NUMERICAL MODELS FOR SPATIAL CORRELATION FUNCTIONS OF WIND WAVE SLOPES AS APPLIED TO SOME PROBLEMS OF SEA OPTICS

V.L. Veber

Institute of Applied Physics,
Russian Academy of Sciences, Nizhnii Novgorod
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Simple numerical spline models for spatial correlation functions for slopes of the one–dimensional and isotropic wind rough sea are developed. The models are based on the modified Pearson–Moskowitz spectrum with the Gaussian cutting function. A parameter of the cutting is chosen so that the fourth moment of the spectrum calculated theoretically and the measured variance of sea surface curvatures would be in a good agreement.

In a theoretical study devoted to the problem of vision, detection and ranging, and communication through a roughed sea surface, a correct choice of correlation functions of the surface slopes plays an important role. It should not be denied, certainly, that approximate models of these functions can be correctly used, especially, to reveal a physical nature of some optical phenomena and estimate a role of that or another factor in the process of optical signal formation. But in these cases a necessity always remains of testing the obtained results on the basis of a sufficiently rigorous numerical model. A goal of this paper is to construct rigorous numerical models of the slope spatial correlation function well agreeing with certain experimental data.

1. SPECTRUM OF WIND ROUGH SEA

Solving many problems of the sea statistical optics the authors (see, for example, Refs. 1 and 2) successfully employ in recent years the spline models of the spatial correlation functions of slopes of isotropic and one–dimensional wind rough sea obtained based on the modified Pearson–Moskowitz spectrum. Let us say at first that the modification of the spectrum is to introduce into the spectrum formula the Gaussian cutting function as a factor. Parameters of such a function provide an agreement between the theoretical and experimental values of the variance of slopes and curvatures of the surface. Let us consider this question in more detail.

The expression for the Pearson–Moskowitz frequency spectrum for the wind rough sea has the following form:

$$S_0(\omega, \Theta) = \beta g^2 \omega^{-5} \exp\left[-0.74\left(\frac{g}{\omega v}\right)^4\right]\varphi(\Theta) , \qquad (1)$$

where ω is the frequency, $\varphi(\Theta)$ is the angular dependence of the spectrum ($\int\limits_{-\pi}^{\pi}\varphi(\Theta)\mathrm{d}\Theta = 1$), v is the wind velocity, $g = 9.8$ m/s^2, and $\beta = 8.1\cdot10^{-3}$.

From the relation

$$S_0(\omega, \Theta)\,\mathrm{d}\omega\,\mathrm{d}\Theta = S_0(\mathbf{k})\,\mathrm{d}\mathbf{k} ,$$

where $\mathbf{k} = (k_x, k_y)$ and $\mathrm{d}\mathbf{k} = \mathrm{d}k_x\,\mathrm{d}k_y$, it follows that

$$S_0(\mathbf{k}) = S_0\,[\omega(\mathbf{k}), \Theta(\mathbf{k})] \begin{vmatrix} \dfrac{\partial\omega}{\partial k_x} & \dfrac{\partial\omega}{\partial k_y} \\ \dfrac{\partial\Theta}{\partial k_x} & \dfrac{\partial\Theta}{\partial k_y} \end{vmatrix} . \qquad (2)$$

Using the dispersion relation for waves on water surface

$$\omega = \sqrt{g\,k},$$

where $k = |\mathbf{k}|$ and $\Theta = \arctan k_y/k_x$, we obtain from Eq. (2) the expression for the Pearson–Moskowitz spectrum as the function of the wave number $\mathbf{k}$

$$S_0(\mathbf{k}) = S_0\,[\omega(\mathbf{k}), \Theta(\mathbf{k})]\; 0.5\, g^{1/2}\, k^{-3/2} . \qquad (3)$$

The expression (3) can be written as

$$S_0(\mathbf{k}) = \frac{\beta}{2k^4}\exp\left(-\frac{\alpha}{k^2}\right)\varphi\left(\arctan\frac{k_y}{k_x}\right), \qquad (4)$$

where $\alpha = 0.74 g^2/v^4$.

The expression (4) for the Pearson–Moskowitz spectrum is written in the Cartesian coordinates. In polar coordinates this expression has the following form:

$$S_0(k, \Theta) = \frac{\beta}{2k^4}\exp\,(-\,\alpha/k^2)\;\varphi(\Theta) \qquad (5)$$

(note that the area element here is $\mathrm{d}\mathbf{k} = k\,\mathrm{d}k\,\mathrm{d}\Theta$).

