INFLUENCE OF THE REFRACTION ON GEOMETRY OF ACOUSTIC SOUNDING OF THE ATMOSPHERE

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Received December 30, 1993

In this paper we discuss the influence of the refraction on the geometry of monostatic and bistatic acoustic sounding of the atmosphere. The relations between the geometry parameters and the profiles of atmospheric temperature and wind velocity were derived using linear approximation of geometric acoustics for an inhomogeneous moving medium. Some results of model calculations made using these relations and numerical estimates of their accuracy characteristics can also be found in the paper.

The regular refraction caused by vertical inhomogeneity of wind velocity **v** and air temperature T fields affects strongly sound propagation in the atmosphere. During acoustic sounding it results in variations of different geometric parameters such as the scattering volume center coordinates $\mathbf{r}'_Q(x'_Q, y'_Q, z'_Q)$, angles of sound scattering θ and its arrival to the receiving antenna α_r and β_r (in two orthogonal planes), and the path length S of sound propagation from the transmitting antenna to $\mathbf{r}'_Q$ and back to the receiving one.

The practice demands a simple analytic relations which could readily and sufficiently accurately assess the refraction variations in the aforementioned parameters with the known profiles of **v** and T. This can be explained by the fact that the refraction variations in the sounding geometry lead to variations in power and frequency of scattered signal measured in the experiment. As a result, some additional systematic errors appear when the atmospheric parameters profiles are reconstructed from the acoustic radar (AR) data. The foregoing relations allow developing an engineering procedure for taking account of these errors.

It is reasonable that the efforts to derive such relations have been already made. The formulas for refraction displacements $\Delta x'_Q$ and $\Delta z'_Q$ of the scattering volume center in a single horizontal and vertical plane for both monostatic and bistatic sounding are given in Refs. 2 and 3. Here too the refraction formula for the angle of scattered sound arrival has been obtained for slant monostatic sounding. These formulas were derived based on the approximate expression for a curvature radius of acoustic beam in the atmosphere borrowed from Ref. 4 where it was obtained with an accuracy to the terms of the order of v/c_0 and $|\Delta c|/c_0$ ($\Delta c = c - c_0$ is the deviation of sonic velocity in air from its value c_0). In this case, a double curvature of acoustic beam in wind (the beam does not lie in a single plane) and angular differences between the directions of the normal to the phase wave front **n** and the unit vector **s** tangent to the beam were not taken into account. It is known[5] that such assumptions lead to errors comparable in magnitude to the terms of the order of v/c_0.

In Ref. 6 the effort was undertaken to make the results[2,3] more accurate by introducing angular corrections for difference between **n** and **s** in the final formulas. Moreover, since in Refs. 2 and 3 the problem was solved for the simplest case of linear altitude profiles of c and v_x (projection of **v** onto the x axis), in Ref. 6 the authors proposed the method of applying these formulas for more complicated stratifications of the atmosphere too. It consisted in dividing the atmosphere into individual layers distinguishing by the altitude. The gradients of c and v_x in each of these layers were assumed constant. The refraction problems in Refs. 7–9 were also solved by linearizing the integral equation of acoustic beam for an inhomogeneous moving medium with respect to the values v/c_0 and $|\Delta c|/c_0$. It was found that this approach enabled the refraction formulas with arbitrary altitude profiles of **v** and c (or T) to be obtained. In Refs. 7–9 the formula for $\Delta z'_Q$ for slant monostatic sounding,[7] the expression for the angle α_r for vertical monostatic sounding,[8] and the expression for the angle and time of the scattered signal arrival for bistatic sounding[9] were obtained.

In this paper we used a similar method for solving the refraction problems. It was proposed to derive the refraction formulas for the entire set of parameters of the acoustic sounding geometry both monostatic and bistatic which could be valid for different orientations of polar diagrams (PDs) of antennas and altitude profiles of **v** and T. Because of cumbersome intermediate computations, only initial equations and final formulas are represented below. At the same time as the derivation of the formulas, the numerical computer solution of the same refraction problems was made based on accurate equations of geometric acoustics for different sounding geometries and types of atmospheric stratification. It enabled us, first, to find and then exclude errors in deriving the final formulas and, second, to assess numerically the accuracy characteristics of these formulas. The numerical results are given below only for a single atmospheric stratification which is the most typical. Since the time of signal propagation along the path τ is more important in practice than path geometric length S, the acoustic analog of S is considered here rather than S.

SYSTEM OF BEAM EQUATIONS FOR ACOUSTIC SOUNDING OF THE ATMOSPHERE AND METHODOLOGY OF ITS SOLUTION

Let the transmitting and receiving AR antennas spaced for a distance d horizontally form a bistatic channel for sounding the atmosphere with the help of their PDs. Monostatic sounding is treated here as a particular case of sounding for $d = 0$, unless otherwise indicated. Due to finite width of antennas PDs inside this channel it is possible to indicate many beam paths of AR signal propagation. Each of these paths contains upward (a direct wave) and downward (a scattered wave) beams which have a point of sound scattering $\mathbf{r}'(x', y', z')$ common for a given pair of the beams.

The condition of intersection at a point $\mathbf{r}'$ of the incident and scattered beams whose parameters are denoted as "t" and "r", respectively, can be formulated as

$$\mathbf{R}(\xi_t, \nu_t, z') = \mathbf{R}(\xi_r, \nu_r, z') , \quad (1)$$

$$\tau = \tau(\xi_t, \nu_t, z') + \tau(\xi_r, \nu_r, z') , \quad (2)$$

where $\mathbf{R}(\xi, \nu, z) = \mathbf{i}\, x(\xi, \nu, z) + \mathbf{j}\, y(\xi, \nu, z)$ is the radius–vector describing the horizontal position of a beam point assigned by the beam parameters ξ and ν with the vertical coordinate z, and $\tau(\xi, \nu, z)$ is the time of sound propagation along a segment of the same beam with boundary points existing at altitudes z_0 (the level of AR antenna aperture over the Earth's surface) and z. The values $\nu_{t,\,r}$ are the angles of azimuthal orientations of normals to the wave front $\mathbf{n}_{t,\,r}(z_0)$ at the points of transmission and reception of the sound, and $\xi_{t,\,r}$ are the tilt angles of these normals in a vertical plane with respect to horizon ($0 < \xi_t \le \pi/2$ and $-\pi/2 \le \xi_r < 0$).

To solve the system of equations (1)–(2) with respect to any characteristics of the beam pair under study, one should specify a type of the functions $\mathbf{R}(\xi, \nu, z)$ and $\tau(\xi, \nu, z)$. For a stratified moving medium they are familiar (see, e.g., Ref. 5). Taking them into account it is possible to write

$\mathbf{R}_{t,\,r}(z') = \mathbf{R}(\xi_{t,\,r}, \nu_{t,\,r}, z')$ and $\tau_{t,\,r}(z') = \tau(\xi_{t,\,r}, \nu_{t,\,r}, z')$:

$$\mathbf{R}_{t,\,r}(z') = \{d\,\mathbf{i}\}_r + \int_{z_0}^{z'} (c(z)\, \boldsymbol{\kappa}_{t,\,r} \cos\xi_{t,\,r} + [c(z)\, A_{t,\,r}(z) - v_z(z)\, \chi_{t,\,r}(z)]\, \mathbf{v}_h(z)/B(z)) / \chi_{t,\,r}(z)\, \mathrm{d}z, \quad (3)$$

$$\tau_{t,\,r}(z') = \int_{z_0}^{z'} \frac{c(z)\, A_{t,\,r}(z) - v_z(z)\, \chi_{t,\,r}(z)}{B(z)\, \chi_{t,\,r}(z)}\, \mathrm{d}z , \quad (4)$$

where $A_{t,\,r}(z) = W_{t,\,r}(z_0) - \mathbf{v}_h(z)\, \boldsymbol{\kappa}_{t,\,r} \cos\xi_{t,\,r}$;
$B(z) = c^2(z) - v_z^2(z)$; $\chi_{t,\,r}(z) = \pm\sqrt{A_{t,r}^2(z) - B(z)\cos^2\xi_{t,\,r}}$;
$\boldsymbol{\kappa}_{t,\,r} = \mathbf{i}\cos\nu_{t,\,r} + \mathbf{j}\sin\nu_{t,\,r}$.

