STATISTICAL ANALYSIS OF THE WAVE FRONT DISTORTIONS IN THE ATMOSPHERE BY HARTMANN FILM

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Idea of representing atmospheric distortions of wave front by a phase screen moving as a whole relative to the aperture is discussed. A technique for determining the probability of the phase screen existence by means of Hartmann film is proposed.

It is characteristic of modern astronomic optics that in order to improve resolution of telescopes the adaptive optics means are used more and more frequently. Creation of effective adaptive systems for the control of atmospheric turbulence requires reliable statistical information about it at the place where the adaptive system is installed. This information determines the requirements to the basic elements of such an adaptive system.

It is important to establish what class of the wave front distortions does the atmosphere introduce and what are the spatial and temporal characteristics of the distorted wave fronts. Investigations into this problem are still urgently needed.

The time of "freezing" of the atmosphere during which the spatial distribution of the atmospheric distortions (AD) could be assumed fixed is an important conception. The freezing time determines the time needed to correct for the AD, and it is equal to 0.01 in accordance with Ref. 1.

Other situation can be assumed, when the atmosphere is considered as a certain phase screen (or a package of screens) which moves as a whole relative to an optical system.

If the hypothesis of the phase screen is valid, having the information on the screen movement relative to an aperture the phase at the point M of the aperture at an instant t along the trajectory of the screen point N_0 which coincides with the point M_0 at the instant t_0 can be determined from the known phase $\Phi(M_0, t_0)$ at the point M_0 of the aperture at the instant t_0.

Hartmann film records the dynamics of the wave front AD, therefore it might contain information about the phase screen, if the latter exists.

In this paper we propose a technique for determination of the existence probability for the phase screen from Hartmann film. Specifically, the case in hand is the Hartmann film recorded at a frequency 48 frames/s with Zeiss–600 telescope of Simeiz scientific base of the former Astrocouncil of the USSR Academy of Sciences. Local wave front tilts due to the atmosphere were observed at 28 points of the aperture (Fig. 1). These points are located at two outlines spaced by 0.1 m. The first results of processing of this film being the temporal correlation characteristics of the Zernike series coefficients by which a two–dimensional distribution of the wave front AD over aperture was approximated are presented in Ref. 2.

In our experiment we have processed 200 frames. Coordinates of the spot intensity centers on the film were determined with a microphotometer IFO–451. Measurement error along the Y–axis was 0.005 mm (with the scale length of displacement indicator of 0.01 mm), and along the X–axis it was 0.025 mm. Since the accuracy along the X–axis was low, processing was carried out solely for the measurements in Y direction.

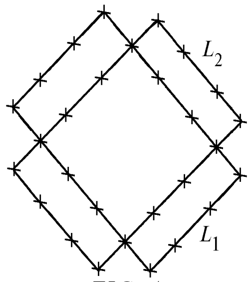

FIG. 1.

Deviation of an intensity center of the ith spot on the film from its theoretical value due to AD was determined in the kth frame by the relation

$$\Delta y(i, k) = y(i, k) - \sum_{j=1}^{200} y(i, j) / 200, \quad k = 1, \ldots, 200.$$

The deviation $\Delta y(i, k)$ determines the wave front local tilt caused by AD in Y direction of ith subaperture to an accuracy of the scale factor.

PROCESSING SCHEME

Figure 2 presents the contour L_1 with 16 subapertures. At an instant $t_k = k\,\Delta t$ ($\Delta t = 1/48$) a movement of the phase screen was specified by the vector **V** being determined by the angle α and the value V can be both negative and positive. For the positive α, as is shown in Fig. 2, the deviation $\Delta y(M_i, t_k)$ at the point M_i of the contour ABC was compared with the deviation $\Delta y(N_i, t_k + \tau_i)$ at the point N_i of the contour ADC at the instant $t_k + \tau_i$ where a lag (advance) τ_i was determined by the equality $\tau_i = |M_i N_i| / V$. Because the point N_i is not necessarily coincident with the center of some subaperture of the contour ADC, and the lag is not necessarily multiple of Δt, then calculation of the deviation $\Delta y(N_i, t_k + \tau_i)$ was made using linear interpolation along the contour and in time. If $\alpha = 0$, points on AB were compared with the points on DC in the same way. For negative α, points from the contour DAB were compared with the points on the contour DCB.

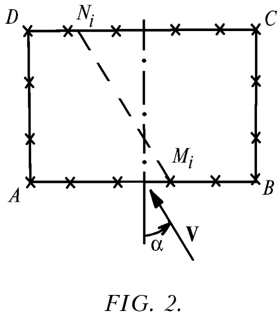

FIG. 2.

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As to the contour L_1 the comparison was made for the angles $|\alpha| \ll \pi/4$. Analogous comparison of deviations was carried out for points from the contour L_2. These versions of the comparison of deviations cover the whole range of the vector **V** variations.

If at instant t_k the phase screen takes place then for a certain vector $\mathbf{V}_k$ the discrepancy

$$\varepsilon^2(\mathbf{V}) = \sum_{i=1}^{n} (\Delta\, y(\, M_i,\ t_k) - \Delta\, y(\, N_i,\ t_k + \tau_i\,))^2 \,/\, n$$

will be about zero. Based on such an approach the problem on the phase screen existence at the time t_k is reduced to solution of the following inequality, with respect to **V**

$$\min \varepsilon\,(\mathbf{V}_k) = \min_{\mathbf{V}} \varepsilon\,(\mathbf{V}) \ll \delta \approx 0. \qquad (1)$$

RESULTS OF PROCESSING THE FILM

The velocity vectors $\mathbf{V}_k$ and corresponding distributions of the deviation $\Delta y(N_i,\ t_k + \tau_i)$ were calculated by the described scheme for the frames k = 25–175. Figure 3 presents examples of the deviation plots: $\Delta y(M_i,\ t_k)$ is shown by solid line, and $\Delta y(N_i,\ t_k + \tau_i)$ is shown by dashed line with the minimum value increasing but not exceeding the limitation δ = 0.023. The value of δ was chosen from the condition that the deviations $\Delta y(N_i,\ t_k + \tau_i)$ follow changes in the deviations $\Delta y(M_i,\ t_k)$.

