BRIGHTNESS FIELD OF REFLECTED SOLAR RADIATION UNDER CONDITIONS OF BROKEN CLOUDS. PART. I. ALGORITHM OF CALCULATION

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Monte Carlo algorithms of calculating the mathematical expectation and the variance of solar reflected intensity in a given direction, developed for a Poissonian model of broken cloudiness, are extended to the case of a three-layer cloudy-aerosol atmosphere located over a Lambertian reflecting underlying surface.

1. INTRODUCTION

The radiation regime and brightness field of the system "atmosphere–underlying surface" are controlled strongly by the variety of forms and strong space–time variability of cloud cover. The spatial and angular structure of radiation fields of cloudy atmosphere, together with their sensitivity to cloud characteristics variations, provides an important information needed for remote optical sensing of the cloudy atmosphere[1,2] and underlying surface, for satellite meteorology, as well as for the reconstrudtion of albedo of the system "atmosphere–underlying surface" from a satellite measurements.[3,4] Most often in theoretical studies this problem is presently treated via solving the transfer equation in a plane–parallel horizontally homogeneous cloud layer covering partially or completely the sky. However, this simplest model disregards an important factors connected with the stochastic geometry of real clouds (irregular and odd shapes, amount, extents, and location of individual clouds) and being of great importance for radiative budget and brightness field formation in cloudy atmosphere.

The presence of drawbacks we outlined above, as well as a variety of unsolved problems connected, e.g., with the interpreting satellite data,[5] all these have recently stimulated the development of radiative transfer theory in stochastic scattering media (see, e.g., Refs. 6–9). Authors of Ref. 10 have treated numerically the angular dependence of reflected solar radiation and first evaluated the effect of random geometry of cloud fields on formation of the mean angular distributions of reflected and transmitted solar radiance. However, the numerical method used by the authors allows the computation of histograms of the mean intensity, but not the mean intensity in a given direction. A finer angular structure of scattered light can be determined using the equations for intensity moments together with appropriate Monte Carlo algorithms.[11–14] In Refs. 11–14 no account is taken of the impact of aerosol and underlying surface on radiative transfer, with results far from exhaustive.

In the present paper, the algorithms for calculation of the statistical characteristics of solar radiation intensity in a given direction are extended to the case of a three–layer aerosol–cloudy atmosphere over reflecting underlying surface.

2. MODEL OF ATMOSPHERE

Let us consider an atmospheric model as consisting of three layers: cloudy (Λ) and above– (Λ_2) and under–cloud (Λ_1) aerosol layers over a Lambertian reflecting underlying surface. The horizontally homogeneous aerosol layers are characterized by the optical thicknesses $\tau_{a,2}$ and $\tau_{a,1}$, single scattering albedos $\lambda_{a,2}$ and $\lambda_{a,1}$, and a common, altitude–independent scattering phase function $g_a(\omega, \omega')$, with $\omega = (a, b, c)$ the unit vector of direction. The optical model of broken cloudiness is defined in the layer Λ as random scalar fields of extinction coefficient $\sigma\kappa(\mathbf{r})$, single scattering albedo $\lambda\kappa(\mathbf{r})$, and scattering phase function $g(\omega, \omega')\,\kappa(\mathbf{r})$, where $\kappa(\mathbf{r})$ is the indicator function of the random set $G \subset \Lambda$ where the cloud matter occurs. The mathematical model of broken cloudiness is generated by the Poisson point fluxes on straight lines.[11] The model input is cloud fraction p, cloud thickness $\Delta H = H - h$, and horizontal size D, determining the correlation function of cloud field.

