DIFFRACTION TOMOGRAPHIC WAVE FRONT SENSOR

V.P. Aksenov, V.A. Banakh, E.V. Zakharova, Yu.N. Isaev, and O.V. Tikhomirova

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk
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The tomographic method for measuring the phase distribution in an optical beam cross section is proposed and tested in the numerical experiment. The phase is reconstructed from the measurement data on intensity integral moments of an image formed by a receiving aperture at different positions of slit and strip diaphragms.

The problem of checking quality of optical elements, mirror and lens surfaces, etc. arises in constructing instruments of astronomical optics and manufacturing other optical instruments. The devices for measuring optical wave phases (the so-called wave front sensors) are an important part of adaptive system. The corresponding meters are well developed.[1–3] The interferometric and diffraction methods of analysis are most widely spread.

Interferometry is able to provide the required accuracy of a control. But its practical use faces serious problems in the case of incoherent radiation and weak fluxes. Besides, the wave front error varying in time leads to interferogram blurring and makes its processing difficult.

The widely spread Hartmann method removes these problems to considerable extent, but in its turn it has a low spatial resolution on the optical surface. Besides, Hartmann diaphragm uses only a part of the light flux incident on the receiving lens what decreases the efficiency of the method for weak light fluxes. If the spatial distribution of the light beam intensity over the subaperture is not homogeneous, Hartmann method makes uncontrollable mistakes. In this paper we propose a method which is, in our opinion, free from the above-mentioned shortcomings.

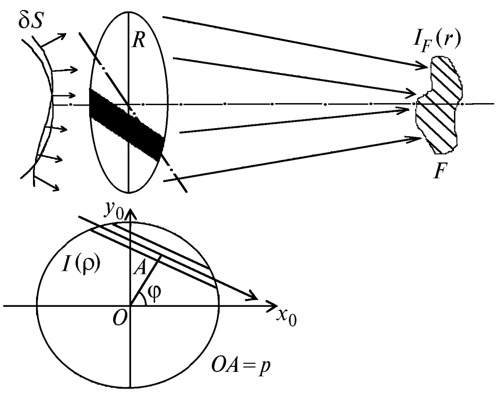

FIG. 1. Measurement scheme in the diffraction tomographic sensor.

Let us consider the measurement scheme presented in Fig. 1. We are interested in the phase distribution of the optical beam $S(\rho_0)$ with the intensity distribution $I(\rho_0)$. The wave front to be measured falls onto a receiving lens of the radius R with a slit aperture diaphragm. The coordinate system $\{x_0, y_0\}$ is connected with the center of the receiving lens. The position of the slit axis is given by the normal equation of the straight line $p - x_0\cos\varphi - y_0\sin\varphi = 0$. The aperture function of the lens adjusted in such a way is denoted by $K(\rho_0)$.

We characterize the intensity distribution $I_F(\rho)$ in the focal plane of the lens by the following integral moments:

$$M_{FR}^0 = \int\int I_F(\rho)\mathrm{d}\rho \,, \tag{1}$$

and

$$\mathbf{M}_{FR} = \int\int I_F(\rho)\rho\, \mathrm{d}\rho \,. \tag{2}$$

It is easy to show[4] that the intensity distribution in the focal plane of the lens and the second moment of the field of the wave incident onto the lens $U(\rho_0)U^*(\rho_0')$ are connected by the formula

$$I_F(\rho)= \frac{k}{4\pi^2F^2}\int\int \mathrm{d}^2\rho_0\int\int \mathrm{d}^2\rho_0' U(\rho_0)U^*(\rho_0')K(\rho_0) \times$$

$$\times K^*(\rho_0')\exp\left\{\frac{ik\rho(\rho_0 - \rho_0')}{F}\right\}, \tag{3}$$