The relations (3) and (4) for the given medium type are accurate. The only restriction imposed on their applicability is a requirement for a short wavelength as compared to a characteristic scale of variations of mean values describing the medium state (c and $\mathbf{v}$). The integrands in them, as compared to the original form in Ref. 5, are normalized for the same constant $\omega/W_{t,\,r}(z_0)$ in the numerator and denominator, where ω is the circular frequency of sound vibrations; $W_{t,\,r} = c + \mathbf{v}_h\, \boldsymbol{\kappa}_{t,\,r} \cos\xi_{t,\,r} + v_z \sin\xi_{t,\,r}$ is the phase velocity of sound; and $\mathbf{v}_h$ and v_z are horizontal and vertical components of wind velocity. For this reason, in contrast to Ref. 5, in Eqs. (3) and (4) the vectors $\boldsymbol{\chi}_{t,\,r}$ as well as $\nu_{t,\,r}$ describing the azimuthal directions $\mathbf{n}_{t,\,r}(z_0)$ are equal to unity by absolute value. Hereinafter the x axis, in the case of bistatic sounding, always coincides with the direction from the AR transmitting antenna to receiving one, and in monostatic sounding it coincides with the azimuthal direction of vertical axis deviation of the antenna, which in this case is unique. When two arithmetic signs are used simultaneously, the upper sign is related to the first index "t" and the lower one to the second index. The term in braces with the index "r" is related to the expression describing the scattered beam only.

The AR antennas are oriented vertically or close to this direction. Therefore, in the experiments the angles α_r and β_r between the normal projection $\mathbf{n}_r(z_0)$ on a planes $y = 0$ and $x = 0$, respectively, and the z axis are under control. Their values are unambiguously related to the angles ξ_r and ν_r via the relations: $\tan\alpha_r = \cos(\nu_r)/\tan\xi_r$ and $\tan\beta_r = \sin(\nu_r)/\tan\xi_r$. Taking this into account it is expedient to introduce new angular beam characteristics (α_t, β_t) and (α_r, β_r) in place of (ξ_t, ν_t) and (ξ_r, ν_r). The system of equations (1)–(2) thus modified can be solved in conjunction with Eqs. (3)–(4) with respect to the three of the following six values: α_t, β_t, α_r, β_r, z', and τ, if the remaining three values are assumed to be known.

Two versions of the refraction problem formulation, as applied to AR, are possible. In the first case, the basic known parameter is the current time τ of signal recording which is counted off from the start of the sounding pulse. The other known parameters can be the angles α_t and β_t with the assumption, e.g., that they coincide with the angles $0 \le \alpha_t^* < \pi/2$ and $0 \le \beta_t^* < \pi/2$, describing the orientation of the transmitting antenna PD axis.* In this case the solution of the refraction problem reduces to finding the angles of scattered signal arrival $\alpha_r(\tau)$ and $\beta_r(\tau)$ recorded at a given moment τ and the height of point of its scattering $z'(\tau)$. Then the remaining coordinates $x'(\tau)$ and $y'(\tau)$ can be calculated by substituting the found values $\alpha_r(\tau)$, $\beta_r(\tau)$, and $z'(\tau)$ into Eq. (3). Such formulation of the refraction problem is used for a monostatic geometry only though it can also be employed in the case of $d \ne 0$.

In contrast to the first version of the refraction problem the second one is used only in bistatic sounding. Of interest in this case are the parameters of the signal arriving from the geometric center of the scattering volume corresponding to a point of actual intersection of PD axes of the transmitting and receiving antennas. In practice the antennas are set to bear on a particular point in the atmosphere, e.g., with the coordinates $\mathbf{r}_Q(x_Q, y_Q, z_Q)$ without considering the effect of refraction. Here the PD axes of both these antennas must lie in a single plane possibly deflected out of vertical for an angle $\beta_Q = \arctan(y_Q/z_Q)$ (as a rule, $\beta_Q = 0$). The refraction results in that the direction of the scattered wave arrival does not coincide with this sounding plane, i.e., the refraction change $\Delta\beta_r$ of the angle β_r is observed. For the same reason the coordinates of real intersection point of the antenna PD axes $\mathbf{r}'_Q$ do not coincide with $\mathbf{r}_Q$ as well. Taking into account the above mentioned remarks, in the second version of the refraction problem the angles $\alpha_t = \alpha_t^*$, $\alpha_r = \alpha_r^* + \pi$, and $\beta_t = \beta_Q$ turn out to be known, and from Eqs. (1) and (2) one must find the angle β_r and time τ of arrival of the signal scattered at a point $\mathbf{r}'_Q$ and, moreover, the displacements of this point coordinates $\Delta x'_Q = x'_Q - x_Q$, $\Delta y'_Q = y'_Q - y_Q$, and $\Delta z'_Q = z'_Q - z_Q$.

*In the atmosphere a real PD of acoustic antenna is affected by wind drift of sound energy. Since this phenomenon is taken into account in Eqs. (3)–(4), we consider here PDs determined by the antenna aperture only.

Equations (1) and (2) do not allow determination of the sound scattering angle θ. Therefore, the reference system of equations must be supplemented by one more equation. The angle θ is, by definition, the angle between the directions of the normals $\mathbf{n}_\mathrm{t}$ and $\mathbf{n}_\mathrm{r}$ at an intersection point of incident and scattered beams. Hence, the value θ must satisfy the relation ($0 < \theta \le \pi$):

$$\sin\theta = |\mathbf{n}_\mathrm{t}(z')\times\mathbf{n}_\mathrm{r}(z')|, \quad (5)$$

where × is the symbol of vector product.

The behavior of $\mathbf{n}_{\mathrm{t,r}}$ as a function of z is described by the law of normal refraction in geometric acoustics.[5] In Ref. 9 this law is written as

$$\cos\xi'_{\mathrm{t,r}}(z) = \mu^{-1}_{\mathrm{t,r}}(z)\cos\xi_{\mathrm{t,r}}\ ;\ \nu'_{\mathrm{t,r}}(z) = \nu_{\mathrm{t,r}}\ , \quad (6)$$

where $\mu_{\mathrm{t,r}}(z) = W_{\mathrm{t,r}}(z_0)/W_{\mathrm{t,r}}(z)$. Here the angles $\xi'_{\mathrm{t,r}}(z)$ and $\nu'_{\mathrm{t,r}}(z)$, as previously $\xi_{\mathrm{t,r}}$ and $\nu_{\mathrm{t,r}}$ for $\mathbf{n}_{\mathrm{t,r}}(z_0)$, describe the orientation of the normals $\mathbf{n}_{\mathrm{t,r}}$ but at the altitude $z \ne z_0$.