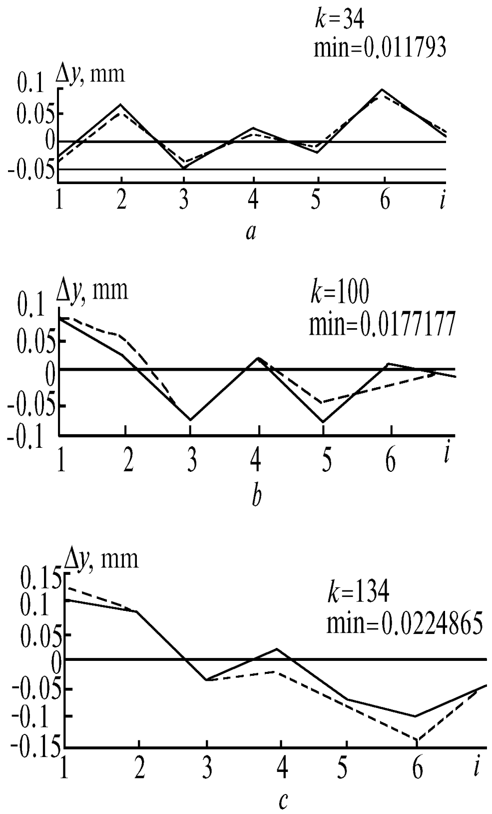

FIG. 3.

Only 20 out of 151 frames satisfied the condition (1). Therefore, the estimation of the probability of the phase screen existence is $p \approx 0.13$.

A comparison of the deviations on the contours ABUCD with the deviations on the contours A′B′UC′D′ (Figs. 4 and 5) for $|\alpha| \ll \pi/4$ was carried out using analogous technique.

Figure 6 presents examples of the deviation plots for the considered scheme: $\Delta y(M_i,\ t_k)$ is shown by solid line, and $\Delta y(N_i,\ t_k + \tau_i)$ is shown by dashed line.

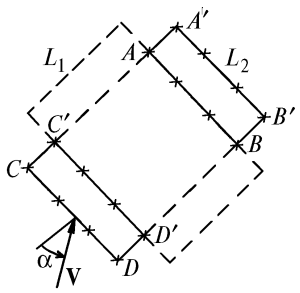

FIG. 4.

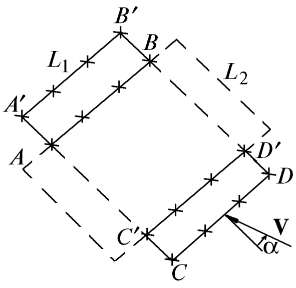

FIG. 5.

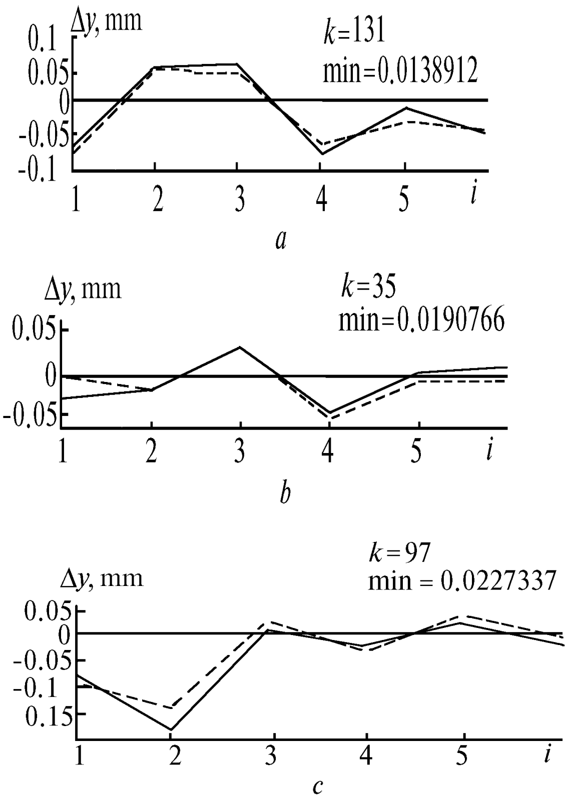

FIG. 6.

Only 17 out of 151 frames satisfied the condition (1). Therefore, the estimation of the probability of the phase screen existence is $p \approx 0.11$. Here we note that the velocity vectors $\mathbf{V}_k$ entering into the condition (1) varied according to different laws in the above schemes, and this fact requires an explanation. In this connection, it should be noted that establishment of the fact of existence of the phase screen by this technique depends on the geometry of a system of points M_i, or, more exactly, on the distance $l = M_i N_i$, which determines the lag τ, the velocity **V**, and the measurement interval Δt. Therefore, for large values of $\mathbf{V}$ and Δt the second scheme does not establish the fact of the phase screen existence. Similarly, if the lifetime of the phase screen is short and its velocity is low then the first scheme also does not establish the fact of the phase screen existence. Hence, a choice of a scheme points for making comparisons introduces selectivity of the choice of the phase screen by its velocity and lifetime. Specifically, the mean modulus of velocities which satisfy the condition (1) equals 4.03 and 1.66 m/s in the above presented schemes, respectively.

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