It is not difficult to make sure that the expressions (1)–(5) cannot be used to calculate the variances of slopes and curvatures of rough sea because the second and the fourth integral moments tend to

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infinity. Let us modify the spectrum (5) by introducing of the Gaussian cutting function

$$S(k,\,\Theta) = S_0(k,\,\Theta)\,\exp\left(-\frac{k^2}{k_m^2}\right). \quad (6)$$

A question on the procedure of the parameter k_m selection was discussed in Ref. 3 where a method of k_m determination has been proposed. According to this method the theoretical value of the slope variance (the second moment of the spectrum (6)) is equated to the corresponding experimental value.[4] But, when analyzing in detail, it appears that such a procedure does not provide a coincidence of the fourth theoretical moment of the spectrum with the measured variance of sea surface curvatures.[5] This variance plays an important role in the problems of the sea surface optics, and an error in its determination is surely undesirable. That is why a principle of agreement of theoretical and experimental estimations of the fourth moment of the rough sea spectrum (the curvature variance) is a basis for the method of determining k_m.

Let us use the data (as the basic experimental dependences) for the rough sea slope variance $\sigma_x^2(v)$ obtained by Cox and Munk[4] and data for the curvature variance $\sigma_{x\,x}^2(v)$ obtained by Burtsev and Pelevin[5]:

variance of the wind wave slopes

$$\sigma_x^2 = (3 + 1.92v)\cdot 10^{-3}\,, \quad (7)$$

variance of the wind wave curvatures

$$\sigma_{x\,x}^2 = (-\,4.13 + 1.23v)^2\ \mathrm{m}^{-2} \quad (8)$$

(formula (8) is valid at $v > 3.5$ m/s).

Generally speaking, we have no exact information about the angular spectrum of the rough sea these experimental data were obtained for. However, since the variances of slopes and curvatures are determined in principle by a high–frequency portion of the rough sea spectrum having a distribution which is close to the isotropic, the dependencies (7) and (8) can be classified as a statistically isotropic rough sea to a certain extent of reliability.

The theoretical values of the variances of slopes and curvatures of a statistically isotropic surface are calculated by formulas

$$\bar{\sigma}_x^2 = \int\limits_{-\infty}^{\infty}\!\!\int k_x^2\, S(\mathbf{k})\,\mathrm{d}\mathbf{k} = \frac{1}{2}\int\limits_0^{\infty} S(k)\,k^3\,\mathrm{d}k\,; \quad (9)$$

$$\bar{\sigma}_{x\,x}^2 = \int\limits_{-\infty}^{\infty}\!\!\int k_x^4\, S(\mathbf{k})\,\mathrm{d}\mathbf{k} = \frac{3}{8}\int\limits_0^{\infty} S(k)\,k^5\,\mathrm{d}k\,. \quad (10)$$

Substituting into these formulas the expression (6) for the rough sea spectrum, we obtain after integration

$$\bar{\sigma}_x^2 = \frac{\beta}{4}\,K_0\left(2k_m\sqrt{\alpha}\right),$$

$$\bar{\sigma}_{x\,x}^2 = \frac{3}{16}\,\beta k_m\sqrt{\alpha}\,K_1\left(2k_m\sqrt{\alpha}\right),$$

where $K_0(\bullet)$ and $K_1(\bullet)$ are the McDonald functions.

For a small argument the McDonald functions can be presented as asymptotic dependences:

$$K_0(x) \approx -\ln x\,, \qquad K_1(x) \approx x^{-1}\,. \quad (11)$$

In this case the expressions for the variances of slopes and curvatures reduce to the forms

$$\bar{\sigma}_x^2 = -\frac{\beta}{4}\ln\left(2\,k_m^{-1}\sqrt{\alpha}\right), \quad (12)$$

$$\bar{\sigma}_{x\,x}^2 = \frac{3}{32}\,\beta\,k_m^2\,. \quad (13)$$

By substituting an experimental value of the curvature variance into the left-hand side of the relation (13), we determine the parameter k_m of the cutting function in the rough sea spectrum (6):

$$k_m = \sqrt{32\,\frac{\sigma_{x\,x}^2}{3\beta}}\,. \quad (14)$$

To examine the assumption (11), the curvature variance for the modified spectrum with the cutting parameter (14) was calculated numerically. It appeared that over the wind velocity range of 4–10 m/s the numerical and experimental estimations are close. To provide the coincidence of the theoretical and experimental estimations of the rough sea slope variances, the parameter β was required to be changed slightly. The best agreement between estimations is achieved at $\beta = 11.5\cdot 10^{-3}$ (Fig. 1).

