

ROLE OF AIR ENTRAINMENT AND MIXING IN FORMATION AND EVOLUTION OF CONDENSATION TRAILS, CLOUDS, AND FOGS

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Basic theoretical concepts about the formation of fogs and aircraft condensation trails under the impact of mixing fluids with different thermogigrometric properties are developed and better understood. In general terms, the conditions are established which favor the condensation of water vapor (into a cloud) in the mixing zone. The cloud liquid water content (LWC) is calculated over the physically representative range of temperature and humidity parameters for the interacting fluids. Given realistic temperature and humidity differences, the mixing (entrainment) produces cloud LWCs well comparing to the value from experiments. The mixing (entrainment) is important for process of cloud formation in cyclones and depressions. Calculated LWCs are especially high in the clouds formed by mixing at high temperatures (25–35°C), and these values are comparable to those found in thick cumulonimbus clouds. Evidently, the horizontal mixing (entrainment), like vertical motions, plays an important part in cloud formation within cyclones and within tropical cyclones (typically not in the lower troposphere) in particular.

The air mixing processes are widespread in many atmospheric phenomena of natural and anthropogenic origin. Of the latter, most studied are the condensation trails behind aircrafts^{1,2} (generally jet airplanes). As known, under favorable conditions the air mixing with combustion products from aircrafts, automobiles, and sea and river ships, as well as industrial and boiler emissions, may substantially alter the total meteorological and ecological situation: spreading condensation trails give rise to fairly dense upper-level clouds; fogs and lower cloudiness may appear at airports, harbors, and much more extended areas; substantial alterations are incurred in haze, fog, and cloud formation in industrial centers.

In nature the mixing is most essential in frontal zones, in formation and growth of convective clouds, in tropical cyclones, etc.

In all the above instances, the knowledge of mixing thermodynamics, as well as application of dynamical approach,³ helps better understanding of the conditions for fog and cloud formation.

We adopt the following notation: m_1 and m_2 are the masses of mixing fluids; T_1 and T_2 are their temperatures; q_1 and q_2 are the mass mixing ratios of water vapor; e_1 and e_2 are the water vapor partial pressures; τ_1 and τ_2 are the dew (ice) points; f_1 and f_2 are the relative humidities of the fluids (prior to mixing). The subscript 1 indicates the colder while 2 the warmer air or gas at the time of its release by an engine.

When the fluids m_1 and m_2 mix by the turbulent exchange, the water vapor mass mixing ratio q and

temperature T of the mixture, subject to the condition of preserving water vapor mass (balance of matter) and heat (thermal balance), will be:

$$q = \frac{m_1 q_1 + m_2 q_2}{m_1 + m_2} \quad \text{or} \quad q = \frac{q_1 + n q_2}{1 + n}; \quad (1)$$

$$T = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2} \quad \text{or} \quad T = \frac{T_1 + n T_2}{1 + n}. \quad (2)$$

Here, parameter $n = m_2/m_1$ shows the mass of warmer fluid mixed per unit mass of colder fluid. In application to a convective cloud or cyclone, this parameter serves as a measure of entrainment of air to the developing cloud or cyclone as a whole ($n^{-1} = m_1/m_2$ is the rate of entrainment).

Formula (2) is strictly applicable to horizontal (more specifically, isobaric) mixing. For vertical mixing, as for horizontal, when considerable pressure differences occur (such as in tropical cyclones), it is convenient to change from kinetic temperatures (T_1, T_2, T) to the potential ($\theta_1, \theta_2, \theta$) or equivalent-potential ones (for mixing of near-saturated fluids).

If already at the initial stage air contains liquid water, by q_1 and q_2 we should mean the specific moisture content, i.e., the sum of water vapor mass mixing ratio and specific LWC.

