OPTIMAL COMBINATION OF ALTERNATIVE METHODS FOR SPATIAL FORECASTING IN PROBLEMS OF ATMOSPHERIC ECOLOGICAL MONITORING. I. METHODOLOGY AND ALGORITHMS

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Received December 29, 1994

An original approach is proposed to the problem of reconstructing mesometeorological fields in the territories uncovered with observational data, which is based on the optimum combination of alternative techniques for spatial prediction. Some grounding in theory of appropriate mathematical methods, such as polynomial fitting, optimal extrapolation, and modified method of clustering of arguments (MMCA) are examined along with an integrated algorithm for spatial prediction of vertical profiles of temperature and wind velocity.

1. INTRODUCTION

An important problem in modern mesometeorology is reliable and adequate reconstruction of vertical structure of mesometeorological fields for regions with coarse (or no) aerological network using atmospheric sensing data for adjacent territories. An optimal solution of this problem is necessary for many applications, among which are:

– estimation of the spatial spread of industrial pollutants in air basins of local areas (such as an industrial zone or an entire region), which is determined by the temperature stratification, influencing the turbulent diffusion of pollutants, and the detailed wind regime inside that territory;

– numerical prediction of mesoscale processes (on horizontal scale of several tens or hundreds of kilometers) and their associated weather for individual regions and locations;

– optimal arrangement of lidar network to be employed for atmospheric monitoring of bounded territories, either on local or regional scale, i.e., territories whose horizontal dimensions vary from 50 to 300 km (Ref. 1).

In practice, however, the optimal reconstruction (spatial prediction) of vertical structure of mesometeorological fields faces a number of problems.

First, the existing world aerological network is extremely nonuniform and coarse (even Europe and North America, most extensively covered with observational data, have closest stations spaced about 300–400 km apart), and obviously does not meet the requirements for an objective analysis of mesometeorological fields (i.e., their construction based on observational data and a certain numerical algorithm). In particular, to make reliable regional and local weather forecasts, one needs aerological observational data at the nodes of a grid whose horizontal step size varies from 50 to 200 km (for regional model) and from 5 to 50 km or even from 1 to 5 km (for local model).[1]

Second, current algorithms for objective analysis of meteorological fields mostly employ polynomial fitting[2–8] and optimal interpolation (for details, see Refs. 9 and 10) that suffer from a number of serious disadvantages. Algebraic polynomials in polynomial fitting, in particular, are arbitrarily chosen, irrelevant to the specific meteorological fields, and are inadequate for a coarse observational grid. Optimal interpolation (extrapolation) technique, on the other hand, calls for preliminary processing of a large body of initial information and calculation of the wanted statistical characteristics (from long–term data), including spatial correlation coefficients.

Third, the objective analysis of altitude dependence of meteorological fields still underexploits nontraditional methods (such as the method of clustering of arguments[11] (MCA)) that are sufficiently efficient under conditions of informational uncertainty and need no preliminary generalization of long–term data.

Under these circumstances, as well as due to the lack of literature on spatial extrapolation of mesometeorological fields, it seems reasonable to solve the problem by an integrated method based on optimal combination of alternative techniques of spatial prediction such as polynomial and optimal extrapolation techniques and MMCA.

This paper discusses this approach with special emphasis on problem formulation, grounding in theory of its numerical solution, principles of implementation of an integrated algorithm, and finally the algorithm by itself for spatial prediction of vertical structure of meteorological fields.

2. PROBLEM FORMULATION AND SOME GROUNDING IN THEORY OF ITS NUMERICAL SOLUTION BASED ON EXPERIMENTAL DATA

Let an experiment output be a parameter $f(r_i)$ (a meteorological parameter, in our case) measured at points $r_i \in W_x \subset R^m$ (with r_i being the radius–vector of a point, $i = 1, 2, \ldots, n$, and n being the number of points in a closed subspace W_x of the finite–dimensional Euclidian space R^m). Then the procedure of extrapolation (spatial prediction) of the parameter f to a point $r_0 \in W_x \subset R^m$, that is, the determination of $f(r_0)$ outside the set W_x knowing f values at points r_i, $i = 1, \ldots, n$ belonging to W_x, is performed on the class of linear models according to the expression

0235-6880/95/07 488-09 $02.00

$$\hat{f}(r_0) = \sum_{i=1}^{n} a_i f(r_i) , \quad (1)$$

where a_i, $i = 1, \ldots, n$ are some weighting factors that do not depend on f values at points r_i, $i = 1, \ldots, n$ and are to be adjusted so that the quality of predictions $\hat{f}(r_0)$ be guaranteed by the confidence inequality

$$E[\hat{f}(r_0) - f(r_0)]^2 \le \varepsilon_i^2 . \quad (2)$$

Here, $E(\cdot)$ denotes the mathematical expectation operator that averages over all $\Delta = \hat{f}(r_0) - f(r_0)$, and ε_i^2 is the acceptable error estimated from the variance of observation errors and maximum permissible error in spatial prediction.