When calculating θ from Eq. (5) we use the resolution of the vectors $\mathbf{n}_{\mathrm{t,r}}(z')$ into orthogonal components with account of Eq. (6) in the form

$$\mathbf{n}_{\mathrm{t,r}}(z') = \boldsymbol{\kappa}_{\mathrm{t,r}}\,\mu^{-1}_{\mathrm{t,r}}(z')\cos\xi_{\mathrm{t,r}} + \mathbf{k}\,(1-\mu^{-2}_{\mathrm{t,r}}(z')\cos^2\xi_{\mathrm{t,r}})^{1/2}. \quad (7)$$

Thus, the effect of refraction on all of the sought geometric parameters is fully described by the system of equations (1), (2), and (5) with the additional use of relations (3), (4), and (7).

The solution of Eqs. (1) and (2) together with (3) and (4) with respect to the sound arrival angles is similar to a standard refraction problem, i.e., pointing of the beam to a point with the known coordinates. It is well known that this problem does not possess accurate analytical solutions. This statement is also valid for the inverse problem of determining the vertical coordinate z' with the known values ξ, ν, x', and y'. Therefore, the equations of the type (3) and (4) are usually solved either numerically with a computer or by means of reduction them to simpler approximated relations. In the last case the expansion of accurate equations into series over the small parameter $\varepsilon = \max\{v/c_0\ ,\ |\Delta c|/c_0\} \ll 1$ is used in atmospheric acoustics.

The assumption of a smallness of ε value at altitudes to which acoustic sounding is carried out (≤1 km) is always fulfilled in the atmosphere for any real variations of the values c and $\mathbf{v}$. The applicability limits of the beam equation expansion over ε with accuracy to terms of the order of ε^1 was considered in Ref. 5. In this case, the relation $|\xi| \gg \sqrt{\varepsilon}$ was obtained which established the limitation on the values of angles $\xi_{\mathrm{t,r}}$ minimum by absolute value. A physical sense of this limitation is that the linear approximation over ε does not allow describing the beam near the point of its turn. In acoustic sounding the sound, as a rule, propagates at large angles with respect to horizon ($|\xi_{\mathrm{t,r}}| \gg \sqrt{\varepsilon}$). Therefore, to obtain the analytical solutions, we use the linearization of expressions (3), (4), and (7) over ε. In addition, we take into account that the relation $\Delta c/c_0 \cong \Delta T/(2T_0)$ (Ref. 9) is fulfilled with good accuracy for air. Since the refraction corrections $\Delta\alpha_\mathrm{r}$ and $\Delta\beta_\mathrm{r}$ to the angles α_r and β_r coincide with ε by the order of magnitude, then the trigonometric functions entering into the reference equations and containing α_r and β_r are also expanded into series where small terms nonlinear with respect to $\Delta\alpha_\mathrm{r}$ and $\Delta\beta_\mathrm{r}$ do not remain.

The main advantage of such approximated equations as compared to the accurate ones lies in the fact that the integration of nonlinear complicated functions of the profiles $c(z)$ and $\mathbf{v}(z)$ in them over z is replaced by separate integration of the profiles $T(z)$ and $\mathbf{v}(z)$. As a consequence, the relations (1)–(3) represent a system of algebraic equations containing the linear combinations of the functionals

$$\Delta\hat{T}(z_Q) = \Delta\overline{T}(z_Q)/(2\,T_0),\ \Delta\hat{\nu}_i(z_Q) = \Delta\overline{\nu}_i(z_Q)/c_0,\ (i = x, y, z)\ ,$$

where

$$\Delta\overline{a}(z_Q) = \frac{1}{(z_Q - z_0)}\int\limits_{z_0}^{z_Q}[a(z) - a_0]\,\mathrm{d}\,z\ ,$$

with constant coefficients which are independent on z and determined only by geometry of sounding. Such a system of equations can always be solved with respect to refraction displacements of geometric parameters. In this case the formulas are derived which are not attached to any type of atmospheric stratification. If necessary, they can be reduced to a more specific form with preliminary calculation of the values $\Delta\hat{T}(z_Q)$ and $\Delta\hat{\nu}_i(z_Q)$.

REFRACTION FORMULAS FOR MONOSTATIC SOUNDING

Let us consider the first version of the refraction problem. The angle of vertical deviation of the antenna axis α is known, and the aforementioned geometric parameters are to be determined for a given instant of time τ. The solution is found in the form of refraction corrections $\Delta\alpha_\mathrm{r}$, $\Delta\beta_\mathrm{r}$, $\Delta z'_Q$, $\Delta x'_Q$, $\Delta y'_Q$, and Δθ to the values of these parameters in the homogeneous immovable medium: $\alpha_\mathrm{r} = \alpha + \pi$; $\beta_\mathrm{r} = \pi$; $\theta = \pi$; $x_Q = c_0\,\tau/2\sin\alpha$; $y_Q = 0$; and, $z_Q = c_0\,\tau/2\cos\alpha + z_0$. As a result, we obtaine:

for refraction corrections to the angles of the scattered signal arrival

$$\Delta\alpha_\mathrm{r}(\tau) \approx 2\,[\Delta\hat{\nu}_x[z_Q(\tau)]\sec\alpha + v_{x\,0}/c_0\cos\alpha - v_{z\,0}/c_0\sin\alpha]\ , \quad (8)$$

$$\Delta\beta_\mathrm{r}(\tau) \approx 2\,\hat{\nu}_y[z_Q(\tau)]\sec\alpha\ ; \quad (9)$$

for displacements of the scattering volume center coordinates

$$\Delta z'_Q(\tau) \approx -(c_0\,\tau/2)\cos\alpha\,[\Delta\hat{T}[z_Q(\tau)]\times$$
$$\times(\tan^2\alpha - 1) + \Delta\alpha_\mathrm{r}(\tau)/2\tan\alpha]\ ; \quad (10)$$

$$\Delta x'_Q(\tau) \approx (c_0\,\tau/2)\cos\alpha\,[2\,\Delta\hat{T}[z_Q(\tau)]\tan\alpha + \Delta\alpha_\mathrm{r}(\tau)/2]\ ; \quad (11)$$

$$\Delta y'_Q(\tau) \approx (c_0\,\tau/2)\cos\alpha\,\Delta\beta_\mathrm{r}(\tau)/2\ ; \quad (12)$$

and, for refraction correction to the sound scattering angle

$$\Delta\theta(\tau) \approx -\left\{\left[\Delta\alpha_\mathrm{r}(\tau) - 2\left(\frac{\Delta\nu_x[z_Q(\tau)]}{c_0}\tan\alpha + \frac{\Delta\nu_z[z_Q(\tau)]}{c_0}\right)\sin\alpha\right]^2 + (\Delta\beta_\mathrm{r}(\tau)\cos\alpha)^2\right\}^{1/2}. \quad (13)$$

For vertical sounding the equations (8) and (9) give the similar relations of the type $\Delta\alpha_{x,y}(\tau) \approx 2\hat{v}_{x,y}(z_Q)$ which coincide with the analogous formulas in Refs. (2) and (8). The expression $\Delta\alpha_{x,y}(\tau) \approx 2\hat{v}_{x,y}(z_Q)\cos\alpha$, markedly differing from Eqs. (8) and (9) was obtained for slant sounding.[3] It was noted in Ref. 6 that the experimental values $\Delta\alpha_{x,y}(\tau)$ are better described by the relation $\Delta\alpha_{x,y}(\tau) \approx 2\hat{v}_{x,y}(z_Q)\sec\alpha$. This contradiction with the results of Ref. 3 the authors of Ref. 6 attributed to the errors of linear approximation over ε. As Eqs. (8) and (9) show, the last statement is not valid.