Since in reality the water vapor condensation progresses when the saturation is reached, for specific (mass) cloud LWC we have

$$\delta = q - q_m, \quad (3)$$

where q is the specific moisture content of the mixture (in which the condensation has occurred) as defined by

Eq. (1); $q_m = q_m(T, p)$ is the mass mixing ratio of saturated water vapor in the mixture of temperature T and pressure p :

$$q_m = 0.622 E(T)/p, \tag{4}$$

with $E(T)$ being the saturation pressure.

Both q and q_m (through T), and hence δ , depend essentially on the parameter n , whose value varies throughout the cloud (fog) and, hence, is poorly known in practice. Value of n can be found for those cloud parts where maximum of LWC occurs. Such n value, and thereby the corresponding maximum LWC, follows by equating, according to the well-known rule, the derivative of δ with respect to n ,

$$\frac{d\delta}{dn} = \frac{dq}{dn} - \frac{dq_m}{dn} = \frac{dq}{dn} - \frac{\partial q_m}{\partial T} \frac{dT}{dn},$$

to zero:

$$\frac{dq}{dn} - \frac{\partial q_m}{\partial T} \frac{dT}{dn} = 0. \tag{5}$$

Henceforth we restrict ourselves to the q_m dependence on T only because the pressure effect on q_m (even in the vertical-mixing case) is much weaker than T effect.

From Eqs. (1) and (2)

$$\frac{dq}{dn} = \frac{q_2 - q_1}{(1+n)^2}, \quad \frac{dT}{dn} = \frac{T_2 - T_1}{(1+n)^2}, \tag{6}$$

and from Clausius-Clapeyron equation

$$\partial q_m / (\partial T) = L q_m / (R_c T^2), \tag{7}$$

so equation (5) with regard for Eq. (4) takes the form

$$L E(T) / (R_c T^2) = B, \tag{8}$$

where L is the specific vaporization (condensation) heat: $L = (2500.6 - 2.71 \cdot T^{\circ}C) 10^3$ J/kg; $R_c = 461.5$ J/(kg·K) is the specific gas constant of water vapor; B is the parameter governing cloud formation:

$$B = \frac{P}{0.622} \frac{q_2 - q_1}{T_2 - T_1}. \tag{9}$$

With $q_i = 0.622E(\tau_i)/p$,

$$B = [E(\tau_2) - E(\tau_1)] / (T_2 - T_1), \tag{10}$$

where τ_1 and τ_2 are dew (ice) points for cold and warm fluids.

Calculations of the product $B(T_2 - T_1)$ for various τ_1 and τ_2 are given in Table I. For given T_1, T_2, τ_1 , and τ_2 , from these tables one immediately obtains B . These values widely vary, from tenths to 250–300 Pa/K.

TABLE I. Value of $B(T_2 - T_1)$, Pa; $\tau_i = \tau_2$ (numerator); $\tau_i = \tau_1$ (denominator).

