ABOUT THE OPTIMIZATION OF PARTIALLY CORRECTING ADAPTIVE OPTICS

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At visible wavelengths, the long exposure point spread function (PSF) of large telescopes or interferometers can be only partially corrected by adaptive optics (AO). For system designed to compensate at infrared wavelengths, the element size or actuator spacing r_E is equal to about 5 times the Fried parameter r_0 in the visible. This is largely outside the range for which the normally used least–squares fits are optimum. Here we present some one dimension numerical experiments confirming that in the case of partially correcting AO, there exists an optimal compensation (here obtained by simulated annealing) that maximizes the Strehl ratio. To evaluate the effects of the atmospheric wave front temporal evolution on the corrected image quality, we calculated the resultant corrected long exposure PSF shape, as a function of the time delay existing between the wave front used for the computation of the correction and the wave front to which the correction is applied. In order to obtain a significant gain using the simulated annealing technique, this optimized correction must be applied within a delay smaller than half the speckle lifetime τ_0.

1. INTRODUCTION

In recent years there has been a strong development of adaptive optics (AO) for real time compensation for the wave front perturbations introduced by the atmosphere. Systems have been developed inside the astronomical community (e.g., COME–ON, University of Hawaii) and also outside it, mainly for military applications. Images close to diffraction limit (although in AO people tend to say "close to diffraction limit" as soon as Strehl ratio is over 30%) are commonly achieved in the infrared (*J* or *K* band) with 3.6 m class telescopes or in the visible with small telescopes. The unique 8 m telescopes of the VLT will feature AO systems providing a close to perfect correction in the *K* band. However, for very large telescopes used at visible wavelengths, a complete correction with AO would require systems with thousands of actuators operated at several hundreds of hertzs and this is at least one magnitude over what will be permitted by technology and funding in a foreseeable future.

However, it has been demonstrated that the partial correction in the visible by systems providing total correction in the infrared, will improve performance of imaging techniques. It is therefore important to characterise the shape of the partial correction resultant point spread function (PSF) to optimize the design of focal instrumentation and to select the optimum image reconstruction strategy. For a long time, there has been a discussion between people claiming that partially correcting AO will result in a seeing improvement (the corrected images will still contain speckles but brighter ones) increasing the performances of speckle related techniques and those explaining that the PSF will feature an Airy size sharp core, to be used for deconvolutions of long exposure images, surrounded by a halo with approximately the seeing disk size (see, e.g., Refs. 5 and 8). This debate is now somewhat obsolete, as there will always be a sharp core. However, it appeared that its height (i.e. the Strehl ratio) is not necessarily optimized by the linear techniques used for the computation for the wave front correcting function which minimizes residual phase from a least–squares point of view (Refs. 1 and 2). Another one unclear point is the behavior of the PSF and its sharp core with the temporal evolution of the incoming wave front which introduces a time delay τ between the measured wave front and the corrected one. The purpose of this work was to contribute to the understanding of this two points through one dimension (1D) numerical simulations. First we simulated wave fronts with a phase structure function as close as possible to the theoretical 5/3 power law. Then we computed the correction function for a rubber mirror deformed by a given number of actuators with several optimization criteria. Last we simulated a temporal evolution of the incoming wave front and studied the influence of the time delay τ on the long exposure PSF.

2. WAVE FRONT PHASE GENERATION AND SIMULATIONS OF AO PARTIAL CORRECTION

In order to obtain a reliable simulation, it is very important to generate carefully an atmospherically perturbed wave front with a phase obeying a $r^{5/3}$ law structure function. To minimize the computing time, we made 1D simulations as if we had a pupil representing a narrow slit telescope. In this case our 1D structure function will be a cut in one direction of the two dimension (2D) theoretical atmospheric phase structure function