It should be noted that the absolute values of errors in the aforementioned approximation, when estimating the angles, amount to the values of the order of ε^2. Therefore, they are usually smaller than the instrumental errors and not detected in measurements.

It can be seen from the analysis of intermediate calculations for obtaining Eqs. (8) and (9) that the multiplier $\cos\alpha$ appears in these formulas if in the reference equations the angular differences in the directions of **n** and **s** are neglected. In this case the account of the angle between **n** and **s** in the final formulas does not provide for a correct result. For the same reason the formulas for $\Delta x'_Q$ and $\Delta z'_Q$ in Ref. 3 do not coincide with Eqs. (10) and (11). The formula for $\Delta z'_Q$ from Ref. 7 differs from Eq. (10) in that there are no terms containing v_{x0} and v_{z0}. As a result, it describes the refraction correctly, but the variation in the direction of the axis of the real antenna PD is not taken into account when a medium flow passes over it.

It should be noted that in the linear approximation over ε the refraction corrections to the angular parameters α_r, β_r, and θ are determined only by vertical distribution of wind velocity in the atmosphere according to Eqs. (8), (9), and (13). Contribution of the temperature refraction to the values of angular parameters is the value of the second order of smallness. What has been said above indicates that a principle of reciprocity of direct and scattered beams connecting the same pair of points holds in an immovable medium. Following this principle, with wind absent, the refraction has an equal effect on trajectories of direct and scattered beams during monostatic sounding. The directions of outgoing direct and incoming scattered beams lie along a single straight line in the opposite direction. In the presence of wind the refraction has an opposite effect on upward and downward beams with common corresponding points. Therefore, the tangents to their trajectories differ in direction by a small angle of the order of ε.

Figure 1 depicts the calculational results of refraction corrections to the parameters of monostatic geometry made using the formulas (8)–(13). From here we represent the calculations for a logarithmic profile of wind velocity[10] and a linear profile of temperature which are described by the relations

$$v(z) = v_m \ln[(z + z_{rou})/z_{rou}]/\ln[(z_m + z_{rou})/z_{rou}],\ \varphi_v(z) = \text{const};$$

$$T(z) = T_m + \gamma(z - z_m),$$

where z_{rou} is the roughness parameter of the Earth's surface; φ_v is the azimuthal direction of wind; γ is the temperature gradient; z_m is the height of extracting the meteorological information; and $v_m = v(z_m)$, $T_m = T(z_m)$.

Figure 1 *a* indicates the almost linear increase of the absolute values of $\Delta x'_Q$, $\Delta y'_Q$, and $\Delta z'_Q$ as a function of sounding altitude z_Q (expected with the neglect of refraction). This is accounted for by the coefficient $z_Q - z_0$ in Eqs. (10)–(12). The nonlinearity of function $v(z)$ in the range of 25 to 500 m altitudes is manifested very slightly. When v_m = 10 m/s the refraction corrections $\Delta x'_Q$, $\Delta y'_Q$, and $\Delta z'_Q$ increase by about 4 m for each 100 m of the z_Q increase. Of particular practical importance here is the effect of refraction on real altitude of sounding z'_Q. In the reconstructing the profiles of atmospheric parameters the value $\Delta z'_Q$ gives the error of fixing their values, measured with AR, to the altitude.

In contrast to $\Delta x'_Q$, $\Delta y'_Q$, and $\Delta z'_Q$ the refraction corrections to angular parameters of scattered signal do not explicitly depend on z_Q. The temperature profile does not affect here too. Therefore, the variations in $\Delta\alpha_r$, $\Delta\beta_r$, and Δθ with the altitude z_Q increase are determined by the wind velocity profile alone. In particular, curves in Fig. 1 *b* reflect the logarithmic dependence of v on z which was assigned in the calculations.

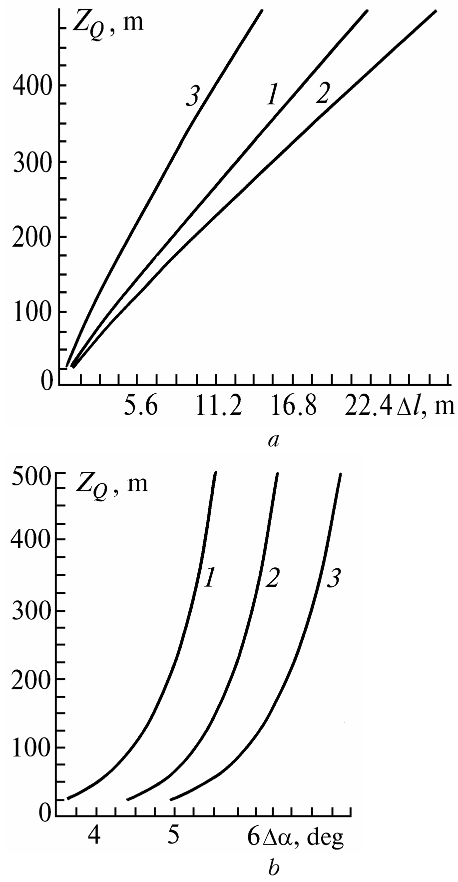

FIG. 1. Refraction variations in the monostatic geometry parameters as a function of sounding altitude z_Q for v_m = 10 m/s, φ_v = 225°, z_{rou} = 2 cm, T_m = 20°C, γ = –6.5°/km, z_m = 1 m, α = 30°, and z_0 = 2 m. a) $\Delta l = \Delta x'_Q$ (1), $\Delta l = \Delta y'_Q$ (2), and $\Delta l = \Delta z'_Q$ (3), b) $\Delta\alpha = \Delta\alpha_r$ (1), $\Delta\alpha = \Delta\beta_r$ (2), and $\Delta\alpha = \Delta\theta$ (3).

The values $\Delta\alpha_r$, $\Delta\beta_r$, and Δθ are known to affect the scattered signal power P_r recorded with AR. So they are one of the main sources of measurement errors of C_2^T (see

Ref. 11). To estimate qualitatively the significance of the values $\Delta\alpha_r$ and $\Delta\beta_r$, they can be most conveniently compared to the angular half–width $\psi/2$ of the antenna PD. In AR $\psi/2$ is 5–7°, as a rule. As follows from Fig. 1 *b*, for such PDs the ground wind velocity of 10 m/s results in substantial decrease of P_r. Moreover, with strong winds the values $\Delta\alpha_r$ and $\Delta\beta_r$ are larger than $\psi/2$. As a result, the failure to detect the signal becomes possible. It should be noted that the refraction corrections in the linear approximation over ε are proportional to v_m. Therefore Fig. 1 can also be used for their estimate with the other values of v_m. In this case, all of the corrections in Fig. 1 should be multiplied by $v_m/10$.