3. MONTE CARLO ALGORITHM OF COMPUTING THE MEAN INTENSITY

The structure of brightness fields in broken clouds is influenced by the processes of scattering and absorption occurring in beyond–cloud atmosphere and on the underlying surface. This influence should be accounted for in algorithms of computing the statistical characteristics of intensity of diffuse solar radiation. Suppose for definiteness that we need to determine the mean intensity of reflected solar radiation $\langle I(H_a, \omega)\rangle$ in the plane $z = H_a$ and in direction ω, with H_a the top of the atmosphere. The mean intensity of reflected solar radiation can be written as

$$\langle I(H_a, \omega)\rangle = T_{a,2}(\omega)\,\langle I(H, \omega)\rangle + i_{a,2}(H_a, \omega)\,. \qquad (1)$$

where $i_{a,2}(H_a, \omega)$ is the solar intensity reflected by the above–cloud aerosol atmosphere, $T_{a,2}(\omega) = \exp\left\{-\sigma_{a,2}\,\dfrac{H_a - H}{c}\right\}$ is the transmission of the layer Λ_2 in direction ω, $\langle I(H, \omega)\rangle = \langle i(H, \omega)\rangle + \langle j_d(H, \omega)\rangle$ is the mean intensity of upward ($c > 0$) solar radiation at cloud top; $\langle i(H, \omega)\rangle$ has the meaning of being the mean intensity formed by scattering in clouds, while $\langle j_d(H, \omega)\rangle$ can be interpreted as the mean intensity of radiation scattered in the under–cloud atmosphere and (or) reflected from the underlying surface and then passed through the cloud layer without scattering. As far as $i_{a,2}(H_a, \omega)$ is trivially calculated (knowing the

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amount of radiant energy coming to Λ_2 from the cloud top) by standard algorithms (see, for example, Ref. 15), we at once address ourselves to the radiative transfer in the layer of broken clouds. From the linearity of radiative transfer equation it follows that the inclusion of the above– and under–cloud atmosphere as well as underlying surface simply changes the boundary conditions in solving the system of equations for the mean solar intensity within the cloud layer Λ. For this reason, we can at once write the solution for the mean intensity $<I(H, \omega)> = <i(H, \omega)> + <j_d(H, \omega)>$ and the function $(H, \omega) = <\kappa(\mathbf{r}_H)\, I(\mathbf{r}_H, \omega)>/p = u(H, \omega) + v_d(H, \omega)$ which is used below to treat the correlation function. The angular brackets indicate the ensemble averages over the cloud field realizations, $\mathbf{r}_H \in z = H$. The derivation of the system of equations for the mean intensity and correlation function of intensity and the algorithms for solving these equations are discussed adequately in Refs. 11–14.

From Refs. 11 and 12 we have

$$<I(H, \omega)> = \frac{\lambda}{2\pi\, c} \int\limits_{E_z} \sum_{i=1}^{2} D_i \exp\left(-\lambda_i \frac{H-\xi}{c}\right) d\xi \times$$

$$\times \int\limits_{4\pi} g(\mu)\, f(\xi, \omega')\, d\omega' + <j_d(H, \omega)>; \quad (2)$$

$$U(H, \omega) = \frac{\lambda}{2\pi\sigma\, p\, c} \int\limits_{E_z} \sum_{i=1}^{2} D_i \lambda_i \exp\left(-\lambda_i \frac{H-\xi}{c}\right) d\xi \times$$

$$\times \int\limits_{4\pi} g(\mu)\, f(\xi, \omega')\, d\omega' + v_d(H, \omega); \quad (3)$$

$$<j_d(H, \omega)> = S_d(h, \omega) \sum_{i=1}^{2} C_i \exp\left(-\lambda_i \frac{H-h}{c}\right); \quad (4)$$

$$v_d(H, \omega) = S_d(h, \omega) \sum_{i=1}^{2} D_i \exp\left(-\lambda_i \frac{H-h}{c}\right), \quad (5)$$

where $S_d(h, \omega)$ is the intensity of diffuse radiation incident upon the lower boundary of cloud layer (diffuse source). We note that $S_d(h, \omega)$ is *a priori* unknown, but it is readily determined in solving the problem of radiative transfer by Monte Carlo method:

$$E_z = (h, H)\,;\ \lambda_{1,2} = \frac{\sigma + A(\omega)}{2} \mp \frac{\sqrt{(\sigma + A(\omega))^2 - 4\, A(\omega)\, p\, \sigma}}{2},$$