$$D_{\Phi}(\mathbf{r}) = 6.88\,(r\,/\,r_0)^{5/3}\,. \qquad (2.1)$$

This power law structure function results from wave front phases containing structures of all scales. To simulate it in the 1D case, we used a modified version of the mid–point displacement technique (e.g., Ref. 4). The first step of the algorithm is the generation of the phase at the edges of the considered segment of length L. These points are Gaussian random numbers with zero mean and a variance equal half the structure function for the distance L (we assume that L is smaller than the outer scale L_0 of the atmospheric turbulence). The next step is generating the point in the middle of the segment as a Gaussian random variable with a mean value equal to

$$\langle\Phi(L/2)\rangle = \frac{\Phi(0) + \Phi(L)}{2} , \tag{2.2}$$

and with a variance given by

$$\mathrm{Var}\,\big(\Phi(L/2)\big) = D_\Phi(L/2) - \alpha\, D_\Phi(L) . \tag{2.3}$$

The α coefficient was adjusted semiempirically and the best fit is given by $\alpha = 0.23$.

The procedure is repeated recursively with each half of the segment until the desired number of points is obtained, i.e., the wave front phase is completed. In Fig. 1 we illustrate the first steps of the algorithm and show the resulting phase.

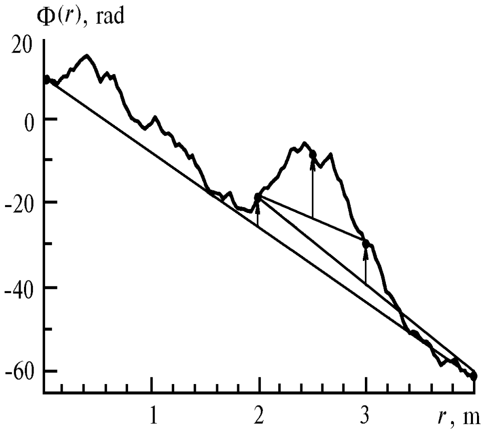

FIG. 1. Simulated atmospheric 1D wave front phase arriving in a pupil of diameter $d = 4$ m *(Fried parameter* $r_0 = 0.1$ m, *computed for a* 16 m *segment out of which a* 4 m *segment is extracted). The first steps of the improved mid–point displacement algorithm used for the simulations are illustrated in a schematic way. The point in the middle of a segment is generated as a Gaussian random variable with a mean value given by Eq. (2.2) and with a variance given by Eq. (2.3). The procedure is repeated recursively with each half of the segment until the wave front phase is completed.*

This procedure yields a very good approximation of the theoretical atmospheric phase structure function as shown in Fig. 2.

In order to simulate the partial correction by AO we first generate a wave front phase and then we apply a correcting phase function, resulting from N actuators with spacing r_E and smoothed triangular influence functions, simulating a continuous deformable mirror. With the residual phase we obtain the instantaneous corrected image as a result of the propagation of the wave between the pupil plane and the focal plane, simulated by an FFT, whose square modulus gives the intensity at the focus of the telescope. The long exposure PSF is generated by averaging hundreds of "partially corrected" images.

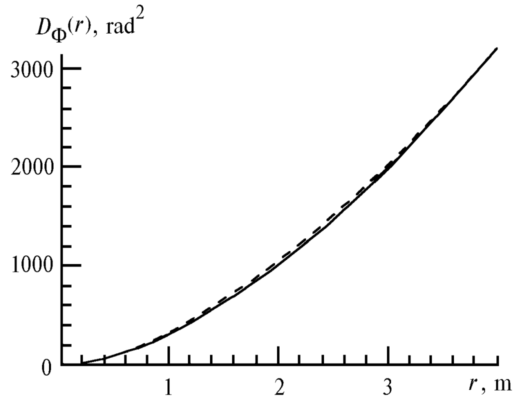

FIG. 2. Comparison of the theoretical atmospheric phase structure function given by Eq. (2.1) (solid curve) and the calculated phase structure function (dashed curve) of the simulated 1D wave front phases with $d/r_0 = 40$.