For the problem just formulated, let us now briefly consider the theoretical grounding of its solution using one of the alternative techniques (polynomial fitting, optimal interpolation, and MCA) that, as mentioned above, were used in construction of the optimal integrated algorithm for meteorological field extrapolation. For complete detail, see Refs. 8–11.

A. Polynomial fitting technique

According to this technique, meteorological parameter f is estimated at a point (x_0, y_0) lying in a given plane (or isobaric surface), from measurements of f at the ith points ($i = 1, \ldots, N$) lying in the vicinity of the point of prediction. We assume that the f values can be represented by a polynomial (algebraic, as is normally the case) of some prescribed functions of coordinates $F_k(x, y)$ in the form[7]

$$f_i = f(x_i, y_i) = \sum_{k=1}^{K} a_k F_{k\,i} \qquad (i = 1, 2, \ldots, N) , \quad (3)$$

where $F_{ki} = F_k(x_i, y_i)$, while $F_k(x_i, y_i)$ for algebraic polynomial have the form

$$F_1(x_i, y_i) = 1 , \quad F_2(x_i, y_i) = x , \quad F_3(x_i, y_i) = y ,$$
$$F_4(x_i, y_i) = x^2 , \quad F_5(x_i, y_i) = x\,y , \quad F_6(x_i, y_i) = y^2 , \ldots , \quad (4)$$

a_k are some weighting factors that are to be determined from N equations (3), and K is the number of coefficients a_k in the given polynomial.

It should be emphasized at once that observation errors can be accounted for only when the number of observations N is larger than K, the number of terms in the polynomial. As a consequence, Eq. (3) turns out to be approximate, and in practice the coefficients a_k are calculated from Eq. (3) by the least squares technique (in full detail, this technique was described in Ref. 12), so that to minimize the sum of squared differences between the right– and left–hand sides

$$E = \sum_{i=1}^{N} \left(f_i - \sum_{k=1}^{K} a_k F_{k\,i} \right)^2 . \quad (5)$$

This condition is satisfied only when all derivatives of E with respect to a_k vanish, that is

$$\frac{\partial E}{\partial a_1} = 0, \ \frac{\partial E}{\partial a_2} = 0, \ \ldots, \ \frac{\partial E}{\partial a_k} = 0. \quad (6)$$

In the present paper, we extrapolate f values from measurements in points distributed nonuniformly in a plane that should be additionally accounted for, e.g., through introducing weights proportional to the distance to the prediction point. The minimum condition (5) then modifies to

$$E' = \sum_{i=1}^{N} S_i \left(f_i - \sum_{k=1}^{K} F_{k\,i} \right)^2 , \quad (7)$$

where the factors S_i are appropriately chosen as functions of the distance

$$r_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} . \quad (8)$$

In addition, to solve the above formulated problem, we require only f_0 rather than f_i. To estimate f_0, the origin of coordinates is traditionally placed at the prediction point, i.e., $x = y = 0$. From Eq. (3) we then obtain

$$f_i(0, 0) = a_1 , \quad (9)$$

that is, only the first coefficient of algebraic polynomial a_k is of necessity.

B. Optimal extrapolation technique

Unlike the polynomial fitting technique, this technique is based on statistical analysis of meteorological fields; so some statistical nomenclature for meteorological fields is presented before proceeding to the technique.

Let f be the value of a meteorological parameter at point i or j, $\overline{f}$ be its average (norm), and $f' = f - \overline{f}$ be deviation from the average. In this case, the average of the square function $D = \overline{f'^2}$ is called the variance of the meteorological parameter f; the average product of f' values at points i and j, $m_{ij} = \overline{f'_i f'_j}$, is called covariance, and being divided by the square root of the product of variances at these points, it yields the normalized autocorrelation function (or autocorrelation moment)

$$\mu_{i\,j} = \frac{\overline{f'_i f'_j}}{\sqrt{\overline{f'^2_i f'^2_j}}} = \frac{m_{ij}}{\sqrt{D_i D_j}} . \quad (10)$$

As is well known, at short distances (of the order of a few hundreds of kilometers[10]) the field of any meteorological parameter f can be assumed isotropic and uniform, and its variances are then identical throughout the field, with $D_i = D_j = d^2$, while the autocorrelation function is dependent solely on the distance between points i and j and is expressed as

$$\mu_{i\,j} = \mu(r_{i\,j}) = \overline{f'_i f'_j} / d^2 . \quad (11)$$

In practice, since the autocorrelation function and the variance are calculated from measurements of meteorological

parameter (taken with an error δ), the empirical autocorrelation function μ_{ij} is written as

$$\mu_{ij} = \frac{1}{N d^2} \sum_{k=1}^{N} \hat{f}'_{ik} \hat{f}'_{jk} , \qquad (12)$$

where $d^2 = \frac{1}{N} \sum_{k=1}^{N} \hat{f}'^2_k - \delta^2$ is the variance of meteorological parameter f, $\hat{f}'_{ik}$ and $\hat{f}'_{jk}$ are its deviations from the average at points i and j, and N is the number of cases used in statistical calculation.