$$C_1 = \frac{\lambda_2 - \sigma\, p}{\lambda_2 - \lambda_1},\ C_2 = 1 - C_1,\ D_1 = \frac{\lambda_2 - \sigma}{\lambda_2 - \lambda_1},\ D_2 = 1 - D_1,$$

$$A(\omega) = A \times (|a| + |b|),\ A = (1.65\,(p - 0.5)^2 + 1.04)/D,$$

for spherical particles

$$g(\omega, \omega') = g(\mu)/2\pi,\ \mu = \omega'\, (\mathbf{r}_H - \mathbf{r}')/|\mathbf{r}_H - \mathbf{r}'|.$$

The function $f(\mathbf{x}) = \sigma<\kappa(\mathbf{r})I(\mathbf{r}, \omega)>$ has the meaning of the mean collision density, and it is the solution of integral equation

$$f(\mathbf{x}) = \int\limits_X k(\mathbf{x}', \mathbf{x})\, f(\mathbf{x}')\, d\mathbf{x}' + \tilde{\psi}(\mathbf{x}) \quad (6)$$

with kernel

$$k(\mathbf{x}', \mathbf{x}) = \frac{\lambda g(\mu) \sum\limits_{i=1}^{2} D_i \lambda_i \exp\{-\lambda_i\, |\mathbf{r} - \mathbf{r}'|\}}{2\pi\, |\mathbf{r} - \mathbf{r}'|^2}\, \delta\left(\omega - \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}\right) \quad (7)$$

and free term

$$\tilde{\psi}(\mathbf{x}) = S_d(\mathbf{x}_z)\, \psi(\mathbf{x}) = S_d(\mathbf{x}_z) \sum_{i=1}^{2} C_i \lambda_i \exp\{-\lambda_i\, |\mathbf{r} - \mathbf{r}_z|\}, \quad (8)$$

where X is the phase space of coordinates and directions, $\mathbf{x} = (\mathbf{r}, \omega)$, and $S_d(\mathbf{x}_z)$ is the intensity of diffuse radiation incident on the cloud upper and lower boundaries, $\mathbf{x}_z = (\mathbf{r}_z, \omega)$, $\mathbf{r}_z \in z = h$ for $c > 0$, $\mathbf{r}_z \in z = H$ for $c < 0$.

We now address ourselves to Monte Carlo computations of linear functionals of the type

$$J_h = (f, h) = \int\limits_X f(\mathbf{x})\, h(\mathbf{x})\, d\mathbf{x}, \quad (9)$$

in particular, $<i(H, \omega)>$ is such a functional. In the space L_1 the kernel (7) satisfies $\|K\| \le \lambda \le 1$ and $\|K^2\| < 1$ for a finite medium, so the Neumann series converges for Eq. (6). This establishes the existence, uniqueness, and positivity of the solution of Eq. (3), and we therefore can apply the Monte Carlo method to estimate the linear functionals of the type (9).

Let us consider the Monte Carlo algorithm of calculating the functional J_h. The Markov chain is determined by initial $\psi(\mathbf{x})$ and transition $k(\mathbf{x}', \mathbf{x})/\lambda$ probabilities ($k(\mathbf{x}', \mathbf{x})$ is the substochastic kernel). To calculate J_h, we have

$$J_h = (f, h) = \int\limits_X f(\mathbf{x})\, h(\mathbf{x})\, d\mathbf{x} = M \sum_{n=0}^{N_1} Q_n\, h(\mathbf{x}), \quad (10)$$

where M is the mathematical expectation over the ensemble of trajectories, N_1 is the random number of the last state of the Markov chain, and the auxiliary weights Q_n are calculated from formulas $Q_0 = S_d(\mathbf{x}_z)$, $Q_n = \lambda Q_{n-1}$. According to Eqs. (2) and (3) the mean specific intensity $<i(H, \omega)>$ and the function $u(H, \omega)$ can be calculated if in Eq. (10) one assumes