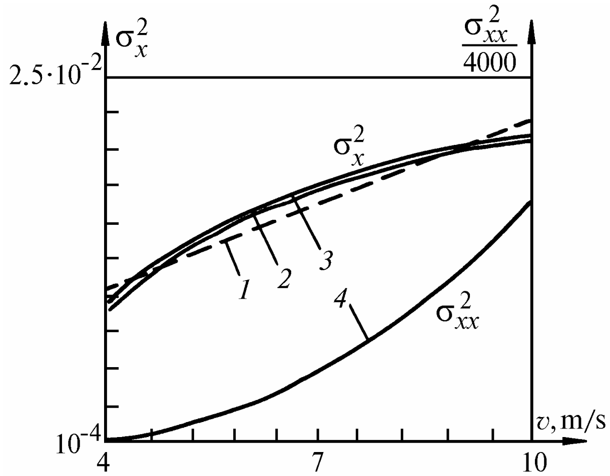

FIG. 1. Variances of slopes σ_x^2 and curvatures $\sigma_{x\,x}^2$ as functions of the wind velocity, σ_x^2: 1) experiment (7), 2) theory (12), and 3) theory (9); $\sigma_{x\,x}^2$, 4) experiment (8) and theory (10) and (13) (deviation is no more than 1 %).

Let us give a summary of the formulas describing the modified version of the Pearson–Moskowitz spectrum of wind rough sea in the final form

$$S(\mathbf{k}) = S(k)\ \varphi(\Theta)\ ,$$

$$S(\mathbf{k}) = \frac{\beta}{2k^4} \exp\left(-\frac{\alpha}{k^2} - \frac{k^2}{k_m^2}\right); \qquad (15)$$

$$\alpha = \frac{0.74g^2}{v^4}\ ; \quad k_m^2 = \frac{32}{2}\frac{\sigma_{x\,x}^2}{\beta}\ ;$$

$$\sigma_{x\,x}^2 = (-\,4.13 + 1.23v)^2\ ; \quad \beta = 11.5\cdot10^{-3}\ .$$

Let us proceed to consideration of the correlation functions of sea surface slopes. Since, in the problems of the sea surface optics, two models (the alternative ones, in certain sense) of the rough sea, isotropic and one–dimensional, are considered most often, we create a numerical model of the spatial slope correlation function for every of the models mentioned.

TABLE I.