To compute the correcting phase, we considered three different methods: *a*) the mean local slope method (Ref. 5), for which the average slope for each sub–aperture is reduced to zero; *b*) the least–squares method, for which mean square of the residual wave front phase is globally minimized; and *c*) the simulated annealing method, for which the best correction for a given wave front and number of actuators is iteratively computed by simulated annealing to obtain the maximum Strehl ratio, that is, the maximum peak intensity in the far field.

In the simulated annealing technique, the actuators are changed randomly. If the resulting Strehl ratio improves, the change is kept. If not, the variation is compared to a function of the "equivalent temperature" of the system. If worse than this value, it is dropped, and the actuator is returned to the former position. As the iteration goes by, the number of accepted movements is reduced as a consequence of a decrease of the system "temperature". At very low temperatures, only actuator steps that improve the Strehl ratio are accepted. This procedure yields, slowly but certainly, to the maximum possible Strehl ratio. This image plane optimization criterion is very similar to the one used in the polysteps or multidithering techniques (e.g., Ref. 6) used at the begining stage of AO development.

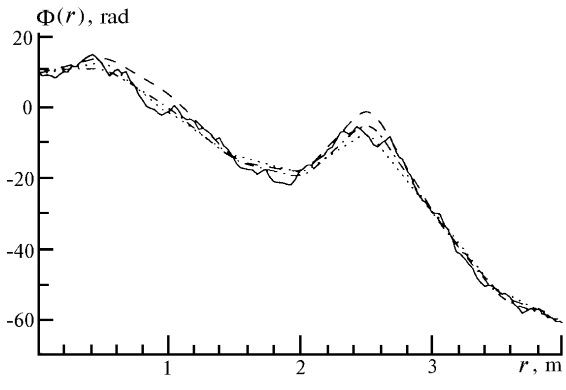

FIG. 3. Comparison of the correcting wave fronts computed using the different techniques. The uncorrected incoming wave front (solid curve), the simulated annealing (dashed curve), the least–squares (dash–and–dot curve), and mean local slope (dotted curve) correcting wave fronts. The correction is applied in the case of an actuator spacing $r_E = 5r_0$ *, and 8 segments in a 4 meter pupil.*

In Fig. 3 we can see the different correcting functions applied to the same wave front phase.

In all this cases the PSF after partial correction shows a sharp core, of Airy size, with a height depending on the correcting technique. As we can see in Fig. 4 the maximum Strehl ratio is obtained by the simulated annealing method. In this case, the gain with regard to the least–squares technique is about 2.5.

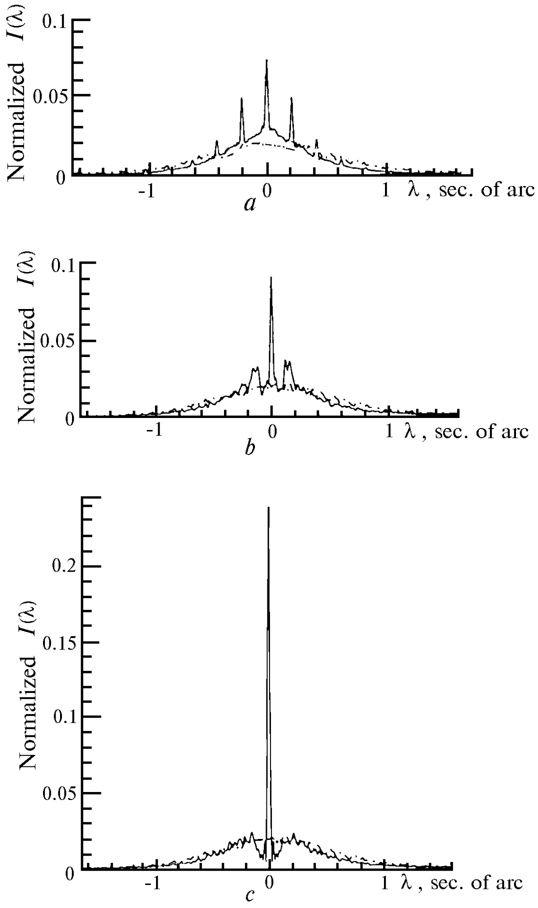

FIG. 4. Long exposure PSF after adaptive optics partial correction. a) mean local slope method, b) least–squares method, and c) simulated annealing method. The uncorrected long exposure image (dot–and–dash curve) is plotted for comparison in each case. Computation parameters are $r_E = 5r_0$, $d/r_0 = 40$, 100 integrated images, and diffraction limited intensity maximum normalized to unity.