After the preliminaries having been completed, let us now proceed to the essence of the optimal extrapolation technique. According to this technique, the meteorological parameter f at the point (x_0, y_0) is determined from its measurements at points i, $i = 1, \ldots, n$, lying in the vicinity of the prediction point, by the relation[10]

$$f_0 = \overline{f}_0 + f' = \overline{f}_0 + \sum_{i=1}^{N} p_i \hat{f}'_i , \qquad (13)$$

where $\overline{f}_0$ is the average of the meteorological parameter, and f'_0 is the deviation of f from the average; $\hat{f}'_i = \hat{f}_i - \overline{f}_i$ is the deviation of the measured $\hat{f}_i$ from the norm $\overline{f}_i$ taken at the ith points; and p_i is an interpolation weight to be adjusted so that to minimize the mean square error of extrapolation

$$E = \overline{(\hat{f}_0 - f_0)^2} = \left[\overline{f'^2_0} - 2 f_0 \overline{\sum_{i=1}^{n} p_i \hat{f}_i} + \overline{\left(\sum_{i=1}^{n} p_i \hat{f}_i \right)^2} \right]. \qquad (14)$$

As in the polynomial fitting, we use the minimum condition

$$\frac{\partial E}{\partial p_i} = 0 \quad (i = 1, 2, \ldots, n) , \qquad (15)$$

take the term–by–term derivative of Eq. (14), and introduce the relative error $\eta = \delta^2/d^2$, which is the ratio of mean square error in measuring meteorological parameter to its variance and is a variability characteristic of f, we finally arrive at the set of equations

$$\sum_{j=1}^{n} p_i \mu_{ij} + p_i \eta = \mu_{0i} \qquad (i = 1, 2, \ldots, n) \qquad (16)$$

for determination of weights p_i.

To calculate the coefficients p_i from Eq. (16), we must know the autocorrelation moments μ_{ij} and μ_{0i}. By virtue of the above assumption on uniform and isotropic field, the autocorrelation moment μ_{ij} is dependent on the distance between points i and j as well as on their distance from the prediction point (x_0, y_0).

In Cartesian coordinates of a geographic map, the said distances are expressed as

$$r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} . \qquad (17)$$

In practice, different analytical expressions are used to evaluate the normalized autocorrelation functions for points i and j as functions of the distance r. For instance, the normalized autocorrelation function of the Earth's surface temperature derived in Ref. 13 has the analytical form

$$\mu(r) = \exp(-0.825\, r^{0.92}) , \qquad (18)$$

and the same function for the wind velocity components found in Ref. 14 is written in the following form:

$$\mu_u(r) = \mu_v(r) = (1 - 0.98\, r) \exp(-0.98\, r) . \qquad (19)$$

Next, the evaluated μ_{ij} values are inserted in the set of equations (16) to yield the weight p_i.

The obtained p_i is inserted in Eq. (13) and, after the norm of $\overline{f}_0$ and $\hat{f}'_0$ being found, the latter by subtracting the norms $\overline{f}_i$ at all the ith points from the corresponding measured $\hat{f}_i$ values, we finally have the desired meteorological parameter f_0 at the point (x_0, y_0).

C. Method of clustering of arguments

This method, in its modified form, is clearly advantageous over the polynomial and optimal extrapolation techniques considered above in its ability to extrapolate spatially not only f_i values but also k–dimensional vectors (vertical profiles) of this same meteorological parameter $f_i(\kappa_k)$, but now resolved into levels by either altitudes $h = 0, 1, \ldots, h_k$ or pressures $p = p_0, \ldots, p_k$. In conformity with Ref. 15, we assume that the vertical local and regional meteorological fields $f_i(h_k)$ are uniform and can be described with a single covariation matrix. Under this condition, the spatial prediction (using MCA) of $f_0(h_k)$ value at the point (x_0, y_0) from known (measured) $f_i(h_k)$ values at the nearest ith point is made for the system of linear regression models of the form[11]

$$Y_0(h, N+1) = \sum_{t=1}^{N^*} A(h, \tau)\, Y_i(h, N+1-\tau) +$$

$$+ \sum_{j=1}^{h-1} B(h, j)\, Y_0(j, N+1) + \varepsilon(h, N+1)$$

$$(h = \overline{h} + 1 , \overline{h} + 2 , \ldots, h_k) \qquad (20)$$

based on the initial experimental data of spatiotemporal observations:

$$\{Y_i(h, t) , \quad h = 0, 1, \ldots, h_k ; \quad t = 1, \ldots, N\} ; \qquad (21)$$

$$\{Y_0(h, t) , \quad h = 0, 1, \ldots, \overline{h} \le h_k ; \quad t = N + 1\} , \qquad (22)$$

where h is the altitude; t is the time of observation; N^* is the time delay ($N^* < [N - h - 1]/2$); $A(h, 1), \ldots, A(h, N)$ and $B(h, 0), \ldots, B(h, h-1)$ are the unknown model parameters; and $\varepsilon(h, N+1)$ is the discrepancy of the model.

An algorithm for choosing the best prognostic model is discussed in section 4.

