since the detected optical signal is proportional to the product of these functions, there appears a variable component in it, its value and sign depending on the phase difference between the functions E_s and E_r, which correspond to beams *1* and *2*. (The constant component of the detected signal is still proportional to e_m.)

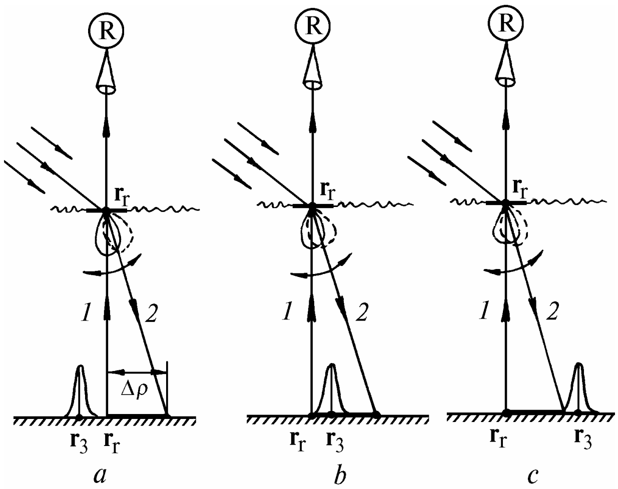

Fig. 3. An explanation of the dependences presented in Fig. 2.

In the case of $|\mathbf{r}_r - \mathbf{r}_3| > 0$ (see Fig. 3*a*), the variable components of the optical signal, corresponding to beams *1* and *2*, change synchronously, so that their averaged product is positive, and, respectively, the addition $\Delta\overline{Q}$ to PSF $\overline{Q}_s$ is positive too. In the case of $0 \le |\mathbf{r}_3 - \mathbf{r}_r| \le \Delta\rho$ (Fig. 3*b*), one can easily see that the variable components of the signal, corresponding to beams *1* and *2*, vary in anti-phase. The product of those signals is then negative, and, respectively, the addition to PSF is negative too. The case of $|\mathbf{r}_3 - \mathbf{r}_r| > \Delta\rho$ (Fig. 3*c*) is similar to that in Fig. 3*a*, so that $\Delta\overline{Q}_s$ is positive again. Such a reasoning illustrates the physics of the process of formation of PSF, as expressed by formulae (9) and (10). If one accounts for the finite size of the area in which the slopes mutually correlate, nothing new adds this reasoning.

Thus, starting with a linear model we managed to demonstrate that in the case of directional irradiation the specific character of distortions of the sea surface PSF may be explained by the combined effect of two factors, namely, mutual correlation between the irradiating and the sighting beams in the vicinity of the entry point of the sighting beam into water, and the dependence of phase difference between the variable components of optical signals from the irradiating and the sighting beams on the difference between the coordinates of the point of observation and the object. Naturally, one should be very carefull with the conclusions that can be drawn based on linear approximation. However, in this case its use is well justified. To prove this, we turn to the second model, which is also approximate, but more rigourous than the linear one.

2. THE MODEL OF THE LOCALLY FLAT INTERFACE

Consider the following observational scheme. An object is at a depth h, and is irradiated with a wide parallel beam through a rough sea surface. As earlier, we assume that the image of the object is formed via spatial scanning of the object by a narrow-beam directional photodetector with a directional pattern $D_r(\mathbf{\Omega})$ ($D_r(0) = 1$). We neglet the effect of deep sea light scattering on the process of formation of the image; in this case $e_m(\mathbf{r}) = \delta(\mathbf{r})$. Note that such a condition is not decisive for the considered model.