After Beckers (Refs. 1 and 2), we confirm that the least–squares method, which is equivalent to the algorithms commonly used for wave front fitting in AO, does not give the optimal correction. The least–squares method gives the best Strehl ratio only if the residual phase variance is small (Ref. 3), which is not the case in foreseeable AO systems used at visible wavelengths. The effect should be even stronger in the 2D case. A simple, maybe oversimplified, analysis indicates that the 2D Strehl ratio should be the square of the 1D Strehl ratio, if we go from a 1D aperture with length d and N actuators with r_E spacing to a 2D pupil with diameter d and about $(\pi/4)\,N^2$ actuators with the same $r_{\rm E}$ spacing: if we assume that the 1D Strehl ratio is roughly the ratio of the fraction of the pupil contributing to the sharp core, of total length l, to the pupil length d, in the 2D case the Strehl ratio might be the ratio $(1/d)^2$ of the "cophased" surface to total surface.

As we can see in Fig. 5, the structure function of the corrected wave fronts has a higher saturation value for the simulated annealing technique but contains smaller amplitude oscillations. Let us imagine that the strong oscillations, with a period slightly larger than r_E, present in the structure function resulting from the least–squares method are responsible for the side lobes which can be seen in the long exposure PSF (Fig. 4*b*). Then, the reduction of these oscillations in the structure function after the simulated annealing correction might explain the reduction of the side lobes in the PSF in Fig. 4*c* and this might be the reason for the Strehl ratio improvement. If this intuitive, and not demonstrated at this point, approach is correct, it might result in a new optimization criteria based on the variance and the covariance of the phase.

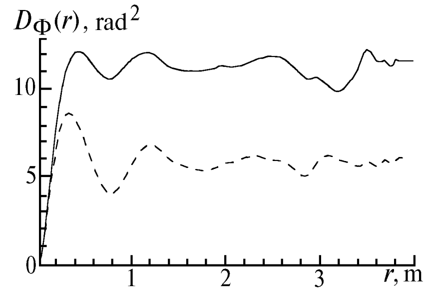

FIG. 5. Resulting structure functions of the corrected wave fronts after the simulated annealing correction (solid curve) and the least–squares correction (dashed curve). Computation parameters are as in Fig. 4. Scale is the same as in Fig. 2.

The difference between the optimum result and the least–squares result increases with the increasing ratio r_E/r_0 of the distance r_E between the actuators to r_0. For very partial correction, i.e., $r_E/r_0 = 10$, it can reach a factor of 4, as illustrated in Fig. 6. This figure present the Strehl ratio gain of the optimal correction with regard to the least–squares one. The circles (o) present the values extracted from the Beckers paper[2] and the crosses (+) result from this work which appears to be in good agreement with the Beckers one (although we obtain systematically slightly higher Strehl ratios with all methods, which might be explained by differences in the wave front simulation algorithm).

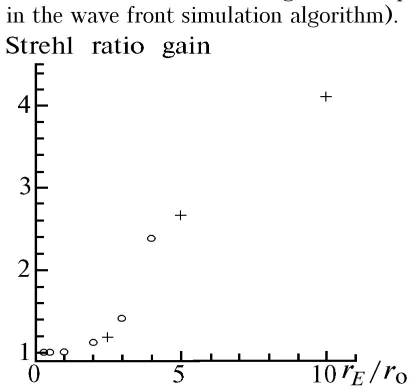

FIG. 6. One dimension Strehl ratio gain (SRG = Strehl simulated annealing / Strehl least–squares) as a function of r_E/r_0. The circles (°) present the extracted data from Ref. 2. The crosses (+) are the points deduced from our simulations.