$$h_i(\mathbf{x}_n) = \begin{cases} \dfrac{g(\mu_n)}{2\pi\, c} \sum\limits_{i=1}^{2} D_i \exp\left\{-\lambda_i \dfrac{H - z_n}{c}\right\}, & c(H - z_n) > 0, \\ 0, & c(H - z_n) < 0. \end{cases} \quad (11)$$

$$h_u(\mathbf{x}_n) = \begin{cases} \dfrac{g(\mu_n)}{2\pi\sigma p\, c} \sum\limits_{i=1}^{2} D_i \lambda_i \exp\left\{-\lambda_i \dfrac{H - z_n}{c}\right\}, & c(H - z_n) > 0, \\ 0, & c(H - z_n) < 0, \end{cases} \quad (12)$$

where $\mu_n = (\omega_n \cdot \omega)$.

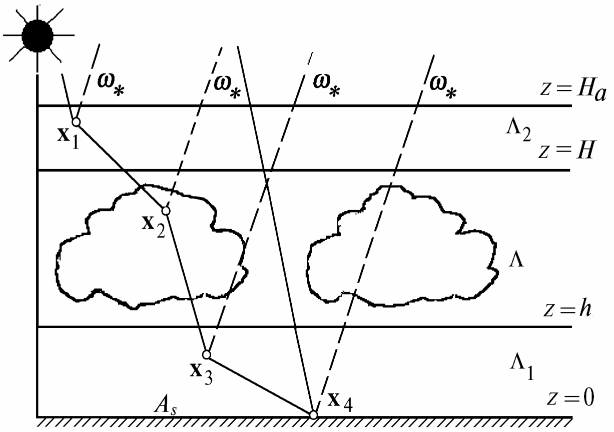

FIG. 1. A schematic illustration of a three-layer model of cloudy atmosphere and a photon trajectory; the open circles show the contributions to mean intensity $\langle I(H_a, \omega)\rangle$ from above– and under–cloud atmosphere, broken clouds, and underlying surface.

The functions $\langle I(H_a, \omega)\rangle$ and $U(H_a, \omega)$ are calculated at the top of the atmosphere as follows. In the layers Λ_i, $i = 1, 2$, as well as at reflection from the underlying surface, the photon trajectories are simulated using the standard algorithms.[15] In the cloud layer, the trajectories are simulated with the algorithms described above. The expressions (11) and (12) for calculating the functions $\langle I(H_a, \omega)\rangle$ (see Eq. (1)) and $U(H_a, \omega)$ are of the following form.

1. Collision in the above–cloud atmosphere (Fig. 1, layer Λ_2, point $\mathbf{x}_1 = (\mathbf{r}_1, \omega_1)$):

$$h_i(\mathbf{x}_n) = \frac{g_a(\mu_n)}{2\pi\, c} \exp\left\{-\sigma_{a,2}\frac{H_a - z_n}{c}\right\}. \tag{13}$$

The function $U(H_a, \omega)$ is estimated by setting $h_u(\mathbf{x}_n) = h_i(\mathbf{x}_n)$.

2. Collision in the clouds (Fig. 1, layer Λ, point $\mathbf{x}_2 = (\mathbf{r}_2, \omega_2)$):

$$h_i(\mathbf{x}_n) = \frac{g_a(\mu_n)}{2\pi\, c}\sum_{i=1}^{2} D_i \exp\left\{-\lambda_i\frac{H - z_n}{c}\right\}\exp\left\{-\sigma_{a,2}\frac{H_a - H}{c}\right\}; \tag{14}$$

$$h_u(\mathbf{x}_n) = \frac{g(\mu_n)}{2\pi\sigma\, p\, c}\sum_{i=1}^{2} D_i\,\lambda_i \exp\left\{-\lambda_i\frac{H - z_n}{c}\right\}\exp\left\{-\sigma_{a,2}\frac{H_a - H}{c}\right\}$$