3. MAIN PRINCIPLES OF CONSTRUCTING AN INTEGRATED ALGORITHM FOR SPATIAL PREDICTION

Let us now discuss general principles of constructing an integrated algorithm obtained by combination of the alternative prognostic methods and intended to reconstruct (spatially predict) vertical profiles of meteorological parameters (in our case, these are the air temperature (T_a) and zonal (U) and meridional (V) wind velocities) at a point (x_0, y_0), using observation data from N closely located aerological stations (or remote sensing stations). In doing this we are guided by the following basic principles:

1. Real–time spatial prediction should be preceded by preliminary estimation of the corresponding mean values both at the initial ith points and at the prediction point (x_0, y_0) with the use of any model (background) values.

2. A prognostic model must be based on a limited volume of real–time information, with no long–term data available.

3. Chosen algorithm must employ optimum (i.e., allowing best extrapolation) combination of alternative techniques of spatial prediction, with a proper allowance for vertical structure of meteorological fields determining the specific weather conditions of local areas.

4. The construction of the integrated algorithm proceeds in too steps. First, we identify level (levels) k, among K levels used, for which the reconstruction error ε is minimum. That is done by means of either polynomial fitting or optimal extrapolation. And only then (after the meteorological parameter f_0 has been reconstructed by polynomial fitting or optimal extrapolation for levels with minimum reconstruction error and at time of observation $t = N + 1$) the spatial prediction by itself is made, based on a statistical sample of spatiotemporal observations, $f_i(h_k, t)$, $t = 1, 2, \ldots, N$ (from Ref. 16, $N = 7–15$) by the MCA to complete the profile $f_0(h_k, N + 1)$.

All these ideas provided a basis for constructing flowchart and integrated algorithm for spatial prediction of altitude structure of mesometeorological fields as applied to the temperature and zonal and meridional wind velocity fields.

4. FLOWCHART AND INTEGRATED ALGORITHM FOR SPATIAL PREDICTION OF VERTICAL STRUCTURE OF MESOMETEOROLOGICAL FIELDS

To understand better the integrated algorithm for spatial prediction of vertical structure of mesometeorological fields, we will consider its individual steps. First we dwell on identifying the level (levels) with minimum prediction error (Fig. 1) by the polynomial fitting or optimal extrapolation. Steps here will be the following.

1. Input of the number of stations, N_s, and the number of altitudes, M, where measurements have been performed.

2. Choice of a spatial prediction technique (between polynomial fitting and optimal extrapolation).

When choosing the polynomial fitting, the temperature (T) and zonal (U) and meridional (V) wind velocity components are calculated for representation of their fields in the vicinity of the point by algebraic polynomial

$$P_1(x, y) = a_0 + a_1 x + a_2 y , \qquad (23)$$

where x and y are coordinates, and $a_0, \ldots, a_2$ are coefficients. Since the origin of coordinates is placed at the point under consideration, $x = y = 0$ and

$$f_i(0, 0) = a_0 , \qquad (24)$$

where f_i is the sought–for value of T, U, or V.

3. Input of T, U, and V values for the measurement points together with their coordinates (x, y) assuming that the wanted point is located at (0, 0), and setting up (by minimizing the sum of squared errors with respect to all coefficients a) the system of linear equations of the third order ($SLE_{3\times3}$)

$$\left.\begin{aligned} a_0 m_{11} + a_1 m_{21} + a_2 m_{31} &= n_1 \\ a_0 m_{12} + a_1 m_{22} + a_2 m_{32} &= n_2 \\ a_0 m_{13} + a_1 m_{23} + a_2 m_{33} &= n_3 \end{aligned}\right\} , \qquad (25)$$

used for determination of the coefficients a at the given point.

4–6. Implementation of cycles for levels $j \le M$, profiles $k \le P$, and stations $i \le N_s$.

7. Calculation of the coefficients in the left–hand side of Eq. (25) using the relations

$$m_{11} = 1 , \quad m_{21} = \frac{1}{N_s}\sum_{i=1}^{N_s} x_i , \quad m_{31} = \frac{1}{N_s}\sum_{i=1}^{N_s} y_i ,$$

$$m_{22} = \frac{1}{N_s}\sum_{i=1}^{N_s} x_i x_i , \quad m_{32} = \frac{1}{N_s}\sum_{i=1}^{N_s} x_i y_i , \quad m_{33} = \frac{1}{N_s}\sum_{i=1}^{N_s} y_i y_i ,$$

$$m_{12} = m_{21} , \quad m_{13} = m_{31} , \quad m_{23} = m_{32} .$$

8. Calculation of the right–hand side of $SLE_{3\times3}$ using the formulas

$$n_1 = \frac{1}{N_s}\sum_{i=1}^{N_s} f_i , \quad n_2 = \frac{1}{N_s}\sum_{i=1}^{N_s} x_i f_i , \quad n_3 = \frac{1}{N_s}\sum_{i=1}^{N_s} y_i f_i .$$