$\tau_i, ^{\circ}C$	$\tau_2 - \tau_1, ^{\circ}C$									
	1	2	3	4	5	6	8	10	15	20
Water-droplet cloud										
40	<u>384</u>	<u>751</u>	<u>1101</u>	<u>1436</u>	<u>1754</u>	<u>2058</u>	<u>2623</u>	<u>3135</u>	<u>4211</u>	<u>5040</u>
	402	824	1265	1726	2208	2712	3788	4962	8368	12548
35	<u>304</u>	<u>593</u>	<u>868</u>	<u>1131</u>	<u>1381</u>	<u>1618</u>	<u>2059</u>	<u>2456</u>	<u>3286</u>	<u>3919</u>
	318	653	1003	1370	1754	2157	3019	3962	6716	10122
30	<u>238</u>	<u>463</u>	<u>678</u>	<u>882</u>	<u>1075</u>	<u>1260</u>	<u>1500</u>	<u>1906</u>	<u>2538</u>	<u>3016</u>
	250	512	788	1077	1381	1699	2383	3135	5342	8096
20	<u>141</u>	<u>247</u>	<u>401</u>	<u>520</u>	<u>633</u>	<u>740</u>	<u>936</u>	<u>1110</u>	<u>1465</u>	<u>1726</u>
	149	306	471	646	830	1024	1442	1906	3286	5040
10	<u>79.8</u>	<u>155</u>	<u>226</u>	<u>293</u>	<u>355</u>	<u>414</u>	<u>522</u>	<u>616</u>	<u>806</u>	<u>941</u>
	84.7	174	270	368	477	590	836	1110	1940	3016
0	<u>43.0</u>	<u>83.2</u>	<u>121</u>	<u>156</u>	<u>189</u>	<u>220</u>	<u>276</u>	<u>324</u>	<u>420</u>	<u>485</u>
	45.8	94.7	147	202	261	324	461	616	1094	1726
-10	<u>21.8</u>	<u>42.2</u>	<u>61.1</u>	<u>78.7</u>	<u>95.1</u>	<u>110</u>	<u>137</u>	<u>161</u>	<u>206</u>	<u>235</u>
	23.4	48.6	75.5	104	135	168	241	324	586	941
-20	<u>10.4</u>	<u>20.0</u>	<u>28.9</u>	<u>37.1</u>	<u>44.7</u>	<u>51.7</u>	<u>64.0</u>	<u>74.5</u>	<u>94.0</u>	<u>106</u>
	11.2	23.4	36.4	50.6	65.8	82.1	119	161	296	485
Crystal cloud										
-10	<u>22.1</u>	<u>42.5</u>	<u>61.3</u>	<u>78.6</u>	<u>94.5</u>	<u>109</u>	<u>135</u>	<u>156</u>	<u>196</u>	<u>222</u>
	24.0	50.0	78.2	109	142	177	258	351	612	967
-20	<u>9.47</u>	<u>18.1</u>	<u>26.0</u>	<u>26.3</u>	<u>39.9</u>	<u>46.0</u>	<u>56.5</u>	<u>65.2</u>	<u>80.8</u>	<u>90.3</u>
	10.4	21.6	34.0	47.4	62.0	77.9	114	156	298	507
-30	<u>3.77</u>	<u>7.19</u>	<u>10.3</u>	<u>13.1</u>	<u>15.6</u>	<u>18.0</u>	<u>21.9</u>	<u>25.1</u>	<u>30.8</u>	<u>34.0</u>
	4.15	8.71	13.7	19.2	25.2	31.9	47.0	65.2	127	222
-40	<u>1.38</u>	<u>2.62</u>	<u>3.74</u>	<u>4.74</u>	<u>5.63</u>	<u>6.44</u>	<u>7.81</u>	<u>8.90</u>	<u>10.7</u>	<u>11.8</u>
	1.53	3.23	5.11	7.19	9.49	12.0	17.9	25.1	50.4	90.3
-50	<u>0.46</u>	<u>0.87</u>	<u>1.23</u>	<u>1.55</u>	<u>1.84</u>	<u>2.10</u>	<u>2.52</u>	<u>2.85</u>	<u>3.39</u>	<u>3.67</u>
	0.51	1.09	1.74	2.46	3.26	4.16	6.28	8.90	18.4	34.0

Significant advantage of the equation (8) is its capability to give air temperature T within mixing zone. As is clearly seen, the left-hand side of Eq. (8) depends only on T . Thus, B and T are uniquely related:

in a water-droplet cloud

$T, ^\circ\text{q}$	40	35	30	25	20	15	10	5	0
$B, \text{Pa}/j$	390	309	242	188	144	109	82.2	60.7	44.3

$T, ^\circ\text{q}$	-5	-10	-15	-20	-25	-30	-35	-40
$B, \text{Pa}/j$	31.9	22.6	15.8	10.8	7.29	4.81	3.11	1.97

in a crystal cloud ($L = 2.837 \cdot 10^6 \text{ J/kg}$)