3. Collision in the under–cloud atmosphere (Fig. 1, layer Λ_1, point $\mathbf{x}_3 = (\mathbf{r}_3, \omega_3)$):

$$h_i(\mathbf{x}_n) = \frac{g_a(\mu_n)}{2\pi\, c}\sum_{i=1}^{2} C_i \exp\left\{-\lambda_i\frac{H - h}{c}\right\}\times$$
$$\times\exp\left\{-\sigma_{a,1}\frac{h - z_n}{c} - \sigma_{a,2}\frac{H_a - H}{c}\right\};$$

(15)

$$h_u(\mathbf{x}_n) = \frac{g_a(\mu_n)}{2\pi\, c}\sum_{i=1}^{2} D_i \exp\left\{-\lambda_i\frac{H - h}{c}\right\}\times$$
$$\times\exp\left\{-\sigma_{a,1}\frac{h - z_n}{c} - \sigma_{a,2}\frac{H_a - H}{c}\right\}.$$

4. At reflection from the underlying surface (Fig. 1, point $\mathbf{x}_4 = (\mathbf{r}_4, \omega_4)$), the particle auxiliary weights are multiplied by $A_s/2\pi$, and the estimates become

$$h_i(\mathbf{x}_n) = 2\sum_{i=1}^{2} C_i \exp\left\{-\lambda_i\frac{H - h}{c}\right\}\exp\left\{-\sigma_{a,1}\frac{h}{c} - \sigma_{a,2}\frac{H_a - H}{c}\right\};$$

(16)

$$h_u(\mathbf{x}_n) = 2\sum_{i=1}^{2} D_i \exp\left\{-\lambda_i\frac{H - h}{c}\right\}\exp\left\{-\sigma_{a,1}\frac{h}{c} - \sigma_{a,2}\frac{H_a - H}{c}\right\}.$$

4. MONTE CARLO ALGORITHM OF CALCULATING THE CORRELATION FUNCTION

Let us have a receiver with the spatial field–of–view angle $\Delta\Omega = 2\pi(1 - \cos\alpha)$. The receiver is located at the point $\mathbf{R} = (\hat{x}, \hat{y}, H_r)$ (see Fig. 2) and measures the quantity

$$F_{\text{mes}}(\mathbf{R}) = \int_{\Delta\Omega} I(\mathbf{R}, \omega)\, d\omega\,. \tag{17}$$

We need to determine the correlation function

$$K_F(\mathbf{R}_1, \mathbf{R}_2) = \langle F_{\text{mes}}(\mathbf{R}_1)\, F_{\text{mes}}(\mathbf{R}_2)\rangle =$$

$$= \int_{\Delta\Omega} d\omega_1 \int_{\Delta\Omega} \langle I(\mathbf{R}_1, \omega_1)\, I(\mathbf{R}_2, \omega_2)\rangle\, d\omega_2. \tag{18}$$

For $\mathbf{R}_1 = \mathbf{R}_2$, the equation (18) yields the variance

$$D_F(\mathbf{R}) = \langle F^2_{\text{mes}}(\mathbf{R})\rangle = \int_{\Delta\Omega} d\omega_1 \int_{\Delta\Omega} \langle I(\mathbf{R}, \omega_1)\, I(\mathbf{R}, \omega_2)\rangle\, d\omega_2. \tag{19}$$

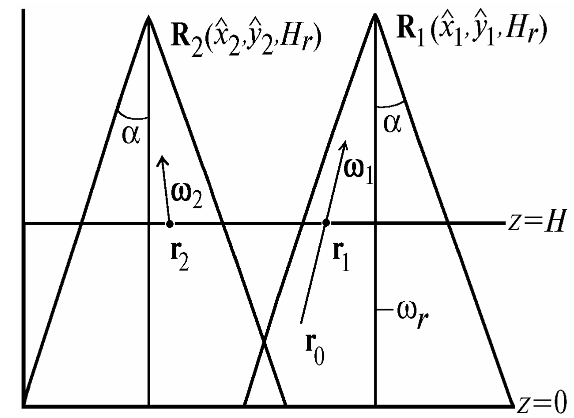

FIG. 2. Scheme illustrating the model geometry.