REFRACTION FORMULAS FOR BISTATIC SOUNDING

Let us consider the second version of the refraction problem. Orientations of transmitting and receiving antenna PD axes assigned below by pairs of angles ($\alpha_t^* \ge 0$, $\beta_t^* = \beta_Q$) and ($\alpha_t^* \le 0$, $\beta_t^* = \beta_Q$), respectively, are taken to be known. The refraction corrections $\Delta x'_Q$, $\Delta y'_Q$, $\Delta z'_Q$, $\Delta\beta_r$, $\Delta\tau'_Q$, and $\Delta\theta$ to the sought parameters without the refraction taken into account are needed to be determined:

$$z_Q = d / (\tan\alpha_t^* - \tan\alpha_r^*) + z_0 ,$$

$$\tau_Q = (z_Q - z_0)(1/\sin\xi_t^* + 1/\sin\xi_r^*)/c_0 ,$$

$$x_Q = (z_Q - z_0)\tan\alpha_t^*,\ y_Q = (z_Q - z_0)\tan\beta_Q,\ \beta_r = \beta_Q + \pi,\ \theta = \theta_0 ,$$

where

$$\sin\theta_0 = (\tan\alpha_t^* - \tan\alpha_r^*)\sin\xi_t^*\sin\xi_r^*/\cos\beta_Q \text{ and}$$

$$\sin\xi_{t,r}^* = (1 + \tan^2\alpha_{t,r}^* + \tan^2\beta_Q)^{-1/2} .$$

The final formulas for this case are obtained:

for vertical displacement of the scattering volume center

$$\Delta z'_Q \approx -(z_Q - z_0)\,\varepsilon_z / (\tan\alpha_t^* - \tan\alpha_r^* + \varepsilon_z) , \tag{14}$$

where

$$\varepsilon_z = \Delta\hat{T}(z_Q)\left(\frac{\tan\alpha_t^*}{\sin^2\xi_t^*} - \frac{\tan\alpha_r^*}{\sin^2\xi_r^*}\right) + \Delta\hat{v}_x(z_Q)\left(\frac{\sec^2\alpha_t}{\sin\xi_t^*} + \frac{\sec^2\alpha_r}{\sin\xi_r^*}\right) +$$

$$+\left(\Delta\hat{v}_y(z_Q)\tan\beta_Q - \frac{v_{z\,0}}{c_0}\right)\left(\frac{\tan\alpha_t^*}{\sin\xi_t^*} + \frac{\tan\alpha_r^*}{\sin\xi_r^*}\right) +$$

$$+\frac{v_{x\,0}}{c_0}\left(\frac{1}{\sin\xi_t^*} + \frac{1}{\sin\xi_r^*}\right);$$

for horizontal displacement of the scattering volume center

$$\Delta x'_Q \approx \Delta z'_Q \tan\alpha_t^* + (z_Q - z_0)\,\varepsilon_x , \tag{15}$$

$$\Delta y'_Q \approx \Delta z'_Q \tan\beta_Q + (z_Q - z_0)\,\varepsilon_y , \tag{16}$$

where

$$\{\varepsilon_x, \varepsilon_y\} = \Delta\hat{T}(z_Q)\frac{\tan\{\alpha_t^*, \beta_Q\}}{\sin^2\xi_t^*} + \{\Delta\hat{v}_x(z_Q), \Delta\hat{v}_y(z_Q)\}\frac{\sec^2\{\alpha_t^*,\beta_Q\}}{\sin\xi_t^*} +$$

$$+\{\Delta\hat{v}_y(z_Q), \Delta\hat{v}_x(z_Q)\}\frac{\tan\beta_Q\tan\alpha_t^*}{\sin\xi_t^*} +$$

$$+\frac{\{v_{x0}, v_{y0}\}}{c_0}\frac{1}{\sin\xi_t^*} + \frac{v_{z\,0}}{c_0}\frac{\tan\{\alpha_t^*,\beta_Q\}}{\sin\xi_t^*};$$

for refraction correction to the angle of scattering signal arrival

$$\Delta\beta_r \approx \frac{1}{2}\sin(2\beta_Q)\left\{\Delta\hat{T}(z_Q)\left(\frac{1}{\sin^2\xi_t^*} - \frac{1}{\sin^2\xi_r^*}\right) + \right.$$

$$+\Delta\hat{v}_x(z_Q)\left(\frac{\tan\alpha_t^*}{\sin\xi_t^*} + \frac{\tan\alpha_r^*}{\sin\xi_r^*}\right) - \left.\frac{v_{z\,0}}{c_0}\left(\frac{1}{\sin\xi_t^*} + \frac{1}{\sin\xi_r^*}\right)\right\} +$$

$$+\left(\Delta\hat{v}_y(z_Q) + \frac{v_{y\,0}}{c_0}\cos^2\beta_Q\right)\left(\frac{1}{\sin\xi_t^*} + \frac{1}{\sin\xi_r^*}\right); \tag{17}$$

for refraction correction to the scattering signal arrival time

$$\Delta\tau'_Q \approx \tau_Q\,[\,\varepsilon_\tau - \varepsilon_z(1 - \varepsilon_\tau)/(\tan\alpha_t^* - \tan\alpha_r^* + \varepsilon_z)\,] , \tag{18}$$

where

$$\varepsilon_\tau = P_T\,\Delta\hat{T}(z_Q) + P_x\,\Delta\hat{v}_x(z_Q) + P_y\,\Delta\hat{v}_y(z_Q) + \varepsilon_0;$$

$$P_T = \tan^2\beta_Q\sin\xi_r^*\left(\frac{1}{\sin\xi_t^*} - \frac{1}{\sin\xi_r^*}\right) +$$

$$+\left(\frac{\cot^2\xi_r^* - 1}{\sin\xi_t^*} + \frac{\cot^2\xi_r^* - 1}{\sin\xi_r^*}\right)\bigg/\left(\frac{1}{\sin\xi_t^*} + \frac{1}{\sin\xi_r^*}\right);$$

$$P_x = \frac{\tan\alpha_t^*\cot^2\xi_t^* - \tan\alpha_r^*\cot^2\xi_r^* + \tan^2\beta_Q\sin\xi_r^*\left(\dfrac{\tan\alpha_t^*}{\sin\xi_t^*} + \dfrac{\tan\alpha_r^*}{\sin\xi_r^*}\right)}{\dfrac{1}{\sin\xi_t^*} + \dfrac{1}{\sin\xi_r^*}};$$

$$P_y = -\tan\beta_Q\left[\frac{1}{\sin\xi_t^*} - \frac{1}{\sin\xi_r^*} + \sec^2\beta_Q\sin\xi_r^*\right];$$

$$\varepsilon_0 = \tan\beta_Q\sin\xi_r^*\frac{v_{y0}}{c_0} - \left(\tan^2\beta_Q\sin\xi_r^* + \frac{1}{\sin\xi_t^*} - \frac{1}{\sin\xi_r^*}\right)\frac{v_{z0}}{c_0};$$

and, for refraction correction to the sound scattering angle

$$\Delta\theta \approx \left\{\tan\beta_Q\,\Delta\beta_r\left[\sec^2\beta_Q\sin^2\xi_t^* - \frac{\tan\alpha_t^*}{\tan\alpha_t^* - \tan\alpha_r^*}\right] + \right.$$

$$+\sin^2\beta_Q\left[g_t(z_Q) + g_r(z_Q)\right] +$$

$$+\cos^2\beta_Q\left[\frac{g_t(z_Q)(\tan\alpha_t^* + \tan\alpha_r^*\cot^2\xi_t^*)}{\tan\alpha_t^* - \tan\alpha_r^*} - \right.$$

$$\left.\left. - \frac{g_r(z_Q)(\tan\alpha_r^* + \tan\alpha_t^*\cot^2\xi_r^*)}{\tan\alpha_t^* - \tan\alpha_r^*}\right]\right\}\tan\theta_0, \tag{19}$$

where

$$g_{t,r}(z) = \frac{\Delta T(z)}{2T_0} \pm$$

$$\pm\left\{\frac{\Delta v_x(z)}{c_0}\tan\alpha_{t,r}^* + \frac{\Delta v_y(z)}{c_0}\tan b_Q^* + \frac{\Delta v_z(z)}{c_0}\right\}\sin\xi_{t,r}^* .$$