The main idea of this model centers around the assumption that water surface is flat within a certain vicinity S_q of the point of the sighting beam entry into it (its slope being random), while outside that area is rough and non-correlated. The boundaries of this area may be described by the function

$$M_q(\mathbf{r}) = \begin{cases} 1, & \mathbf{r} \in S_q, \\ 0, & \mathbf{r} \notin S_q. \end{cases}$$

The formula for image transfer in this case will take the form

$$P(\mathbf{r}_r) = \frac{B_s\,\Delta_s}{\pi\, m^2} \int\limits_{-\infty}^{\infty}\!\!\int R_0(\mathbf{r}_3)\, E_s(\mathbf{r}_3)\, E_r(\mathbf{r}_3)\, \mathrm{d}\mathbf{r}_3\,, \tag{12}$$

where

$$E_s(\cdot) = E_1(\cdot) + E_2(\cdot);$$

$$E_1(\cdot) = \int\limits_{-\infty}^{\infty}\!\!\int M_q(\mathbf{r}_2 - \mathbf{r}_r)\, \delta(\mathbf{r}_3 - \mathbf{r}_2 - \Delta\boldsymbol{\rho} - a\mathbf{q}_0)\, \mathrm{d}\mathbf{r}_2;$$

$$E_2(\cdot) = \int\limits_{-\infty}^{\infty}\!\!\int [1 - M_q(\mathbf{r}_2 - \mathbf{r}_r)]\, \delta[\mathbf{r}_3 - \mathbf{r}_2 - \Delta\boldsymbol{\rho} - a\mathbf{q}(\mathbf{r}_2)]\, \mathrm{d}\mathbf{r}_2;$$

$$E_r(\cdot) = \Sigma_r \int\limits_{-\infty}^{\infty}\!\!\int D_r(\mathbf{\Omega})\, \delta\,(\mathbf{r}_3 - \mathbf{r}_r - L\,\mathbf{\Omega} - a\mathbf{q}_0)\, \mathrm{d}\mathbf{\Omega};$$

$$\Delta\boldsymbol{\rho} = h_0\, \mathbf{\Omega}_s,\ L = H + h_0,\ a = h_0\,(m - 1),\ h_0 = h/m.$$

Here $E_{1,2}$ describe the distributions of irradiation produced within the object plane by solar beams passing through the correlated area S_q and outside it, respectively.

By averaging expression (12) over the set of slopes of the sea surface $\mathbf{q}_0$ and $\mathbf{q}(\mathbf{r}_2)$ (assuming that such realizations mutually correlate) and making some simple transformations we obtain the following expression for the averaged image of a subwater object:

$$\bar{P}(\mathbf{r}_{\mathrm{r}}) = P_0 \int\int_{-\infty}^{\infty} R_0(\mathbf{r}_3)\, \bar{Q}(\mathbf{r}_{\mathrm{r}} - \mathbf{r}_3;\, \Delta\rho)\, \mathrm{d}\mathbf{r}_3\,, \qquad (13)$$

where

$$\bar{Q}(\mathbf{r};\, \Delta\rho) = \bar{Q}_{\mathrm{s}}(\mathbf{r}) + \Delta\bar{Q}(\mathbf{r};\, \Delta\rho);$$

$$\bar{Q}_{\mathrm{s}}(\mathbf{r}) = (2\pi)^{-2} \int\int_{-\infty}^{\infty} F_{\mathrm{r}}(L\,\mathbf{k})\, \theta_1(a\,\mathbf{k})\, \mathrm{e}^{i\mathbf{kr}}\, \mathrm{d}\mathbf{k};$$

$$\Delta\bar{Q}(\mathbf{r};\, \Delta\rho) = (2\pi)^{-4} \int\int_{-\infty}^{\infty} F_q(\mathbf{k}_1)\, F_{\mathrm{r}}(L\,\mathbf{k}_2)\, [\theta_1(a\,\mathbf{k}_1 + a\,\mathbf{k}_2) - $$

$$- \theta_1(a\,\mathbf{k}_1)\, \theta_1(a\,\mathbf{k}_2)]\, \exp\,(-\, i\,\mathbf{k}_1\,\Delta\rho - i(\mathbf{k}_1 + \mathbf{k}_2)\,\mathbf{r})\, \mathrm{d}\mathbf{k}_{1,2}\,;$$

θ_1 is the single-point characteristic function of the distribution of surface slopes; F_q and F_{r} are the Fourier-images of the functions M_q and D_{r}.