9. Reduction of $SLE_{3\times3}$ to $SLE_{2\times2}$, that is, the derivation of the linear system of equations of the form

$$\begin{aligned} a_0 M_1 + a_1 M_2 + N_1 , \\ a_0 M_3 + a_1 M_4 + N_2 , \end{aligned} \qquad (26)$$

and calculation of the left–hand–side coefficients of the $SLE_{2\times2}$ using the relations

$$M_1 = m_{12} - m_{13} m_{32} / m_{33} , \quad M_2 = m_{22} - m_{23} m_{32} / m_{33} ,$$

$$M_3 = m_{11} - m_{13} m_{31} / m_{33} , \quad M_4 = m_{21} - m_{23} m_{31} / m_{33} .$$

10. Calculation of the right–hand side of $SLE_{2\times2}$ using the relations

$$N_1 = n_2 - n_3 m_{32} / m_{33} , \quad N_2 = n_1 - m_{31} n_3 / m_{33} .$$

11. Evaluation of a_0 from the expression

$$a_0 = (N_1 - M_2 N_2 / M_4) / (M_1 - M_2 M_3 / M_4) , \qquad (27)$$

derived from $SLE_{2\times2}$, and its subsequent insertion in Eq. (24) to obtain the desired T, U, and V.

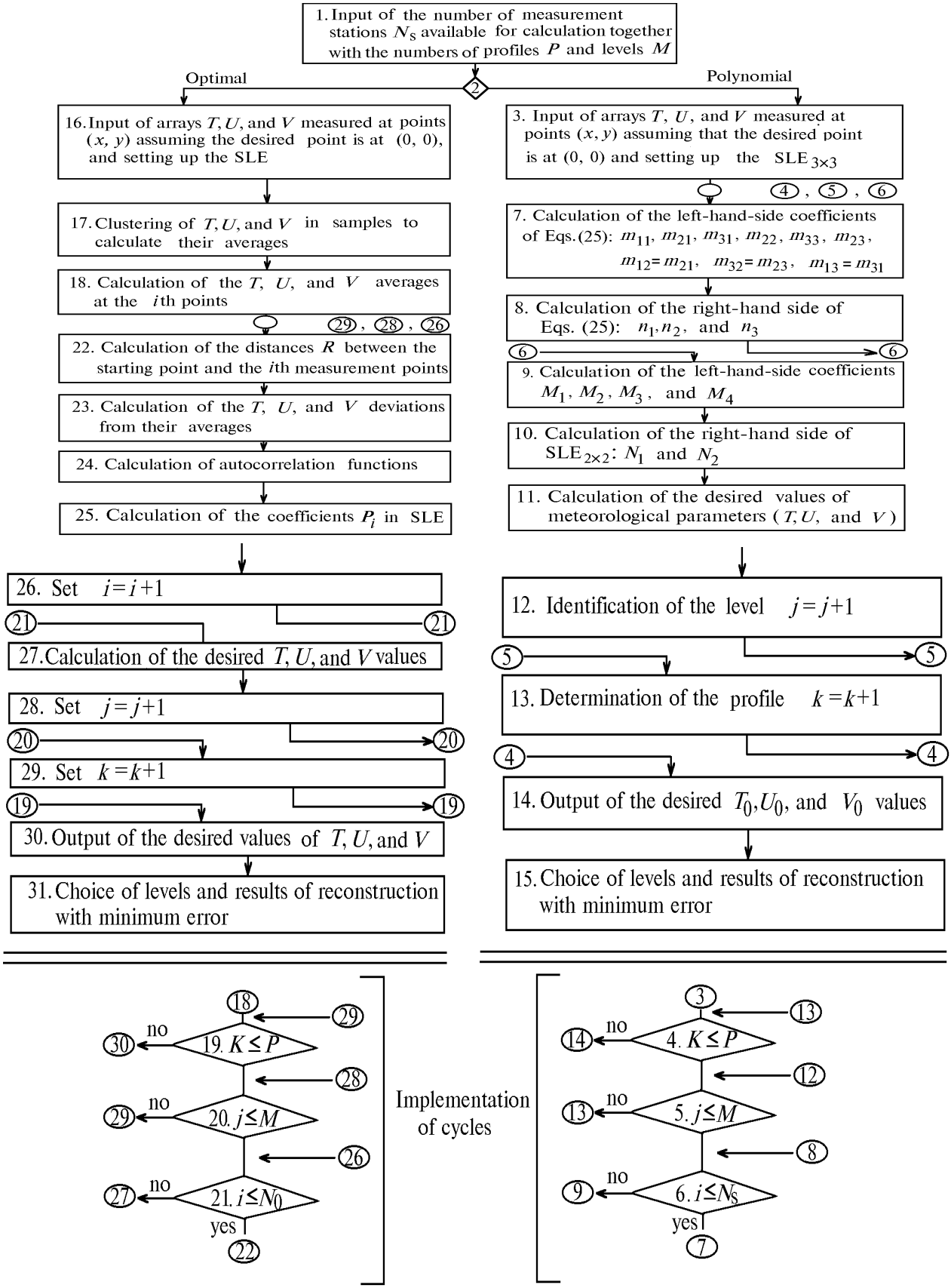

FIG. 1. Flowchart of spatial prediction by polynomial and optimal extrapolation techniques.