Let us consider the calculation of the variance and correlation function of solar radiation in a three–layer atmospheric model, bounded at its bottom by the underlying surface. Consider first the effect of the above–cloud atmosphere (layer Λ_2). The layer Λ_2 is optically thin (for the background model $\tau_{a,2} < 1$), so that the radiation escaping through the cloud top can be assumed to propagate unscattered in the layer. Then the random intensity of solar radiation can be written as

$$I(\mathbf{R}_1, \omega) = T_{a,2}(\omega)\, I(\mathbf{r}_1, \omega) + \tilde{i}_{a,2}(\mathbf{R}_1, \omega). \tag{20}$$

Here $\tilde{i}_{a,\,2}(\mathbf{R}_1,\,\omega)$ has the meaning of intensity of the diffuse solar radiation scattered by the above–cloud atmosphere when the upper boundary of the layer Λ_2 is only illuminated. In the approximation considered, the transmission $T_{a,\,2}$ and the intensity $\tilde{i}_{a,\,2}(\mathbf{R}_1,\,\omega)$ are both nonrandom functions. With this in mind, putting (20) in (18) we obtain

$$K_F(\mathbf{R}_1,\,\mathbf{R}_2) = \int\limits_{\Delta\Omega}\int\limits_{\Delta\Omega} \langle[T_{a,\,2}(\omega_1)\,I(\mathbf{r}_1,\,\omega_1) + \tilde{i}_{a,\,2}(\mathbf{R}_1,\,\omega_1)] \times$$

$$\times [T_{a,\,2}(\omega_2)\,I(\mathbf{r}_2,\,\omega_2) + \tilde{i}_{a,\,2}(\mathbf{R}_2,\,\omega_2)]\rangle\,\mathrm{d}\omega_1\,\mathrm{d}\omega_2 =$$

$$= \int\limits_{\Delta\Omega}\int\limits_{\Delta\Omega} \{T_{a,\,2}(\omega_1)\,T_{a,\,2}(\omega_2)\,\langle I(\mathbf{r}_1,\,\omega_1)\,I(\mathbf{r}_2,\,\omega_2)\rangle +$$

$$+ T_{a,\,2}(\omega_1) < I(\mathbf{r}_1,\,\omega_1) > \tilde{i}_{a,\,2}(\mathbf{R}_2,\,\omega_2) + \tilde{i}_{a,\,2}(\mathbf{R}_1,\,\omega_1) \times$$

$$\times T_{a,\,2}(\omega_2) < I(\mathbf{r}_2,\,\omega_2) > + \tilde{i}_{a,\,2}(\mathbf{R}_1,\,\omega_1)\tilde{i}_{a,\,2}(\mathbf{R}_2,\,\omega_2)\}\,\mathrm{d}\omega_1\,\mathrm{d}\omega_2. \quad (21)$$

The calculation of the last three terms in formula (21) presents no serious difficulties, and it is done using the presented above algorithms of calculating the mean intensity. Let us consider in more detail the calculation of the first term.

As was noted above in treating the mean intensity, the upward radiation scattered by the under–cloud atmosphere and (or) reflected by the underlying surface can be interpreted as a source of diffuse radiation; this determines the boundary conditions at cloud base, and the intensities $I(\mathbf{r}_n,\,\omega_n)$, $n = 1, 2$ can be written as

$$I(\mathbf{r}_n,\,\omega_n) = i(\mathbf{r}_n,\,\omega_n) + j_d(\mathbf{r}_n,\,\omega_n). \quad (22)$$

According to Eq. (22), for the first term of Eq. (21) we have

$$\int\limits_{\Delta\Omega}\int\limits_{\Delta\Omega} T_{a,\,2}(\omega_1)\,T_{a,\,2}(\omega_2)\,\{< i(\mathbf{r}_1,\,\omega_1)\,i(\mathbf{r}_2,\,\omega_2) > +$$

$$+ < j_d(\mathbf{r}_1,\,\omega_1)\,i(\mathbf{r}_2,\,\omega_2) > + < i(\mathbf{r}_1,\,\omega_1)\,j_d(\mathbf{r}_2,\,\omega_2) > +$$

$$+ < j_d(\mathbf{r}_1,\,\omega_1)\,j_d(\mathbf{r}_2,\,\omega_2) > \}\,\mathrm{d}\omega_1\,\mathrm{d}\omega_2. \quad (23)$$