The main peculiarity of bistatic sounding as compared to monostatic one is the substantial contribution of temperature profile of the order of ε^1 to angular parameters. Another important feature consists in that the final formulas for x'_Q, y'_Q, z'_Q, and τ'_Q cannot consistently be linearized with respect to ΔT and ν or, more particularly, with respect to ε_z even though the condition $|\xi_{t,\,r}| \gg \sqrt{\varepsilon}$ is fulfilled. The numerical estimates indicate that the trigonometric coefficients for $\Delta\hat{\nu}_x(z_Q)$ and ν_{x0}/c_0 in the expressions for ε_z in Eq. (14) can be much larger than unity at certain bistatic geometries. As a result, when the wind is strong and longitudinal, $\nu_x \geq 10$ m, the parameter ε_z becomes comparable, by magnitude, with the difference $\tan\alpha^*_t - \tan\alpha^*_r$. Therefore, further simplifications of the formulas (14)–(16) and (18) by neglecting the terms of the order of $(\varepsilon_z)^2$, $(\varepsilon_z\varepsilon_\tau)^1$ and higher are possible only when the condition $(\varepsilon_z/(\tan\alpha^*_t - \tan\alpha^*_r)| \ll 1$ holds. When the value ε_z is fixed this inequality is violated with simultaneous decrease of $|\alpha^*_t|$ and $|\alpha^*_r|$ or accordingly increase of $|\xi_t|$ and $|\xi_r|$. By this is meant that the applicability limits of linearization with respect to ε in the final refraction formulas for bistatic geometry parameters are narrower than those in the initial equations (3) and (4).

It should be noted that in the literature there is no evidence for usage of slant bistatic geometry in the experimental studies of the atmosphere. The account of the case $\beta_Q \neq 0$ in Eqs. (14)–(19) is necessary mainly to the development of algorithms for estimating the effect of refraction on the other, nongeometric, parameters of the AR signal. For example, in the AR signal power calculations one must carry out the integration over the scattering volume. In this case it is divided into a great number of elementary volumes, the most part of which is always related to the case of $\beta_Q \neq 0$. In these calculations it is necessary to assess the different characteristics of signals appearing during scattering of sound in each elementary volume. Such calculations, without considering the refraction, can be found in Refs. 12 and 13.

When, with $\beta_Q = 0$, the effect of refraction only on geometric parameters must be assessed, formulas (14)–(19) are simplified substantially. In addition it is possible to take into account the fact that the value of ν_z is usually small as compared to ν_h. When $\beta_Q = 0$ and $\nu_z = 0$, very simple relations for angular parameters are derived:

$$\Delta\beta_r \approx \hat{\nu}_y(z_Q)\,(\sec\alpha^*_t + \sec\alpha^*_r)\,;$$

$$\Delta\theta \approx -\frac{\Delta T(z_Q)}{2T_0}(\tan\alpha^*_t - \tan\alpha^*_r) - $$
$$-\frac{\Delta\nu_x(z_Q)}{c_0}(\tan\alpha^*_t\sin\alpha^*_t + \tan\alpha^*_r\sin\alpha^*_r)\,.$$

Comparing our formulas with the known ones, we can note that the relations for $\Delta x'_Q$ and $\Delta z'_Q$ in Ref. 2 differ from Eqs. (14) and (15) by the value of one order of smallness with the considered terms (ε^1). The formula for $\Delta\beta_r$ in our earlier paper[9] fully coincides with Eq. (17) and for $\Delta\tau'_Q$ it coincides with the relation

$$\Delta\tau'_Q \approx \tau_Q\left[\varepsilon_\tau - \varepsilon_z/(\tan\alpha^*_t - \tan\alpha^*_r)\right],$$

which is valid when the condition $|\varepsilon_z/(\tan\alpha^*_t - \tan\alpha^*_r)| \ll 1$ holds.

The refraction corrections to the bistatic geometry parameters were calculated using formulas (14)–(19). In so doing the profiles $T(z)$ and $\nu(z)$ were specified similarly to the case of monostatic sounding. Below the calculational results are represented for rectangular geometry which is most frequently used in bistatic ARs. In this geometry the distance between the antennas d does not change; the PD axis of one of the antennas (in our case it is a transmitting one) is constantly oriented vertically ($\alpha^*_t = 0$, $\beta_Q = 0$); and the sounding altitude, without considering the refraction z_Q, is specified by the changeable angle of the PD axis tilt of the second antenna ($\alpha^*_r = \alpha^*_r(z_Q)$). In this geometry for small values of z_Q, the scattered beam possesses a small slant angle over the horizon. As a result, the condition $|\xi_r| \approx \pi/2 - |\alpha^*_r| \ll \varepsilon^{1/2}$ of applicability of linear approximation over ε to its equation $\mathbf{R}_r(z)$ can be unreal. Therefore, the calculational results obtained from these formulas at these altitudes are liable to breakdown. To exclude this case, the calculations for altitudes lower than 100 m ($\xi_r < 18.5°$) were not carried out.

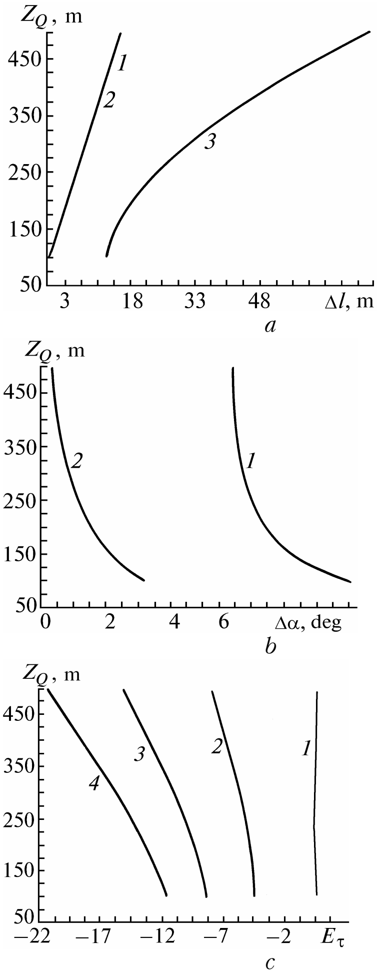

FIG. 2. Refraction variations in bistatic geometry parameters as a function of sounding altitude z_Q for $\varphi_\nu = 225°$, $z_{rou} = 2$ cm, $T_m = 20°C$, $\gamma = -6.5°/km$, $z_m = 1$ m, $\alpha^*_t = 0°$, $\beta_Q = 0°$, $d = 300$ m, and $z_0 = 2$ m. a) at $\nu_m = 10$ m/s $\Delta l = \Delta x'_Q$ (1), $\Delta l = \Delta y'_Q$ (2), and $\Delta l = \Delta z'_Q$ (3); b) at $\nu_m = 10$ m/s $\Delta\alpha = \Delta\alpha_r$ (1), $\Delta\alpha = \Delta\beta_r$ (2), and $\Delta\alpha = -\Delta\theta$ (3); c) at $\nu_m = 0$ (1), 5 (2), 10 (3), and 15 m/s (4).