where F is the focal length of the lens, $k = 2\pi/\lambda$, λ is the wavelength. Now let us obtain the value of the moments (1) and (2) in the focal plane of the lens. Substituting Eq. (3) into Eqs. (1) and (2) and using well-known relations

$$\int\limits_{-\infty}^{\infty}\int \exp\left\{\frac{ik\,\rho(\rho_0 - \rho_0')}{F}\right\}\mathrm{d}^2\rho =$$

$$= \frac{4\pi^2F^2}{k^2}\delta(x_0 - x_0')\,\delta(y_0 - y_0');$$

$$\int_{-\infty}^{\infty}\!\!\int \exp\left\{\frac{ik\rho(\rho_0 - \rho_0')}{F}\right\}\rho d^2\rho =$$

$$= \frac{4\pi^2 F^3}{ik^3}\,\delta(y_0 - y_0')\delta'(x_0 - x_0')\mathbf{l} + \delta(x_0 - x_0')\delta'(y_0 - y_0')\mathbf{m}\ ,$$

where **l** and **m** are unit basic vectors in the plane of the lens, we obtain

$$M^0_{FR} = \int\!\!\int K(\rho_0)I(\rho_0)d^2\rho_0; \qquad (4)$$

$$\mathbf{M}_{FR} = \frac{F}{ik}\int\!\!\int K(\rho_0)U(\rho_0)\nabla_\perp U^*(\rho_0)d^2\rho_0\ . \qquad (5)$$

Taking into account that the value $\mathbf{M}_{FR}$ is real, one can write

$$\mathbf{M}_{FR} = \frac{F}{2ik}\int_{-\infty}^{\infty}\!\!\int K(\rho_0)\{U^*(\rho_0)\nabla_\perp U(\rho_0) -$$
$$- U(\rho_0)\nabla_\perp U^*(\rho_0)\}d^2\rho_0\ .$$

Since the value

$$\frac{1}{2i}\,(U^*\nabla_\perp U - U\nabla_\perp U^*) = I(\rho_0)\ \nabla_\perp S(\rho_0)$$

is the transverse component of the Umov–Poynting vector, we finally obtain

$$\mathbf{M}_{FR} = -\,\frac{F}{k}\int_{-\infty}^{\infty}\!\!\int K(\rho_0)I(\rho_0)\nabla_\perp S(\rho_0)d^2\rho_0. \qquad (6)$$

Approximating the slit diaphragm (whose length is limited by the lens size) by the function

$$K(x_0,y_0) = \text{rect}\left(\sqrt{x_0^2 + y_0^2} - R\right) \times$$
$$\times\ \delta(p - x_0\cos\varphi - y_0\sin\varphi),$$

where $\text{rect}(\xi) = 1$ for $\xi < 0$ and $\text{rect}(\xi) = 0$ for $\xi > 0$, from Eqs. (4) and (5) we have

$$M^0_{FR}(p,\varphi) = \int\!\!\int dx\ dy\ \text{rect}(\sqrt{x^2+y^2} - R)\ \times$$
$$\times\ I(x,y)\delta(p - x\cos\varphi - y\sin\varphi), \qquad (7)$$

$$\mathbf{M}_{FR}(p,\varphi) = \int\!\!\int dx\ dy\ \text{rect}(\sqrt{x^2+y^2} - R)I(x,y)\ \times$$
$$\times\ \nabla S(x,y)\ \delta(p - x\cos\varphi - y\sin\varphi). \qquad (8)$$

The integrals (7) and (8) are Radon transformation[5] of the two-dimensional intensity distribution and the transverse component of the Umov–Poynting vector.

