# EFFECT OF WIND VELOCITY FLUCTUATIONS ON THE SPATIOTEMPORAL STRUCTURE OF AEROSOL LIDAR RETURNS

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*An analytic study has been carried out of the influence of the wind velocity fluctuations on the cross–correlation fluctuation characteristics of aerosol lidar returns. The influence of the degree of anisotropy of the spatial and temporal wind velocity fluctuations is analyzed. It is shown that for strong fluctuations of wind velocity the phase spectrum becomes nonlinear.*

The spatiotemporal structure of lidar returns is used for the study of dynamics of atmospheric processes and for remote sensing of the wind velocity field. Until quite recently laser radar measurements of the wind velocity used primarily the correlation technique[1] based on general considerations of correlation between the spatiotemporal signal structure and wind velocity field. A more realistic model of the spatiotemporal fluctuations of the wind velocity was used in Ref. 2. In that paper a conclusion that spectral processing of signals was preferred for retrieval of the mean wind velocity was made and, in particular, the stability of the phase spectrum of signals with respect to the small wind fluctuations was pointed out. However, in Ref. 2 an isotropic model of the spatiotemporal structure of the wind velocity field was considered. According to modern theoretical and experimental data[3] the spatial spectrum of the wind velocity fluctuations in the boundary atmospheric layer is anisotropic (tensorial) with parameters depending on stratification of the atmosphere and being the subject of investigation.

The variations of the echo–signal power are assumed to be determined by the fluctuations of particle concentration in the scattering volume, and the hypothesis of local frozen turbulence[4] is assumed to be true for the field of concentration. On this assumption the expression for the spatiotemporal correlation function was obtained in Ref. 2 in the form

$$B(\mathbf{r},\tau) = A \int d^3r_1\, d^3r_2\, d^3\kappa\, f(\mathbf{r}_1) f(\mathbf{r}_2 + \mathbf{r}) \times$$

$$\times\, \Phi_N(\kappa) \exp\left[i\kappa(\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{v}\tau)\right], \qquad (1)$$

where $A = a^2 (TL^{-1})^4\, \bar{\sigma}^2$, $a$ is the instrumental constant determined by the parameters of a transmitting lidar system, $T$ is the transparency of a sounded layer, $L$ is the path length, is the backscattering cross section averaged over particle size, $f(\mathbf{r}) = (2\pi a_\nu^2)^{-3/2} \exp(-r^2/2a_\nu^2)$ is the filtering function determining the dimensions of an isotopically scattering volume $a_v$, $\mathbf{r}$ is the distance between the volume centers, $\Phi_N(\kappa) = 0.033\, C_N^2\, \kappa^{-11/3}\left[1 - \exp(-\kappa^2 \kappa_0^{-2})\right]$ is the spatial spectrum of the concentration fluctuations, $\kappa_0 = 2\pi/L_0$, $L_0$ is the turbulence outer scale, $\mathbf{v}_0$ is the wind velocity, and $\tau$ is the time delay.

Then the Cartesian components of the wind velocity $\nu_x$, $\nu_y$, and $\nu_z$ are assumed to be distributed according to the normal law with the mean values $\nu_{0x}$, $\nu_{0y}$, and $\nu_{0z}$ and variances $\sigma_x^2$, $\sigma$, and $\sigma$. Upon integrating over the spatial variables $\mathbf{r}_1$ and $\mathbf{r}_2$ in Eq. (1) and by averaging over the wind velocity fluctuations we obtained

$$B(\mathbf{r},\tau) = A \int d^3\kappa\, \Phi_N(\kappa) \exp\left[-\left(a_\nu^2 + \frac{1}{2}\sigma_\nu^2\tau^2\right)\kappa^2 + i(\mathbf{r} - \mathbf{v}_0\tau)\kappa\right], \qquad (2)$$

where $\nu_0^2 = \nu_{0x}^2 + \nu_{0y}^2 + \nu_{0z}^2$ and $\sigma_v^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2$.

In the majority of meteorological situations, an essential distinction between the variances of the horizontal and vertical wind velocity fluctuations is observed. To estimate an effect of this factor on the cross correlation function, the case in which the vertical wind velocity fluctuations are different from the fluctuations in the horizontal plane will be considered, i.e., $\sigma_x = \sigma_y = \sigma \neq \sigma_z$. In most cases the horizontal velocities that are of principal interest for applications are much larger than the vertical velocity of air flows. Therefore, to simplify calculations, the vertical component of the mean wind velocity $v_{0z}$ is assumed to be small as compared with the horizontal one and can be neglected.