Depicted in Figs. 2 *a* and *b* are the values $\Delta x'_Q$, $\Delta y'_Q$, $\Delta z'_Q$, $\Delta\beta_r$, and $\Delta\theta$ as functions of z_Q at v_m = 10 m/s. These plots differ qualitatively from the analogous ones for monostatic sounding. This is accounted for by the fact that in the calculational formulas there are coefficients varying as functions of altitude z_Q in combination with the angle α_r^*. As a rule, they decrease by absolute value with the increase of z_Q. For this reason in Fig. 2 *b* $\Delta\beta_r$ and $\Delta\theta$, in formulas for which there is no multiplier $z_Q - z_0$, also decrease by absolute value with the increase of z_Q. Formula (14) for $\Delta z'_Q$ is a special case where the decrease of the α_r^* angle with the increase of z_Q leads to violation of the inequality $|\varepsilon_z/\tan\alpha_r^*| \ll 1$. As a result, starting from some values of z_Q, the parameter $|\varepsilon_z/\tan\alpha_r^*|$ is not a small value of the order of ε^1. Therefore, very large values of $\Delta z'_Q$ can be observed (see Fig. 2 *a*). It should be noted that the curves in Figs. 2 *a* and *b* can also be used for different values of v_m not equal to 10 m/s taking account of proportionality of refraction corrections to this value. Nonlinearity of formulas (14)–(16) and (18) with respect to v does not manifest strongly in this case.

Figure 2 *c* depicts the z_Q–dependences of relative values $E_\tau = \Delta\tau'_Q/\tau_Q \cdot 100$ of refraction displacement of the scattered signal arrival time in per cent, which differ by different values of v_m. Curve *1* in this figure relates to the case of refraction due to temperature gradient only. In this case the values $|E_\tau|$ do not exceed one per cent. In the presence of wind the value $|E_\tau|$ sharply increases attaining the value of 20% at maximum altitude z_Q = 500 m and v_m = 15 m/s. What this means is a dominating effect of wind in atmospheric refraction of sound.

The behavior of curves as a function of z_Q depicted in Fig. 2 with other bistatic geometries can differ in some way. However, the absolute values of refraction corrections coincide by the order of magnitude.

ACCURACY CHARACTERISTICS OF REFRACTION FORMULAS

It is interesting to estimate numerically the accuracy in describing the effect of refraction on geometry of acoustic sounding with formulas of linear approximation over ε (in those cases where the condition of their applicability $|\xi_{t,r}| \gg \sqrt{\varepsilon}$ holds). To this end, the aforementioned calculations of refraction corrections to geometric parameters were also carried out based on accurate equations (1)–(5) and (7). In this case Eqs. (1) and (3) were reduced to two scalar equations: $x_t(z'_Q) = x_r(z'_Q)$ and $y_t(z'_Q) = y_r(z'_Q)$. The system of three equations incorporating both this pair of equations and Eq. (2) was computed on a personal computer with respect to three parameters the choice of which depended on the used version of refraction problem statement. The values z'_Q, α_r, β_r and z'_Q, β_r, τ'_Q were found for monostatic and bistatic sounding, respectively.

The method of dichotomy[14] (halving the segment) was used for numerical solution of accurate equations with respect to z'_Q and angular parameters. The initial boundaries of searching real values of these parameters were specified taking account of the value $|\varepsilon|$ in the vicinity of their values when there was no refraction. A need for multiple calculation of integrals of the type (3) and (4) during realization of this algorithm made the computational time much longer. This, in its turn, restricted the real possibilities of decreasing the calculational errors. In the case of monostatic geometry, we used a three–fold method of dichotomy for simultaneous determination of the three unknown parameters. The accuracies of calculations of z'_Q and angles α_r and β_r were specified as 5 cm and 0.05°, respectively. When calculating the bistatic geometry, we used only a two–fold method of dichotomy for simultaneous determining the values z'_Q and β_r. Here the third unknown parameter τ'_Q is described by Eqs. (2) and (4) in an explicit form. Therefore, the higher calculational accuracy was specified here: for z'_Q – better than 1 cm and for β_r – better than 0.01°. After calculations of these parameters we found the values $x_t(z'_Q)$ and $y_t(z'_Q)$ from Eq. (3) and the angle $\theta(z'_Q)$ from Eqs. (5) and (7). The absolute errors in estimating the parameters were determined as a difference between their values calculated from approximate and accurate formulas ($\delta x = x_L - x_T$).

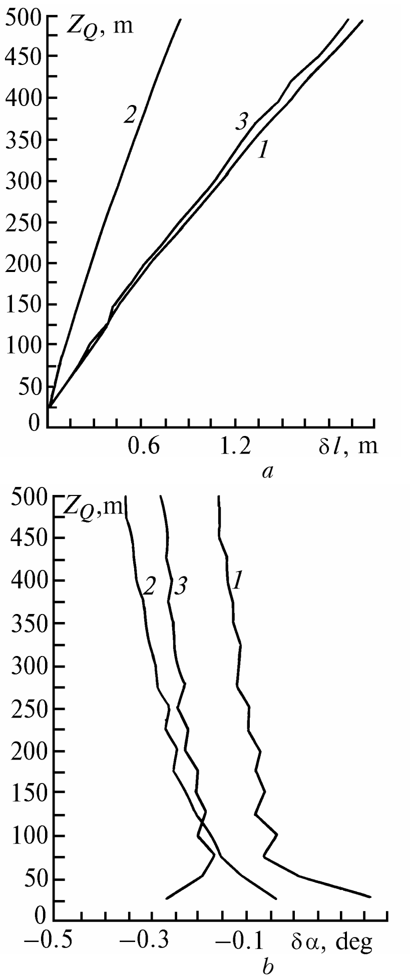

FIG. 3. Errors in estimating the refraction changes in the monostatic geometry parameters via formulas of linear approximation as a function of sounding altitude z_Q *for* v_m= 10 m/s, φ_v= 225°, z_{rou}= 2 cm, T_m= 20°C, γ = –6.5°/km, z_m = 1 m, α = 30°, and z_0 = 2 m. *a)* $\delta l=\Delta x'_L-\Delta x'_T$ *(1),* $\Delta y'_L-\Delta y'_T$ *(2), and* $\Delta z'_L-\Delta z'_T$ *(3); b)* $\delta\alpha=\Delta\alpha_{r\,L}-\Delta\alpha_{r\,T}$ *(1),* $\Delta\beta_{r\,L}-\Delta\beta_{r\,T}$ *(2), and* $\Delta\theta_L-\Delta\theta_T$ *(3).*

Figure 3 depicts the calculational results of these errors for monostatic sounding. Here the errors in determining the refraction displacements of the scattering volume center coordinates $\delta x'_Q$, $\delta y'_Q$, and $\delta z'_Q$ turn out to be proportional to $z_Q - z_0$ and v_m as well as their values. When $v_m = 10$ m/s and $z_Q = 500$ m the value $\delta z'_Q$ in Fig. 3*a* is maximum, i.e., 2 m, that is not more than 10% of $\Delta z'_Q$ (see Fig. 1 *a*) or 0.6% of z'_Q. Thus, the relative error in determining z'_Q from the approximate formulas is about $0.6 \cdot 10^{-2}\, v_m$ (in %).