Therefore, conducting the measurement of the values $M^0_{FR}(p,\varphi)$ and $\mathbf{M}_{FR}(p,\varphi)$ in the focal plane and using then the algorithms of the Radon inversion one can reconstruct the distribution of the intensity $I(x,y)$ for $\sqrt{x^2+y^2} \le R$ and that of the value $I(x,y)\nabla S(x,y)$, $\sqrt{x^2+y^2} \le R$. Note that, "synthesizing" the slit by consecutive subtraction of the values M^0_{FR} and $\mathbf{M}_{FR}$ for different values of the parameter p of the normal straight line $p - x\cos\varphi - y\sin\varphi = 0$, one can use a half-plane or a strip as a diaphragm in order to use as large area of the lens as possible. Dividing the components of the Umov–Poynting vector by the intensity we obtain the values of the derivative (local tilts) of the wave front in the receiving aperture plane. In order to reconstruct the phase from measurements of its tilts we use the results from Ref. 7.

Now let us describe the results of numerical experiment on the phase reconstruction. A single-mode Gaussian beam with the field distribution in the lens plane of the following form

$$U(\rho) = U_0\exp\left\{-\frac{\rho^2}{2a_e^2} - i\,\frac{k\rho^2}{2F_e}\right\} \qquad (9)$$

with the effective radius $a_e = 0.05$ m and the focal length $F_e = -600$ m, $U_0 = \text{const}$, was chosen as the initial model for reconstruction. The receiving aperture was limited by the radius $R = 0.05$ m.

The form of a tomographic projection of the intensity $M^0_{FR}(p,\varphi)$ and two components of the Umov–Poynting vector $M^x_{FR}(p,\varphi)$ and $M^y_{FR}(p,\varphi)$ are presented in Fig. 2. The algorithm using the analytical formula of the inverse Radon transformation[6]

$$I(x,y) = -\,(2\pi)^{-2}\int_0^{\pi}\int_{-\infty}^{\infty}\frac{M^0_{FR}(p,\varphi)}{(p - x\cos\varphi - y\sin\varphi)^2}\,dp d\varphi. \qquad (10)$$

was used to reconstruct $I(\rho)$, $I(\rho)\dfrac{\partial S(\rho)}{\partial x}$, and $I(\rho)\dfrac{\partial S(\rho)}{\partial y}$. The integrals over the variables p and φ were calculated by the rectangular method. The reconstruction of tomograms was made by projections obtained from 20 directions which were evenly distributed in the angle range 0°–180°. The reconstruction was made in a unit circle. The dimension of the image matrix was 21×21, and the grid step was $h = 0.05$. Figure 3 presents one of the phase derivatives obtained by dividing the component of the Poynting vector by intensity. In accordance with the chosen model (9) the intensity and the components of the Poynting vector vanish at the periphery of the beam. This tendency turns to be uneven for the nominator and denominator due to computational errors. It is the factor that causes the artifacts in the picture of the components of the phase gradient (Fig. 3*a*). In order to remove these artifacts filtration of the calculated values of the derivative was made. The component of the filtered phase gradient is shown in Fig. 3*b*. The reconstruction of the phase distribution by measuring its partial derivatives was made on the basis of algorithms described in Ref. 7. The picture of the initial model and the reconstructed phase distribution is shown in Fig. 4. The comparison of the initial and the reconstructed phase distributions

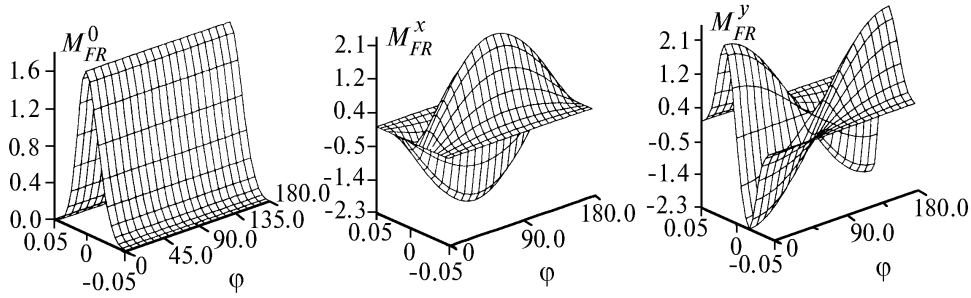

FIG. 2. The tomographic projections of intensity and the components of the Umov–Poynting vector.