From Eq. (23) it follows that the calculation of $K_F(\mathbf{R}_1,\,\mathbf{R}_2)$ requires the knowledge of the correlation functions $\langle i(\mathbf{x}_1)\,i(\mathbf{x}_2)\rangle$, $\langle i(\mathbf{x}_1)\,j_d(\mathbf{x}_2)\rangle$, $\langle j_d(\mathbf{x}_1)\,i(\mathbf{x}_2)\rangle$, and $\langle j_d(\mathbf{x}_1)\,j_d(\mathbf{x}_2)\rangle$. These in turn are calculated as follows.[13,14]

Correlation $\langle i(\mathbf{x}_1)\,i(\mathbf{x}_2)\rangle$:

$$\langle i(\mathbf{x}_1)\,i(\mathbf{x}_2)\rangle = \langle i(H,\,\omega_1)\rangle\,\langle i(H,\,\omega_2)\rangle +$$

$$+ [u(H,\,\omega_2) - \langle i(H,\,\omega_2)\rangle]\,M\sum_{n=0}^{N_1} Q_n\,h_n(\mathbf{x}_n,\,\mathbf{x}_1) \times$$

$$\times \exp(-A_x\,\Delta x_0 - A_y\,\Delta y_0), \quad (24)$$

where $h_n(\mathbf{x}_n,\,\mathbf{x}_1)$ is determined from the formula (11); $\Delta x_0 = |x_0 - x_2|$; $\Delta y_0 = |y_0 - y_2|$; $\mathbf{r}_0 = (x_0,\,y_0,\,z_0) = \mathbf{r}_1 + \dfrac{z_0 - H}{c_1}\,\omega_1$ is the point of initial collision in the cloud layer (Fig. 2).

Correlation $\langle j_d(\mathbf{x}_1)\,i(\mathbf{x}_2)\rangle$

$$\langle j_d(\mathbf{x}_1)\,i(\mathbf{x}_2)\rangle = < j_d(H,\,\omega_1) > < i(H,\,\omega_2)> +$$

$$+ [S_d(h,\,\omega_1) - < j_d(H,\,\omega_1) >]\,[u(H,\,\omega_2) - < i(H,\,\omega_2) >] \times$$

$$\times \exp\{-A_x\,\Delta x_0 - A_y\,\Delta y_0\}. \quad (25)$$

Correlation $\langle i(\mathbf{x}_1)\,j_d(\mathbf{x}_2)\rangle$

$$< i(\mathbf{x}_1)\,j_d(\mathbf{x}_2) > = < i(H,\,\omega_1) > \langle j_d(H,\,\omega_2)\rangle +$$

$$+ [v_d(H,\,\omega_2) - < j_d(H,\,\omega_2) >]\,M\sum_{n=0}^{N_1} Q_n\,h_i(\mathbf{x}_n,\,\mathbf{x}_1) \times$$

$$\times \exp\{-A_x\,\Delta x_0 - A_y\,\Delta y_0\}\,. \quad (26)$$

Correlation $< j_d(\mathbf{x}_1)\,j(\mathbf{x}_2) >$. To treat the space–angle correlation function $< j_d(\mathbf{r}_1,\,\omega_1)\,j_d(\mathbf{r}_2,\,\omega_2) >$, closed equations[16] have been obtained which depend on the location of points $\mathbf{r}_i$ and on directions ω_i, $i = 1, 2$. These equations are readily solved, e.g., through the Laplace transform; however, the ultimate formulas are very intricate and then not given here.