Transferring to the spherical coordinate system in Eq. (2) and integrating over all variables, we obtain an expression for the normalized spatiotemporal correlation function of backscattered radiation in the form

$$\frac{B(\mathbf{r},\tau)}{B(0,0)} = B^{-1}\left\{ \frac{(a_\nu^2 + k_0^{-2} + \frac{1}{2}\sigma^2\tau^2)^{5/6}}{(a_\nu^2 + k_0^{-2} + \frac{1}{2}\sigma_z^2\tau^2)^{1/2}} \times \right.$$

$$\times\, \theta_1\left(\frac{1}{2}, -\frac{1}{3}, \frac{11}{6}; \frac{3}{2}; \frac{-\frac{1}{2}(\sigma^2 - \sigma_z^2)\tau^2}{a_\nu^2 + k_0^{-2} + \frac{1}{2}\sigma_z^2\tau^2}, \frac{-(\mathbf{r} - \mathbf{v}_0\tau)^2}{4(a_\nu^2 + k_0^{-2} + \frac{1}{2}\sigma^2\tau^2)}\right) -$$

$$- \frac{(a_\nu^2 + \frac{1}{2}\sigma^2\tau^2)^{5/6}}{(a_\nu^2 + \frac{1}{2}s_z^2\tau^2)^{1/2}} \times$$

$$\left. \times\, \theta_1\left(\frac{1}{2}, -\frac{1}{3}, \frac{11}{6}; \frac{3}{2}; \frac{-\frac{1}{2}(\sigma^2 - \sigma_z^2)\tau^2}{a_\nu^2 + \frac{1}{2}\sigma^2\tau^2}, \frac{-(\mathbf{r} - \mathbf{v}_0\tau)^2}{4(a_\nu^2 + \frac{1}{2}\sigma^2\tau^2)}\right)\right\}, \qquad (3)$$

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where $B = (a_v^2 + \kappa_0^{-2})^{1/3} - a_v^{2/3}$ and $\theta_1(a, a', b; c; t, x)$ is the hypergeometric function of two variables.[5] We note that when the variances of the wind velocity components are the same in Eq. (3), i.e., $\sigma_z = \sigma$, then $t = 0$ and $\theta_1(a, a', b; c; 0, x) = {}_1F_1(a'; c; x)$. In this case Eq. (3) simplifies and takes the form

$$\frac{B(\mathbf{r},\tau)}{B(0,0)} = B^{-1}\left\{ (a_v^2 + k_0^{-2} + \frac{1}{2}\sigma^2\tau^2)^{1/3} \times {}_1F_1\left(-\frac{1}{3}; \frac{3}{2}; \frac{-(\mathbf{r}-\mathbf{v}_0\tau)^2}{4(a_v^2+k_0^{-2}+\frac{1}{2}\sigma^2\tau^2)}\right) - (a_v^2+\frac{1}{2}\sigma^2\tau^2)^{1/3}\, {}_1F_1\left(-\frac{1}{3};\frac{3}{2};\frac{-(\mathbf{r}-\mathbf{v}_0\tau)^2}{4(a_v^2+\frac{1}{2}\sigma^2\tau^2)}\right)\right\}, \quad (4)$$

where ${}_1F_1(a; c; x)$ is the confluent hypergeometric function.[5] The formula (4) coincides with the expression for the case of the spatial isotropic fluctuations in the wind velocity.

The behavior of the ratio $B(\mathbf{r}, \tau)/B(0, 0)$ calculated by formula (3) for two limiting cases $\sigma_z = 0$ and $\sigma_z = \sigma = 1$ m/s for the values $a_v = 1$ m, $L_0 = 80$ m, $r = 5$ m, and $v_0 = 1$ m/s is shown in Fig. 1. It is seen that anisotropy in the wind velocity fluctuations (curve *1*) influences the cross correlation function in the following manner: a decrease in the variance of the wind velocity vertical component $\sigma$ as compared with the horizontal one $\sigma^2$ leads to a displacement of the correlation maximum toward larger $\tau$, causes broadening of the correlation function and increase in the maximum degree of correlation as compared with the case of the isotropic fluctuations (curve *2*).

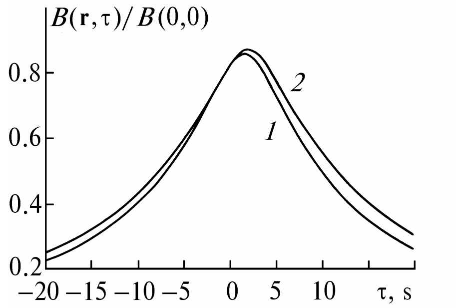


*FIG. 1. Effect of anisotropy in the wind velocity fluctuations on the cross correlation function of lidar returns: 1)* $\sigma_x = \sigma_y = \sigma_z = \sigma$ *and 2)* $\sigma_x = \sigma_y = \sigma$ *and* $\sigma_z = 0$.