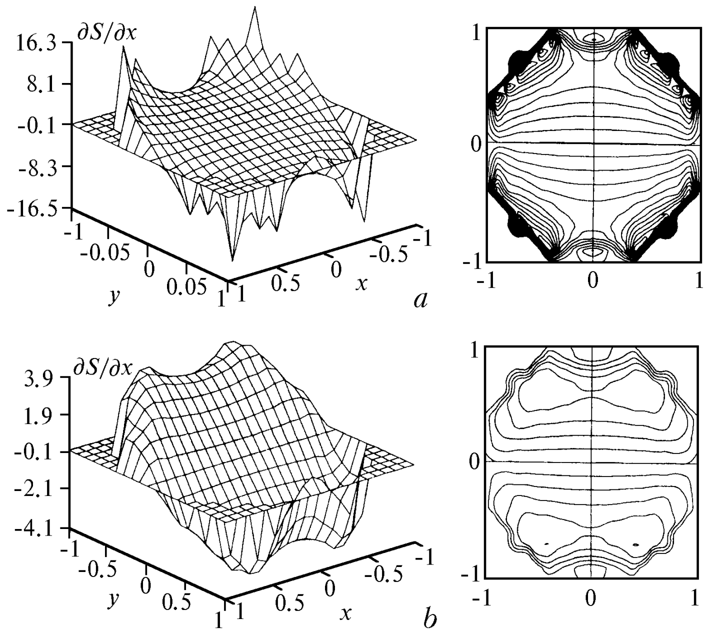

FIG. 3. The reconstructed component of the phase gradient: without (a) and with filtration (b).

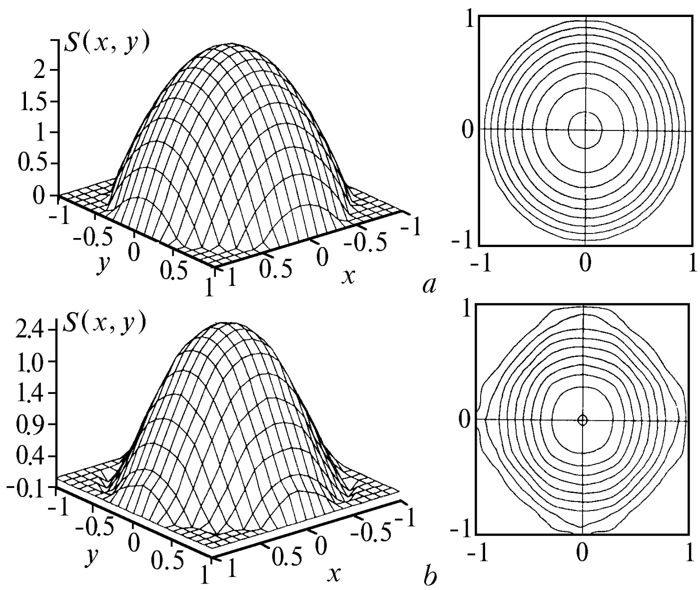

FIG. 4. Initial (a) and reconstructed (b) phase distribution of the Gauss optical beam.

makes it possible to conclude that the proposed method can be interesting for practical use. However, in order to realize the theoretical reasons one should solve preliminary the main theoretical calculational and engineering problems, namely,

1. to estimate the reconstruction error quantitatively and to make the algorithms of the Radon inversion and Hamilton's operator inversion to the values corresponding to the purposes of the control and to the parameters of measured objects with respect to accuracy and spatial resolution;

2. to provide the measurement of integral moments in the focal plane of a lens with the accuracy necessary to reach the given accuracy in phase measurement;

3. to give the coordinates of the position of the diaphragm and its axis in such a way that the error is minimum.

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