3. SIMULATING THE TEMPORAL EVOLUTION OF THE WAVE FRONTS

In order to simulate the influence of the temporal evolution of the wave front on the long exposure PSF for the corrected images, we introduced a time delay τ between the wave front used for the computation of the correcting function and the wave front actually corrected.

During the delay, the incoming wave front is modified by a shift of the turbulent layers with regard to the telescope pupil, as if it were drifted by the wind without being otherwise modified.

The temporal scale was calibrated through an estimation of the lifetime of the simulated speckles (τ_0 = equivalent width of the temporal autocorrelation function) as Ref. 7. This makes it possible to give the delay τ in speckle lifetime units.

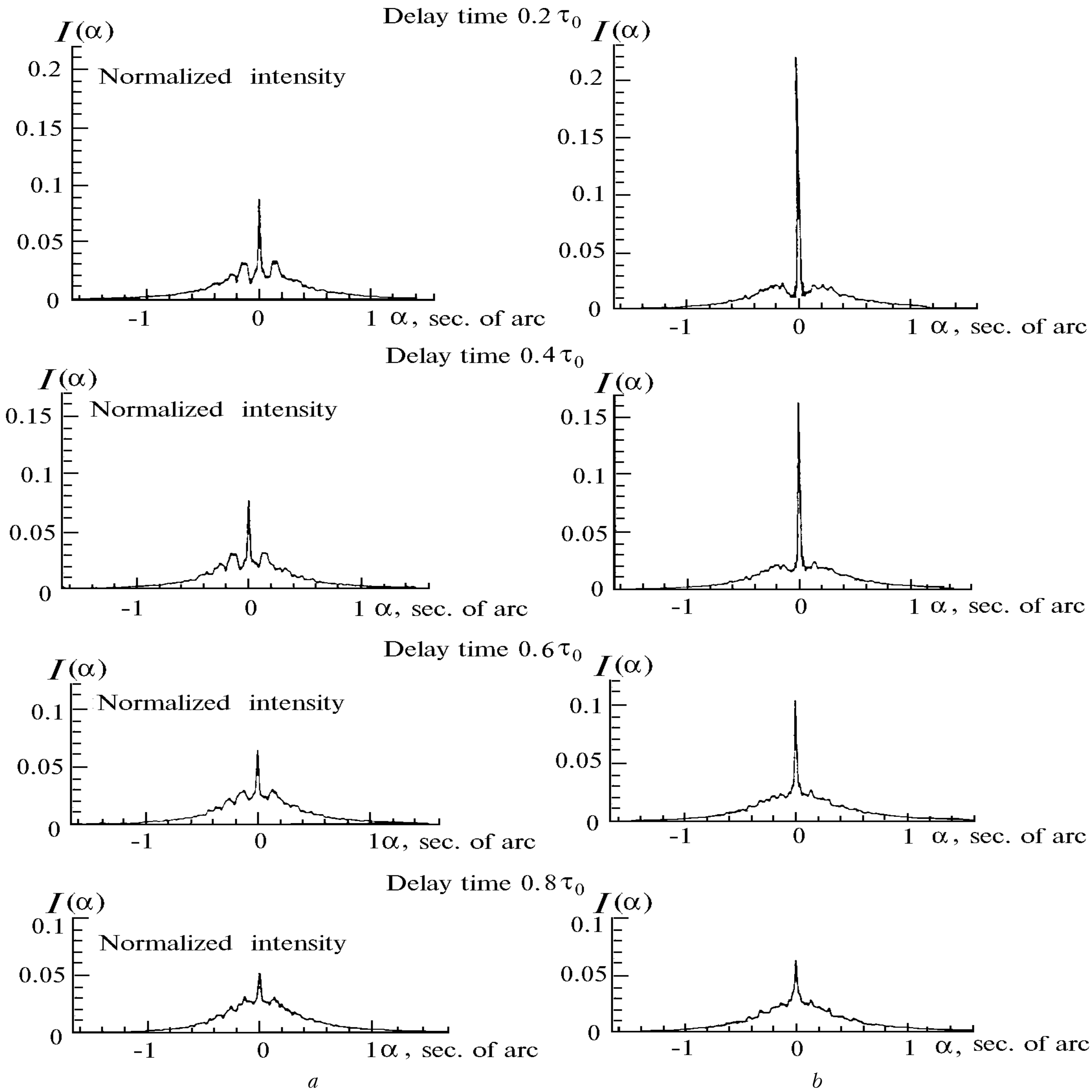

FIG. 7. The effect, in the PSF, of time delay between the wave front sensing and the AO correction, using the least–squares method (a) and simulated annealing method (b). The atmospheric wave front temporal evolution was simulated by shifting in opposite directions and then adding two phase screens with $d/r_0 = 40$. The parameter τ_0 characterizes the speckle lifetime.

We considered the case of one turbulent layer (Taylor model) and two turbulent layers (an approximation of the speckle boiling model, e.g., Ref. 7). As we verified, there exists a critical dependence on the atmospheric model used. For the little realistic Taylor atmosphere the Strehl ratio decays and the sharp–core of the PSF disappears much faster than for the two–layer model. For the two–layer model which is more realistic, we can see in Fig. 7 that the decay of the PSF obtained by simulated annealing is faster than this of the PSF obtained by a least–squares minimization of the wave front. For a time delay approaching the speckle lifetime, the advantage of the simulated annealing technique is lost as illustrated in Fig. 8.