ρ, m	R_1	R_1'	ρ, m	R_2	R_2'
0.00 + 0	1.000 + 0	0.000 + 0	0.00 + 0	1.000 + 0	0.000 + 0
3.00 − 2	9.261 − 1	− 3.487 + 0	3.00 − 2	8.171 − 1	− 7.474 + 0
2.10 − 1	6.100 − 1	− 7.928 − 1	6.00 − 2	6.672 − 1	− 3.381 + 0
3.70 − 1	5.075 − 1	− 5.110 − 1	1.30 − 1	5.179 − 1	− 1.382 + 0
7.00 − 1	3.925 − 1	− 2.456 − 1	3.30 − 1	3.470 − 1	− 5.288 − 1
1.40 + 0	2.703 − 1	− 1.241 − 1	5.50 − 1	2.556 − 1	− 3.244 − 1
2.70 + 0	1.625 − 1	− 5.559 − 2	8.00 − 1	1.904 − 1	− 2.128 − 1
4.80 + 0	8.227 − 2	− 2.545 − 2	1.25 + 0	1.162 − 1	− 1.277 − 1
8.00 + 0	3.101 − 2	− 9.442 − 3	2.00 + 0	4.615 − 2	− 6.778 − 2
1.20 + 1	7.879 − 3	− 3.112 − 3	2.90 + 0	5.924 − 4	− 3.679 − 2
1.50 + 1	1.657 − 3	− 1.260 − 3	3.00 + 0	− 2.973 − 3	− 3.455 − 2
1.60 + 1	5.803 − 4	− 9.055 − 4	4.20 + 0	− 3.170 − 2	1.554 − 2
1.70 + 1	− 1.808 − 4	− 6.307 − 4	5.80 + 0	− 4.594 − 2	− 3.867 − 3
2.00 + 1	− 1.236 − 3	− 1.416 − 4	9.00 + 0	− 4.274 − 2	3.867 − 3
2.40 + 1	− 1.254 − 3	8.132 − 5	1.30 + 1	− 2.523 − 2	4.308 − 3
3.00 + 1	− 6.143 − 4	1.048 − 4	1.60 + 1	− 1.390 − 2	3.204 − 3
3.80 + 1	− 5.957 − 5	3.746 − 5	2.20 + 1	− 1.180 − 3	1.181 − 3
4.00 + 1	2.307 − 6	2.498 − 5	2.40 + 1	7.019 − 4	7.251 − 4
4.40 + 1	6.532 − 5	8.092 − 6	2.80 + 1	2.333 − 3	1.559 − 4
5.00 + 1	7.293 − 5	− 3.231 − 6	3.40 + 1	2.072 − 3	− 1.628 − 4
5.80 + 1	3.483 − 5	− 4.923 − 6	4.00 + 1	1.004 − 3	− 1.690 − 4
6.60 + 1	5.344 − 6	− 2.423 − 6	4.60 + 1	2.007 − 4	− 9.652 − 5
6.80 + 1	1.111 − 6	− 1.825 − 6	4.80 + 1	3.283 − 5	− 7.195 − 5
7.00 + 1	− 2.013 − 6	− 1.311 − 6	5.00 + 1	− 8.934 − 5	− 5.076 − 5
7.80 + 1	− 6.447 − 6	3.430 − 9	6.00 + 1	− 2.343 − 4	9.077 − 6
8.40 + 1	− 5.232 − 6	3.313 − 7	6.60 + 1	− 1.541 − 4	1.543 − 5
9.20 + 1	− 2.367 − 6	3.338 − 7	7.40 + 1	− 4.286 − 5	1.109 − 5
1.00 + 2	− 3.173 − 7	1.768 − 7	7.80 + 1	− 6.305 − 6	7.291 − 6
1.04 + 2	2.337 − 7	1.034 − 7	8.00 + 1	6.559 − 6	5.588 − 6
1.12 + 2	6.340 − 7	7.396 − 9	9.40 + 1	2.615 − 5	− 1.175 − 6
1.20 + 2	5.108 − 7	− 3.053 − 8	1.04 + 2	1.101 − 5	− 1.506 − 6
1.24 + 2	3.838 − 7	− 3.044 − 8	1.12 + 2	1.406 − 6	− 8.726 − 7
1.28 + 2	2.559 − 7	− 3.889 − 8	1.16 + 2	− 1.416 − 6	− 5.462 − 7
1.32 + 2	1.000 − 7	− 2.685 − 8	1.32 + 2	− 3.180 − 6	1.566 − 7
1.36 + 2	5.000 − 8	− 8.099 − 9	1.36 + 2	− 2.496 − 6	1.753 − 7
1.40 + 2	1.000 − 8	− 8.250 − 9	1.40 + 2	− 1.755 − 6	2.111 − 7
1.50 + 2	0.000 + 0	0.000 + 0	1.50 + 2	0.000 + 0	0.000 + 0

The values in Table I are presented by two numbers. The first number is the mantissa and the second number is the order (for example, 1.40+2 means $1.4\cdot10^2$ and 9.652–5 means $9.652\cdot10^{-5}$).

2. CORRELATION FUNCTIONS OF THE ISOTROPIC ROUGH SEA SLOPES

As known, the spatial correlation function of elevations of two–dimensional random surface is determined from its spatial spectrum as follow:

$$M_\varsigma(\rho) = \int_{-\infty}^{\infty}\int S(\mathbf{k}) \cos\mathbf{k}\rho \, \mathrm{d}\mathbf{k} . \qquad (16)$$

For the statistically isotropic surface ($\varphi(\Theta) = 1/2\pi$) it follows from Eq. (16) that

$$M_\varsigma(\rho) = \int_0^\infty S(k) \, J_0(k\,\rho) \, k \, \mathrm{d}k , \qquad (17)$$

where $\rho = |\rho|$, and $J_0(\ldots)$ is the Bessel function.