Let us show that the effect of the PSF shape change under a directional illumination is not less distinct in this model. We specify the functions entering into expression (13). First, we approximate the functions F_q and F_{r} by expressions of the form

$$F_q(\mathbf{k}) = S_q \exp\left(-\frac{S_q}{4\,\pi}\, k^2\right),$$

$$F_{\mathrm{r}}(L\,\mathbf{k}) = \exp\left(-\frac{S_{\mathrm{r}}}{4\,\pi}\, k^2\right), \qquad (14)$$

where $S_{\mathrm{r}} = \Delta_{\mathrm{r}}\, L^2$.

In the case of one-dimensional roughness the characteristic function of wave slopes has the form

$$\theta_1(a\,\mathbf{k}) = \exp\left(-\frac{\Sigma_q}{4\,\pi}\, k_x^2\right). \qquad (15)$$

Substituting the relations (14) and (15) into the expression for $\Delta\bar{Q}$ (13) at $y = 0$ and integrating it over the variables $\mathbf{k}_{1,2}$ we obtain, after certain transformations, an expression for the "correlated" component of the PSF

$$\Delta\bar{Q}(\xi;\, \delta) = \frac{1}{\Sigma_q} \sqrt{\frac{\alpha_q}{\alpha_{\mathrm{r}}}}\, \mathrm{e}^{-\xi^2/(1+\alpha_{\mathrm{r}})}\, (\Delta Q_1 - \Delta Q_2), \qquad (16)$$

where

$$\Delta Q_1 = \frac{1}{\sqrt{\alpha_0 - 1}} \exp\left\{-\frac{[\alpha_{\mathrm{r}}\xi + (1 + \alpha_{\mathrm{r}})\, \mathrm{d}]^2}{(1 + \alpha_{\mathrm{r}})\, (\alpha_0 - 1)}\right\},$$

$$\Delta Q_2 = \frac{1}{\sqrt{\alpha_0}} \exp\left[-\frac{(\xi + \delta)^2}{1 + \alpha_q}\right],\ \xi = \sqrt{\pi/\Sigma_q}\; x,\ \delta = \sqrt{\pi/\Sigma_q}\; \Delta\rho,$$

$$\alpha_0 = (1 + \alpha_{\mathrm{r}})\, (1 + \alpha_q),\ \alpha_{\mathrm{r}} = S_{\mathrm{r}}/\Sigma_q,\ \alpha_q = S_q/\Sigma_q.$$

Computations according to (16) will be performed below. Meanwhile we only mention that the results obtained in this section coincide with the results published in Ref. 4. Note that an expression similar to (16) was derived there via formal factorization of a two-point characteristic slope function of sea surface. In our study we arrived at that result by analyzing a simple physical model of the air–water interface. It follows, in particular, that the description used in Ref. 4 is in fact an approximation of a locally flat air–water interface. The model proposed in this section may serve as a basis for physical interpretation of the results from Ref. 4.

The next question we address is how well the above models describe actual process of PSF formation, quantitatively and qualitatively.

3. COMPARISON OF MODELS

Reference 5 presents some results of an exact solution of the problem on the effect of roughness and conditions of illumination on the formation of PSF of the sea surface. Below we employ these results to estimate the quality of the above approximate models. Naturally, the main attention will be paid to the correlated component of the PSF. Let us write the expression for the correlated component which has been derived in Ref. 5 based on an exact model

$$\Delta\bar{Q}(\xi;\, \delta) = \frac{1}{\Sigma_q \sqrt{\pi\, \alpha_{\mathrm{r}}\, (1 + \alpha_{\mathrm{r}})}}\, \mathrm{e}^{-\xi^2/(1+\alpha_{\mathrm{r}})} \times$$

$$\times \int_{-\infty}^{\infty} \left(\frac{\mathrm{e}^{-\rho_1^2/A_1}}{\sqrt{A_1}} - \frac{\mathrm{e}^{-\rho_0^2/A_0}}{\sqrt{A_0}}\right) \mathrm{d}\rho, \qquad (17)$$