Another technique for lidar return signal processing to retrieve the wind velocity is based on the use of the coherent analysis methods. In Ref. 2 an account of the wind velocity fluctuation effect on the spectral characteristics of lidar return signals was carried out for the asymptotic case $\sigma^2/v \ll 1$. However, for a number of practical situations the ratio $\sigma^2/v$ can be of the order of 1 or more. An account of the wind velocity fluctuations for an arbitrary ratio $\sigma/v_0$ in the cross correlation spectrum $W(\mathbf{r}, \omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} B(\mathbf{r}, \tau) \exp(-i\omega\tau)\, d\tau$ leads to the expression

$$W(\mathbf{r}, \omega) = A\cdot 0.033\, \pi\, C_N^{-2} \frac{1}{v_0} \exp(-i\omega\, r/v_0) \times \int_0^{\infty} \kappa\, d\kappa\, \Phi_N(\kappa) \exp\left[-\left(a_v^2 + \frac{1}{2}\frac{\sigma^2 r^2}{v_0^2}\right) k^2\right] \times \left\{ \mathrm{erf}\left(\frac{v_0}{\sqrt{2}\,\sigma} - \frac{\omega}{\sqrt{2}\,k} + i\frac{\sigma k\, r}{\sqrt{2}\, v_0}\right) + \mathrm{erf}\left(\frac{v_0}{\sqrt{2}\,\sigma} + \frac{\omega}{\sqrt{2}\,k} - i\frac{\sigma k\, r}{\sqrt{2}\, v_0}\right)\right\}, \quad (5)$$

where $\omega = 2\pi f$ and $f$ is the frequency, in Hz. (Here for simplicity the distance between the scattering volumes $\mathbf{r}$ is oriented along the direction of the mean wind velocity $v_0$ and $\sigma_x = \sigma_y = \sigma_z = \sigma$.)

The coherence spectrum

$$\gamma(\mathbf{r}, \omega) = |\, W(\mathbf{r}, \omega)| \,/\, |\, W(0, \omega)|$$

and the phase spectrum

$$\varphi(\mathbf{r}, \omega) = \arctan \frac{\mathrm{Im}\; W(\mathbf{r}, \omega)}{\mathrm{Re}\; W(\mathbf{r}, \omega)}$$

are shown in Fig. 2. The theoretical curves in Fig. 2 have been obtained by numerical integrating of expression (3) for $a_v = 0.2$ m, $L_0 = 80$ m, and $r = 5$ m. Curves *1–3* describe the decrease of $\gamma^2(\mathbf{r}, \omega)$ due to the wind velocity fluctuations. In the region of low frequencies the coherence spectrum is saturated at a level depending on the ratio $3\sigma^2/v$, and in the region of high frequencies it tends to zero fast. We note that without wind velocity fluctuations the coherence spectrum does not depend on the frequency and equals to 1, while the phase spectrum varies linearly (see curves *4'* and *5'*). The wind velocity fluctuations cause steeper slope of the phase spectrum proportional to the value $1 + 3\sigma^2/v$ (see curves *1'*, *2'*, and *3'*). Moreover, for large ratios $\sqrt{3}\sigma/v_0$ (curve *2*) a pronounced nonlinear behaviour of $\varphi(\mathbf{r}, \omega)$ is observed.

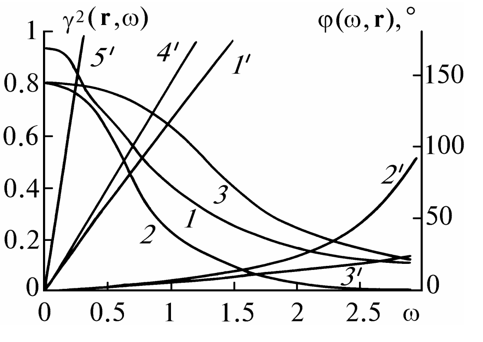


*FIG. 2. Phase and coherence spectra for strong wind velocity fluctuations: 1–3) coherence spectra and 1'–5') phase spectra. a)* $v_0 = 2$ m/s: *4')* $\sqrt{3}\sigma/v_0 = 0$; *1 and 1')* $\sqrt{3}\sigma/v_0 = 0.5$; *3 and 3')* $\sqrt{3}\sigma/v_0 = 5$; *b)* $v_0 = 0.5$ m/s: *5')* $\sqrt{3}\sigma/v_0 = 0$, *2 and 2')* $\sqrt{3}\sigma/v_0 = 10$.

Thus, the above studies allow a conclusion that spatiotemporal anisotropy in the wind velocity fluctuations affects appreciably the shape of the correlation function of lidar returns. What is more, in parallel with a decrease of the degree of coherence, the wind velocity fluctuations cause an essential change in the phase spectrum slope. Therefore, to increase the accuracy of measurements of the wind velocity characteristics, allowance must be made for the factors mentioned above when interpreting the data of lidar sounding.

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