12. Setting $j = j + 1$, go through the cycle for all levels adopted.

13. Setting $k = k + 1$, go through the cycle for all profiles.

14. Output of the desired values in a table format.

15. Identification of levels and results of most accurate reconstruction.

When choosing the optimal extrapolation, T, U, and V are determined by another flowchart (see Fig. 1):

16. Input of arrays T, U, and V measured at points (x, y) together with their coordinates assuming the desired point to be at the origin of coordinates (0, 0).

17. Data conversion and clustering of initial sample T, U, and V to calculate their averages.

18. Calculation of averages $\overline{T}$, $\overline{U}$, and $\overline{V}$ at all the ith points.

19–21. Implementation of cycles for profiles $k \le p$, levels of measuring T, U, and V $j \le M$, and stations $i \le N_s$.

22. Calculation of the distances R from the initial point to the point of measuring T, U, and V by the formula

$$R = \sqrt{X^2 + Y^2} . \tag{28}$$

23. Calculation of deviation of measured T, U, and V from their averages by the formula

$$f' = f'_{\text{meas}} - \overline{f} . \tag{29}$$

24. Calculation of the autocorrelation function by the analytical expressions

$$\mu(R) = \exp(-0.825\, r^{0.92}) \tag{30}$$

for temperature and

$$\mu_u(R) = \mu_v(R) = (1 - 0.98\, R) \exp(-0.98\, R) \tag{31}$$

for wind velocity.

25. Calculation of the coefficients p_i in the system of equations (16) by the Gauss method accomplished in two steps.

First, we eliminate successively the unknowns from the equations with the help of relation

$$p_i = \left(\mu_{0i} - \sum_{j=1}^{n} p_i\, \mu_{ij} - p_i\, \eta \right) \Big/ \mu_{ij} , \tag{32}$$

where μ_{0i} and μ_{ij} are the autocorrelation functions, p_i is the weight, and η is a so–called measure of measurement error.

According to Eq. (32), each top–row element of the extended matrix of SLE must be divided by the diagonal element

$$\mu_{ij} = \mu_{ij} / \mu_{ii} . \tag{33}$$

Next, we substitute Eq. (32) into the other equations of the system thereby eliminating corresponding p_i from each equation. The elements of extended matrix are transformed as

$$\mu_{0i} = \mu_{ij} - \mu_{ii} / \mu_{jj} . \tag{34}$$

The above calculations result in SLE with top–triange matrix whose under–diagonal elements are equal to zero.

Next step is to solve SLE by successive determination of the unknown coefficients p_i from formula (16).

26. Setting $i = i + 1$, implement a cycle for all stations.

27. Calculation of the desired T, U, and V at the point (x_0, y_0) with the help of climatic average $\overline{a}_0$ (e.g., taken for regional model of altitude distribution of temperature and wind velocity components or obtained otherwise) by the formula

$$a_0 = \overline{a}_0 + \sum_{i=1}^{N_s} p_i\, a_i . \tag{35}$$

28. Setting $j = j + 1$, implement a cycle for all levels.

29. Setting $k = k + 1$, implement a cycle for all profiles.

30. Output of the results (desired T, U, and V) in a table format.

31. Identification of levels and results of the most correct reconstruction.

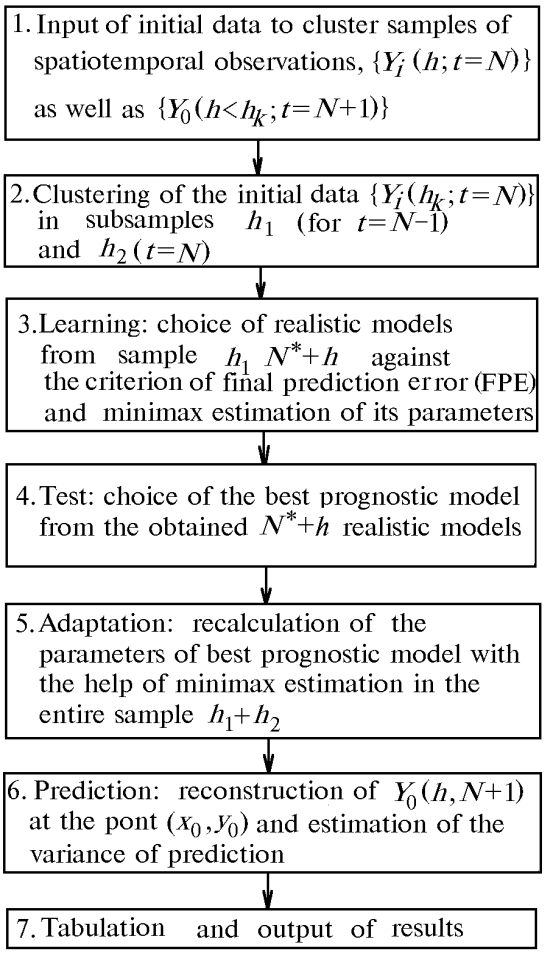

FIG. 2. Flowchart of reconstruction of vertical profiles of the meteorological parameters by modified MCA.