The slope correlation functions $M_x(\rho)$, $M_y(\rho)$, and $M_{xy}(\rho)$ of the sea surface are expressed in the case of the isotropic rough sea from two general functions M_1 and M_2 (Ref. 6):

$$M_x(\rho) = \rho_x^2 \, M_2(\rho) + \rho_y^2 \, M_1(\rho) ,$$

$$M_y(\rho) = \rho_x^2 \, M_1(\rho) + \rho_y^2 \, M_2(\rho) ,$$

$$M_{x\,y}(\rho) = \rho_x \, \rho_y \, [M_2(\rho) - M_1(\rho)] ,$$

where

$$M_1(\rho) = -\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho} M_\varsigma(\rho) ; \;\; M_2(\rho) = -\frac{\mathrm{d}^2}{\mathrm{d}\rho^2} M_\varsigma(\rho) .$$

By substituting the expression (17) into the relations for $M_{1,2}$ we obtain:

$$M_1(\rho) = \frac{1}{\rho}\int_0^\infty k^2 \, S(k) \, J_1(k\,\rho) \, \mathrm{d}k ; \qquad (18)$$

$$M_2(\rho) = \int_0^\infty k^2 \, S(k) \, J_0(k\,\rho) \, k \, \mathrm{d}k - M_1(\rho) . \qquad (19)$$

The relations (18) and (19) are the Fourier–Bessel transforms of the spectrum of the statistically isotropic rough sea. The correlation functions M_1 and M_2 were computed with the use of the spectrum (15) for several values of the wind velocity (4, 6, and 8 m/s). The obtained data array for every ρ describes the functions (18) and (19) at a more than 200 points (nonequidistant sequence) over the range ρ = 0–270 m. To compute the function between the nodes, we used the method of cubic spline–interpolation[7]

$$y(x) = y_i + a_i \, h_i \left[y_i' + a_i \, (c_i + a_i \, b_i) \right] , \qquad (20)$$

where

$$a_i = (x - x_i) \; / \; h_i ; \;\; h_i = x_{i+1} - x_i ;$$

$$b_i = y_{i+1}' - y_i' - 2 \, (y_{i+1} - y_i) \; / \; h_i ;$$

$$c_i = - b_i + (y_{i+1} - y_i) \; / \; h_i - y_i' .$$

The spline derivatives were computed by the procedures of direct and back pass of the initial array of the function values.[7] These procedures provide a continuity not only of the first but of the second derivatives at the spline nodes. The author of the present paper optimized the data array intended for the spline–interpolation in the sense of minimization of the node number and their optimal arrangement along the coordinate ρ. This optimization allowed the node number necessary for interpolation to be reduced essentially without the loss in accuracy of determination of the function values. Table I presents the values of normalized functions

$$R_1(\rho) = M_1(\rho)/\bar{\sigma}_x^2 , \;\; R_2(\rho) = M_2(\rho)/\bar{\sigma}_x^2$$

and their derivatives at the nodes ρ_i for the wind velocity of 6 m/s. Analysis has shown that the use of this data in the formula (20) makes the interpolation error not more than 1%.

3. CORRELATION FUNCTION OF THE ONE–DIMENSIONAL ROUGH SEA SLOPES

In the case of a one–dimensional rough sea ($\varphi(\Theta) = \delta(\Theta)$) we obtain from Eq. (16) a formula for the correlation function of the sea surface elevations:

$$M_\varsigma(\rho) = \int_0^\infty S(k) \cos k\rho \; k \, \mathrm{d}k ,$$

where $\rho = \rho_x$ is the coordinate along the wave propagation direction.

Hence, it is not very difficult to obtain an expression for the spatial correlation function of the surface slopes

$$M_x(\rho) = \frac{\mathrm{d}^2}{\mathrm{d}\rho^2} M_\varsigma(\rho) = \int_0^\infty k^3 \, S(k) \cos k\rho \, \mathrm{d}k . \qquad (21)$$

The theoretical values of the slope and curvature variances of one–dimensional wind rough sea can be calculated by the formulas

$$\bar{\sigma}_x^2 = \int_0^\infty k^3 \, S(k) \, \mathrm{d}k , \qquad (22)$$

$$\bar{\sigma}_{x\,x}^{\,2} = \int_0^\infty k^5 \, S(k) \, \mathrm{d}k . \qquad (23)$$

Note, by the way, that the slope variance of one–dimensional rough sea is two times as much as the analogous value for the isotropic rough sea, and the value of curvatures variance of one–dimensional rough sea is 8/3 of the corresponding value for the isotropic rough sea. That can be easily tested by comparing the expressions (9) and (22), (10) and (23).