The calculations of $\delta\alpha_r$, $\delta\beta_r$, and $\delta\theta$ depicted in Fig. 3*b* did not reveal somewhat marked z_Q–dependence for them. Their values at $v_m = 10$ m/s were about 0.1–0.2°. Since these values did not substantially exceed the calculational errors in α_r, β_r, and θ using the accurate formulas (0.05°), then these errors are seen as fluctuations of the curves in Fig. 3*b*. It turned out that the relation $\delta\alpha \approx 2 \cdot 10^{-2}\, v_m$ (in degrees) can be used for rough estimation of the absolute errors in determining the angular parameters.

The calculational results of accuracy characteristics of approximate formulas for bistatic sounding are represented in Fig. 4. It was found for rectangular geometry that $\delta x'_Q$ and $\delta y'_Q$ are very small (not more than 20 cm), and practically undependent on z_Q at altitudes between 100 and 500 m (see Fig. 4*a*). This is accounted for by the fact that in rectangular geometry there is practically no refraction distortion of a vertical beam ($\xi_t = \pi/2$). The value $\delta z'_Q$ in this case decreases monotonically with z_Q increase from 1.7 to –1.6 m and passes zero when $z_Q \approx d/2$. This behavior of the error $\delta z'_Q$ is caused by the effect of $z_Q - z_0$ multiplier in Eq. (14) at high altitudes z_Q and by weakening of inequality $\xi_r \gg \sqrt{\varepsilon}$ at lower altitudes. The errors in determining the angular parameters $\delta\beta_r$ and $\delta\theta$ do not usually exceed 0.3° by absolute value in Fig. 4*b*. The exception is the $\delta\beta_r$ behavior at $z_Q < 150$ m where the constraints on applicability of expansion over ε to the equation for a scattered beam can start their effect in estimating β_r. With the wind velocity v_m decrease the errors depicted in Figs. 4*a* and *b* reduce proportionally.

Figure 4*c* represents a relative error in estimating the scattered signal arrival time via the approximate formulas $\delta E_\tau = (\tau'_{QL} - \tau'_{QT}) \cdot 100$ at different wind velocities v_m. In the presence of refraction caused only by temperature gradient this error at all of the calculated altitudes proved to be practically zero. When there was wind at altitude $z_Q \approx d/2$, it was also close to zero, and at altitudes $z_Q > d/2$ it increases with the z_Q increase proportionally to $v(z_Q)$ (according to the logarithmic law). It is seen from the comparison between the values δE_τ in Fig. 4*c* and E_τ in Fig. 2*c* at $z_Q = 500$ m for different v_m that the relation $P_{\tau\max} \approx 0.25 \cdot v_m$ (in %) is valid for an approximate estimate of the maximum value of the parameter $P_\tau = |\delta\tau'_Q / \Delta\tau'_Q| \cdot 100$ (in %) which characterizes a relative error in determining the refraction displacement $\Delta\tau'_Q$.

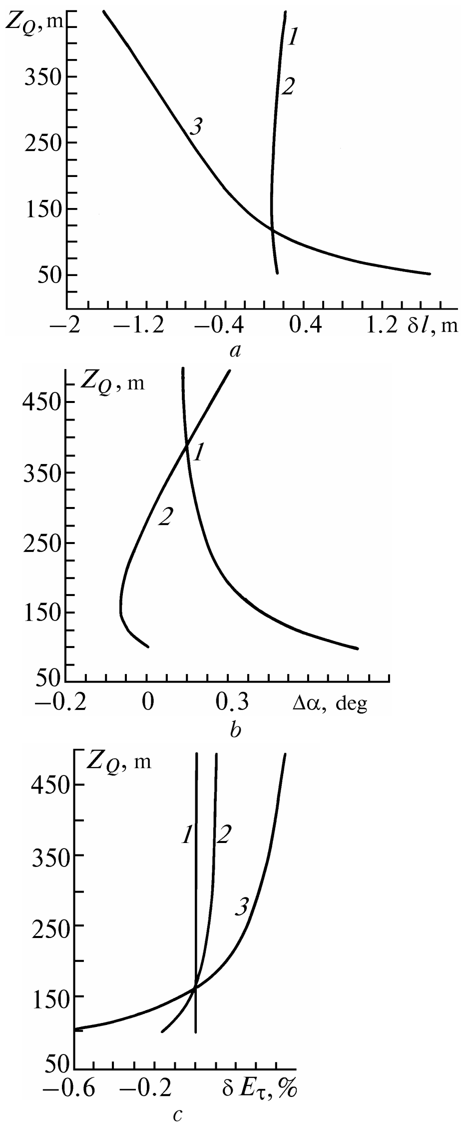

FIG. 4. Errors in estimating the refraction changes in the bistatic geometry parameters via formulas of linear approximation as a function of sounding altitude z_Q for $\varphi_v = 225°$, $z_{rou} = 2$ cm, $T_m = 20°$C, $\gamma = -6.5°$/km, $z_m = 1$ m, $\alpha^*_t = 0°$, $\beta_Q = 0°$, $d = 300$ m; $z_0 = 2$ m. *a) at* $v_m = 10$ m/s $\delta l = \Delta x'_{QL} - \Delta x'_{QT}$ *(1),* $\Delta y'_{QL} - \Delta y'_{QT}$ *(2), and* $\Delta z'_{QL} - \Delta z'_{QT}$ *(3); b) at* $v_m = 10$ m/s, $\delta\alpha = \Delta\beta_{r\,L} - \Delta\beta_{r\,T}$ *(2), and* $\delta\alpha = \Delta\theta_L - \Delta\theta_T$ *(3); and, c) at* $v_m = 0$ *(1),* 5 *(2),* 10 *(3),* and 15 m/s *(4).*

CONCLUSION

In this paper we present a system of accurate equations which takes into account the relations of geometric acoustic of stratified moving medium which enables one to analyze and assess numerically the effect of refraction on parameters of acoustic sounding geometry of the atmosphere. After linearization of accurate equations with respect to $|\Delta c|/c_0$ and v/c_0 we derived the analytical solution for these parameters which are valid for any real temperature profile and wind velocity in the atmosphere and sounding geometry with the exception of small altitudes of sounding using bistatic geometry. They provide, within their applicability limits, a good accuracy in estimating these parameters at calculational time shorter by one–two orders of magnitude than the interval between two consecutive transmission of sounding acoustic pulses into the atmosphere. Therefore, they can be used in algorithms for processing the experimental data of AR in real time. Since the effect of temperature refraction is relatively weak one should take into account the refraction caused by wind alone.

At the same time, to solve these problems directly from an accurate equations, we must employ the numerical methods. As a result, the geometric parameters obtained in this case are also approximate. It should be noted that the equal accuracy in estimating these parameters from the accurate formulas requires the calculational time by two orders of magnitude longer than that for the estimates from the approximate formulas. The decrease of this time by a factor of about n results in the n^3– and n^2 –fold increase of errors in estimating the required parameters for monostatic and bistatic sounding, respectively. For this reason, the numerical solution of accurate equations can be used only in some theoretical studies.

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