After the level (levels) with the least error in T, U, and V reconstruction has been identified, MMCA[11] is invoked to complete all levels. The algorithm proceeds as follows (Fig. 2).

1. Input of the arrays T, U, and V measured at the nearest (to the desired point) ith point (station), together with T, U, and V values reconstructed at the levels with the least ε by the optimal or polynomial extrapolation,

and clustering of the samples of spatiotemporal observations in the forms of Eqs. (21) and (22).

2. Clustering of the initial sample given by Eq. (21) in subsamples n_1 including observations made until $t = N - 1$ and n_2, including solely the observations at $t = N$ (test sample).

3. Learning block. From subsample n_1 we select $N^* + h$ realistic models (20) against Akaike's criterion (FPE)

$$\mathrm{FPE} = \frac{(N - N^* - 1) + s}{(N - N^* - 1) - s}\,\mathrm{RSS}(s)\,, \qquad (36)$$

where

$$\mathrm{RSS}(s) = \sum_{j=1}^{N-N^*-1} \left[Y^{(i)}_{h,\,N-j} - \hat{Y}^{(i)}_{h,\,N-j}(s) \right]^2$$

is the residual sum of squares for the current model $\hat{Y}^{(i)}_{h,\,N-j}(s)$ containing s nonzero estimates of the parameters

$$\hat{Y}^{(i)}_{h,\,N-j} = X\,\hat{\theta}\,,\quad X \in M_{(N-N^*-1)(N^*+h)}\,,\quad \hat{\theta} \in R^{N^*+h}\,,$$

where

$$\hat{\theta} = \left[\hat{A}_{h,\,1} \ldots \hat{A}_{h,\,N^*} \quad \hat{B}_{h,\,0} \ldots \hat{A}_{h,\,h-1} \right]^{\mathrm{T}} \qquad (37)$$

is the minimax estimate on the subsample n_1 calculated by formulas given below, R^k is Euclidean space of k–dimensional vectors, and $M_{m\times p}$ is the space of matrices of order $m\times p$.

4. Test block. From $N^* + h$ realistic models we select the only one (hereafter, the best model) with minimal prediction error on the test subsample n_2

$$\left| Y^{(i)}_{h,\,N} - \hat{Y}^{(i)}_{h,\,N}(s) \right| \to \min\,,$$

where minimum is taken over all $N^* + h$ structures, each of which corresponds to its own model $\hat{Y}^{(i)}_{h,\,N}(s)$.

Matrix of input variables (regressors) $X^*(s) \in M_{(N-N^*-1)s}$ of the best structure has the form

$$X^*(s) = \left[x_{p_1} \vdots x_{p_2} \vdots \ldots \vdots x_{p_s} \right], \qquad (38)$$

where $x_{p_1}, \ldots, x_{p_s}$ are the columns of the X matrix,

$$X = \begin{bmatrix} Y^{(i)}_{h,\,N-2} \ldots Y^{(i)}_{h,\,N-N^*-1} & \vdots & Y^{(i)}_{0,\,N-1} \ldots Y^{(i)}_{h-1,\,N-1} \\ \cdots & \vdots & \cdots \\ Y^{(i)}_{h,\,N^*} \ldots Y^{(i)}_{h,\,1} & \vdots & Y^{(i)}_{0,\,N^*+1} \ldots Y^{(i)}_{h-1,\,N^*+1} \end{bmatrix}, \qquad (39)$$

and $p_1, \ldots, p_s$ are the serial numbers of nonzero components of the vector–estimator.

5. Adaptation block. The vector $\hat{\theta} \in R^s$ of the parameters of the best structure (37) is recalculated (adapted) on the entire sample $n_1 + n_2$ of initial data (21)–(22) according to the formulas of the algorithm of minimax estimation:

$$\theta^* = \gamma\tilde{\theta}\,,\quad \tilde{\theta} = [\tilde{X}^{\mathrm{T}}\,\tilde{X}]^{-1}\,\tilde{X}^{\mathrm{T}}\,\tilde{Z}\,, \qquad (40)$$

where the vector–estimator

$$\gamma = \Delta^2_{h,\,N+1} \,/\, (\Delta^2_{h,\,N+1} + u_h)\,,\quad u_h = \tilde{x}^{\mathrm{T}}_h\,[\tilde{X}^{\mathrm{T}}\,\tilde{X}]^{-1}\,\tilde{x}_h\,\sigma^2\,, \qquad (41)$$

$\Delta_{h,\,N+1}$ are maximum permissible values of the predictant $Y^{0}_{n,\,N+1}$, which are either calculated by the formula

$$\Delta_{h,\,N+1} = \max_{t=1,\,\ldots,\,N} \left| Y^{(i)}_{h,\,t} \right| \qquad (h = 0,\ 1,\ \ldots,\ h^*) \qquad (42)$$

or specified by user, while the matrix $\tilde{X}$ of input variables and the vector of output variables $\tilde{Z}$ are defined as

$$\tilde{Z} = [\tilde{z}_i] = \begin{bmatrix} Y^{(i)}_{h,\,N-1} \\ \cdot \\ \cdot \\ \cdot \\ Y^{(i)}_{h,\,N^*+1} \\ \cdot \\ \cdot \\ \cdot \\ Y^{(i)}_{h,\,N} \end{bmatrix} \in R^{N-N^*}\,,$$

$$\tilde{X} = [\tilde{x}_{i\,j}] = \begin{bmatrix} X^*(s) \\ \cdot \\ \cdot \\ \cdot \\ \tilde{x}^{\mathrm{T}}_h \end{bmatrix} \in M_{(N-N^*)\,s}\,, \qquad (43)$$