The relation (21) is the Fourier transform of the one-dimensional rough sea spectrum. The correlation function M_x was calculated numerically with the use of the spectrum (16) for the wind velocities of 4, 6, and 8 m/s. The obtained data array for every value of ρ describes the values of the function (21) at more than 120 points over the range ρ = 0–190 m. To calculate the functions between the nodes, the method of cubic spline interpolation described above is used. The spline derivatives are computed using known procedure of direct and back pass of the initial array of the function values. The data array for the spline interpolation were optimized.

Table II presents the values of the normalized function $R_x(\rho) = M_x(\rho)/\bar{\sigma}_x^2$ and its derivatives at the nodes ρ_i when the wind velocity is 6 m/s. The use of these data in the formula (21) makes the interpolation error which does not exceed 1 %.

TABLE II.

ρ, м	R_x	R'_x
0.00 + 0	1.000 + 0	0.000 + 0
3.00 − 2	7.773 − 1	− 8.575 + 0
6.00 − 2	6.273 − 1	− 2.967 + 0
1.40 − 1	4.680 − 1	− 1.341 + 0
3.00 − 1	3.292 − 1	− 5.742 − 1
5.00 − 1	2.380 − 1	− 3.603 − 1
9.00 − 1	1.374 − 1	− 1.786 − 1
1.50 + 0	5.776 − 2	− 9.682 − 2
2.30 + 0	2.155 − 3	− 4.910 − 2
2.40 + 0	− 2.586 − 3	− 4.573 − 2
3.50 + 0	− 3.630 − 2	− 1.903 − 2
5.00 + 0	− 5.156 − 2	− 3.569 − 3
8.00 + 0	− 4.424 −	5.782 − 3
1.20 + 1	− 2.043 − 2	5.385 − 3
1.70 + 1	− 1.272 − 3	2.362 − 3
1.80 + 1	8.285 − 4	1.854 − 3
2.40 + 1	5.489 − 3	6.184 − 6
2.90 + 1	4.155 − 3	− 4.262 − 4
3.50 + 1	1.582 − 3	− 3.788 − 4
4.00 + 1	1.105 − 4	− 2.108 − 4
4.50 + 1	− 5.746 − 4	− 7.207 − 5
5.00 + 1	− 7.043 − 4	1.022 − 5
5.50 + 1	− 5.549 − 4	4.319 − 5
6.00 + 1	− 3.225 − 4	4.610 − 5
6.50 + 1	− 1.181 − 4	3.451 − 5
7.00 + 1	1.735 − 5	1.974 − 5
8.00 + 1	9.876 − 5	− 5.219 − 7
9.00 + 1	5.691 − 5	− 5.785 − 6
1.00 + 2	7.905 − 6	− 3.594 − 6
1.05 + 2	− 6.013 − 6	− 2.027 − 6
1.10 + 2	− 1.281 − 5	− 7.305 − 7
1.15 + 2	− 1.427 − 5	− 8.139 − 9
1.20 + 2	− 1.243 − 5	9.931 − 7
1.30 + 2	0.000 + 0	0.000 + 0

Figure 2 presents the dependence $R_x(\rho)$ obtained from the data of Table II with the use of the interpolation formulas (20).

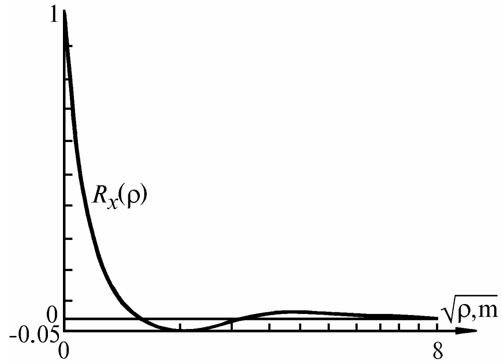

FIG. 2. Normalized correlation function of slopes of the one–dimensional rough sea for the wind velocity of 6 m/s.

A peculiarity of this dependence is the region in the vicinity of small ρ, where this dependence quickly varies and also the presence of a comparatively slow oscillations at large argument. The first is explained by effect of the short gravitation and capillary waves (ripple) on the slope correlation function, and the second is explained by the action of the long (power carrying) gravitation waves at the sea surface.

It should be noted in conclusion that with refinement of the model of the wind rough sea and obtaining of new (more accurate) experimental data on the variance of slopes and curvatures of the rough sea surface our model of the slope correlation functions will be also refined. But it will not require any changes in the technique discussed above.

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