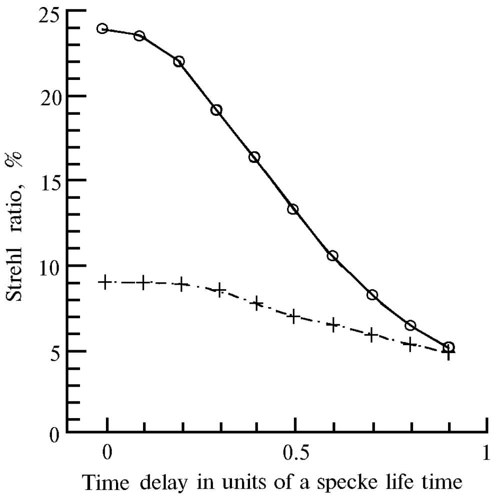

FIG. 8. One dimension Strehl ratio decay as a function of the time delay in the application of the correction. Circles (o) and solid curve represent the decay of the Strehl ratio after the correction by the simulated annealing method and the respective delay. Crosses (+) and dash–dotted curve represents the decay in the case of the least–squares correction.

4. CONCLUSIONS

In the case of partially correcting AO there exists an optimal compensation that maximizes the Strehl ratio. This optimization, here done with the simulated annealing technique, was obtained with little information from the image plane (only one sensor). However, it is difficult to achieve in practice because it requires many iterations to converge and sufficient flux in the sensor, all this is to be done during a delay time shorter than half the speckle lifetime.

The 1D optimal point spread function obtained by simulated annealing, is substantially better than the one resulting from a least–squares minimization of the residual wave fronts. This effect will be stronger in the 2D case.

One might think out an optimization method that uses the information provided by a wave front sensor. The good sampling of the incoming wave fronts, and the application of some sorting algorithms of the "easy–to–cophase" wave front points will allow the optimal solution to be obtained in a substantially shortened time.

We can think out other techniques to accelerate the process, these will include: massive parallelization, point selection, classification and weighting algorithms, and finally a more detailed knowledge of the incoming information by doing a wavelets analysis of the atmospheric wave fronts.

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