$\tilde{x}_h = [\tilde{x}_{p_1}, \tilde{x}_{p_2}, \ldots, \tilde{x}_{p_s}]^{\mathrm{T}} \in R^s$, $\tilde{x}_{p_1}, \ldots, \tilde{x}_{p_s}$ are observations from the array

$$\left[Y^{(i)}_{h,\,N-1}\ Y^{(i)}_{h,\,N-2} \ldots Y^{(i)}_{h,\,N-N^*} \vdots Y^{(i)}_{0,\,N} \ldots Y^{(i)}_{h-1,\,N} \right] \qquad (44)$$

numbered by $p_1, \ldots, p_s$, respectively,

$$\sigma^2 = \frac{1}{N - N^* - s} \sum_{i=1}^{N-N^*} \left(\tilde{z}_i - \sum_{j=1}^{s} \tilde{x}_{ij}\,\tilde{\theta}_j \right)^2. \qquad (45)$$

6. Prediction block gives $Y^{*0}_{h,\,N+1}$ value at the point $(h,\ N + 1)$

$$Y^{*0}_{0,\,N+1} = \tilde{x}^{\mathrm{T}}_h\,\theta^* \qquad (46)$$

and the statistical estimate of the maximum prediction variance

$$E\,[E(Y(Y^{0}_{h,\,N+1}) - Y^{*0}_{h,\,N+1}]^2 \le \delta_{h,\,N+1}\,,$$

$$\delta_{h,\,N+1} = u_h\,\Delta^2_{h,\,N+1} \,/\, (u_h + \Delta^2_{h,\,N+1})\,, \qquad (47)$$

where u_h is evaluated from Eq. (41).

7. Output of the results of spatial prediction of the vertical profile $Y_0(h,\ N + 1)$ completing the vertical profile f.

Thus, the problem of spatial prediction of the vertical profiles of temperature and wind velocity components for regions uncovered with temperature and wind sensing data has been completely solved. Summarizing, it may be said that efficiency of the integrated method just described can be evaluated only from the results of numerical experiments based on the data of vertical sensing performed at different points of a mesometeorological polygon. This will be the subject of the second part of the paper published in this issue.

REFERENCES

1. P.N. Belov, E.P. Borisenkov, and B.D. Panin, *Numerical Methods of Prediction* (Gidrometeoizdat, Leningrad, 1989), 376 pp.
2. B. Gilchrist and G.P. Cressman, Tellus **6**, No. 4, 309–318 (1954).
3. F.H. Bushby and V.M. Huckle, Quart. J. Roy. Meteorol. Soc. **83**, No. 356, 232–247 (1957).
4. G.A. Corby, Quart. J. Roy. Meteorol. Soc. **87**, No. 371, 34–42 (1961).
5. G.A. Corby, Tellus **15**, No. 4, 432–438 (1963).
6. V.V. Bykov and G.P. Kurbatkin, Izv. Akad. Nauk SSSR, Ser. Geofiz., No. 2, 307–318 (1961).
7. V.V. Bykov, G.P. Kurbatkin, and N.V. Gorel'sheva, Trudy MMT, No. 4, 56–72 (1964).
8. P.N. Belov, *Applications of Numerical Weather Forecast* (Gidrometeoizdat, Leningrad, 1967), 355 pp.
9. S.L. Belousov, L.S. Gandin, and S.A. Mashkovich, *Processing of Real–Time Meteorological Information on a Computer* (Gidrometeoizdat, Leningrad, 1968), 282 pp.
10. L.S. Gandin and R.L. Kagan, *Statistical Methods of Interpolation of Meteorological Data* (Gidrometeoizdat, Leningrad, 1976), 359 pp.
11. Yu.L. Kocherga, Avtomatika, No. 5, 80–87 (1991).
12. M.G. Papazov and S.G. Mogil'nyi, *Error and Least–Square Method Theories* (Nedra, Moscow, 1968), 303 pp.
13. D.I. Kazakevich, *Foundations of the Theory of Random Functions and its Application to Hydrometeorology* (Gidrometeoizdat, Leningrad, 1977), 319 pp.
14. Yu.M. Liberman, in: *Statistical Methods for Meteorological Application* (Gidrometeoizdat, Moscow, 1978), pp. 162–165.
15. V.E. Zuev and V.S. Komarov, *Statistical Models of Temperature and Gaseous Components of the Atmosphere* (Gidrometeoizdat, Leningrad, 1986), 264 pp.
16. V.S. Komarov, V.I. Akselevich, and A.V. Kreminskii, Atmos. Oceanic Opt. **7**, No. 2, 121–124 (1994).