where

$$\rho_1 = \rho - \nu\xi - \delta,\ \rho_0 = \rho - \xi - \delta,\ \nu = \frac{1 + \alpha_{\mathrm{r}} - R_q(\rho)}{1 + \alpha_{\mathrm{r}}},$$

$$A_1 = \frac{1 + \alpha_{\mathrm{r}} - R_q^2(\rho)}{1 + \alpha_{\mathrm{r}}},\ R_q(\rho) = \exp\left(-\frac{\rho^2}{\alpha_q}\right),$$

$$A_0 = A_1\ (\rho \to \infty).$$

Note that the parameter α_q, present in all the above expressions, is a value inverse to the parameter of focusing $\gamma = \Sigma_q/S_q$ (Ref. 6), which characterizes the lens-like effect of a rough sea surface on the radiation penetrating.

Figure 4 brings together computational results for the functions $\Delta\bar{Q}(\xi,\, \delta)$, as yielded by expressions (11), (16), and (17). They are presented as isolines in the ξ, δ plane for the following values of corresponding parameters: $\alpha_q = 1$ (maximum focusing depth) and $\alpha_{\mathrm{r}} = 0.1$. It follows from the obtained dependences that the correlated component of the PSF $\Delta\bar{Q}$ is a nonmonotonic and, generally speaking, sign-alternating function of both the coordinate of observation point ξ and of the "discrepancy" angle δ.

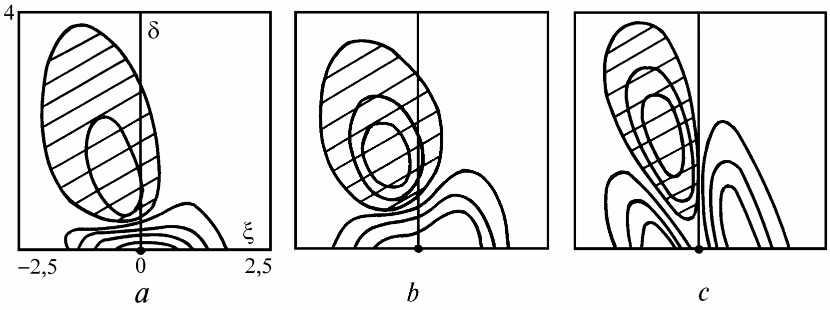

Fig. 4. "Correlated" component of a PSF vs. spatial coordinate ξ and angle of irradiation δ in cases of exact model (a); model of the locally flat interface (b); and, linear approximation model (c). Shadowed areas correspond to negative $\Delta\overline{Q}$.

For large δ the value of $\Delta\overline{Q}$ is close to zero. Note that the function $\Delta\overline{Q}(\xi, \delta)$ is centrally symmetric with respect to point $\xi = \delta = 0$.

Comparing the trends of $\Delta\overline{Q}$ for various models it is easy to see that the model of locally flat interface is in a qualitatively good agreement with the exact model. The model based on the approximation which is linear in wave slopes offers much coarser description of the PSF (it is wrong, in particular, that $\Delta\overline{Q} = 0$ for $\xi = 0$ and $\xi = -\delta$), although this model is rather adequate in presenting the overall picture. As to the quantitative agreement between the considered models, our computations show that the values of $\Delta\overline{Q}$ from the exact model exceed the approximate ones, particulary in the range of small angles δ. What might be an explanation of this fact? Clearly, the linear model cannot provide a basis for quantitative estimates, since it is, essentially, a model of weak fluctuations of the field of radiation. The merit of this model is that it helps one to understand the physical nature of PSF distortions in case of directional irradiation, and nothing more. The model of the locally flat interface is closer to the exact model, but produces wrong results as well, when one tries to retrieve the absolute values of a PSF. To explain the latter fact we recall that such a model does not account for the lens effects at the rough air–water interface, while just these effects are essential in formation of a signal coming through such an interface.[6,7]

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