To calculate the variance (19), it is necessary to integrate the expressions (24)–(26) and the formula for $\langle j_d(\mathbf{r}_1,\,\omega_1)\,j_d(\mathbf{r}_2,\,\omega_2)\rangle$ over the receiver spatial field–of–view angle $\Delta\Omega$. The integration over a large spatial angles $\Delta\Omega$ is computationally expensive. The situation significantly simplifies in a particular, but a very important case of receiver with a small field of view. The point is that all terms in formulas (24)–(23) and in expression for $\langle j_d(\mathbf{r}_1,\,\omega_1)\,j_d(\mathbf{r}_2,\,\omega_2)\rangle$, describing different components of the mean intensity, are smooth functions which vary slightly with the small changes of viewing angle.[12,13,16] Therefore, without a significant loss in accuracy, these functions can be treated as constants within a small spatial angle $\Delta\Omega$ and hence can be removed from the integral over $\Delta\Omega$. We denote the direction of the receiver optical axis by $-\omega_r$ (see Fig. 2). Then, for the correlation function (24) we obtain

$$K(\mathbf{r}_1,\,\mathbf{r}_2) = < i(H,\,\omega_r) >^2 (\Delta\Omega)^2 +$$

$$+ [u(H,\omega_r) - < i(H,\omega_r) >]\,M\sum_{n=0}^{N_1} Q_n\,h_i(\mathbf{x}_n,\,\mathbf{x}_1)\,J(\Delta\hat{x},\,\Delta\hat{y}), \quad (27)$$

where

$$J(\Delta\hat{x},\,\Delta\hat{y}) = \int\limits_{\Delta\Omega} \mathrm{d}\omega_1 \int\limits_{\Delta\Omega} \exp\{-A_x\,\Delta x_0 - A_y\,\Delta y_0\}\,\mathrm{d}\omega_2; \quad (28)$$

$$\left.\begin{aligned} \Delta x_0 &= \left|\Delta\hat{x} + (H_r - H)\left(\frac{a_2}{c_2} - \frac{a_1}{c_1}\right) - \frac{a_1}{c_1}(H - z_0)\right|; \\ \Delta y_0 &= \left|\Delta\hat{y} + (H_r - H)\left(\frac{b_2}{c_2} - \frac{b_1}{c_1}\right) - \frac{b_1}{c_1}(H - z_0)\right|; \end{aligned}\right\} \quad (29)$$

If the receiver is located on a high–flying aircraft or satellite, then $(H_r - H) > H - z_0$ and hence the last terms in Eq. (29) can be neglected. Expression (24) then becomes

$$K(\mathbf{r}_1, \mathbf{r}_2) = < i(H, \boldsymbol{\omega}_r) >^2 (\Delta\Omega)^2 + [u(H, \boldsymbol{\omega}_r) -$$

$$- < i(H, \boldsymbol{\omega}_r) >] < i(H, \boldsymbol{\omega}_r) > J(\Delta\hat{x}, \Delta\hat{y}) , \quad (26')$$

where $J(\Delta\hat{x}, \Delta\hat{y})$ is calculated from Eq. (28) taking

$$\Delta \mathbf{x}_0 = \left| \Delta\hat{x} + (H_r - H) \left(\frac{a_2}{c_2} - \frac{a_1}{c_1} \right) \right| ,$$

$$(29')$$

$$\Delta \mathbf{y}_0 = \left| \Delta\hat{y} + (H_r - H) \left(\frac{b_2}{c_2} - \frac{b_1}{c_1} \right) \right| .$$

The integral (28) is calculated by the Monte Carlo method. Clearly, the discussed above simplification can also be used to integrate over the angle of receiver field of view the other three correlations $< i(\mathbf{x}_1)\, j_d(\mathbf{x}_2) >$, $< j_d(\mathbf{x}_1)\, i(\mathbf{x}_2) >$, $< j_d(\mathbf{x}_1)\, j_d(\mathbf{x}_2) >$ (the three last terms in formula (23)).

Thus, the Monte Carlo algorithms are developed for calculating the mathematical expectation and the variance of intensity of reflected solar radiation in a three–layer cloudy–aerosol atmosphere over the Lambertian reflecting underlying surface. A notable feature of the algorithms is their capability of calculating the statistical characteristics of intensity in a given direction, thus capturing rather fine features in angular structure of scattered light.

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