$T, ^\circ\text{q}$	-5	-10	-15	-20	-25
$B, \text{Pa}/j$	34.3	23.0	15.2	9.89	6.31

$T, ^\circ\text{q}$	-30	-35	-40	-45	-50
$B, \text{Pa}/j$	3.94	2.42	1.45	0.850	0.486

Now let us calculate cloud LWC. From equation (2) it follows that

$$n = (T - T_1)/(T_2 - T). \tag{11}$$

By substituting Eq. (11) into Eq. (1) and the resulting q into Eq. (3) we have

$$\delta = [q_1(T_2 - T) + q_2(T - T_1)] / (T_2 - T_1) - q_m(T, p). \tag{12}$$

Replacing mass mixing ratios q_1 and q_2 with the corresponding dew points τ_1 and τ_2 , we obtain from Eq. (12)

$$\delta = \frac{0.622}{P} \left[\frac{E(\tau_1)(T_2 - T) + E(\tau_2)(T - T_1)}{T_2 - T_1} - E(T) \right]. \tag{13}$$

Then, changing from $e(\tau_2)$ to B according to Eq. (10), and from mass (specific) LWC δ to volume (absolute) LWC $\delta^* = \delta\rho$ (where $\rho = p/RT$ is the air density), we obtain

$$\delta^* = (0.622/RT)[B(T - T_1) - (E(T) - E(\tau_1))]. \tag{14}$$

Repeating these steps now for $E(\tau_1)$, the alternative expression for δ^* is

$$\delta^* = (0.622/RT)[(E(\tau_2) - E(T)) - B(T_2 - T)]. \tag{15}$$

Thus cloud LWC depends on three parameters, namely T_1 , τ_1 , and B as in formula (14), or T_2 , τ_2 , and B as is given by Eq. (15).

Parameters B and T , the air temperature in mixing zone, are uniquely related, thus being interchangeable.

Results of calculating δ^* are presented in Table II. Specifically, first variant (δ^* determination from T_1 , τ_1 , and B) assumes saturation ($\tau_1 = T_1$) of the cold fluid, while the second (δ^* determination from T_2 , τ_2 , and B) the saturation ($\tau_2 = T_2$) of the warm fluid (the state for the other fluid is arbitrary in both cases).

Table II presents both variants: the value in the numerator (above the line) is δ^* calculated with T and

$\Delta T = T - T_1$ (or $T_1 = T - \Delta T$), while the value in the denominator (under the line) is that with T and $\Delta T = T_2 - T$ (or $T_2 = T + \Delta T$). Clearly, the lower temperature of the colder fluid implies the higher cloud LWC for a fixed temperature T in the mixing zone. Furthermore, δ^* depends strongly on T_1 : as the difference $T - T_1$ increases by a factor of 10 (from 1 to 10°C), cloud LWC increases by a factor of 78, 92, and 126 for T equaling -20, 10, and 40°C, respectively.

The dependence of δ on the warm fluid temperature T_2 is also rather strong: as the difference $T_2 - T$ increases by a factor of 10 (from 1 to 10°C), cloud LWC increases by a factor of 133, 111, and 88 for T equaling -20, 10, and 40°C, respectively.

For fixed $T - T_1$ and $T_2 - T$, cloud LWC is a strong function of temperature T in the mixing zone. For instance, at $T - T_1 = 5^\circ\text{C}$, LWC grows from 0.08 to 0.96 $\text{g}\cdot\text{m}^{-3}$ as T increases from -20 to 30°q, while at $T_2 - T = 5^\circ\text{C}$, there is a 0.1 to 1.22 $\text{g}\cdot\text{m}^{-3}$ increase in LWC for analogous T increase.

We also note that a few degrees difference $T - T_1$ (or $T_2 - T$) is already sufficient for mixing-generated cloud LWC to reach 0.2–0.4 $\text{g}\cdot\text{m}^{-3}$ for $T \leq 0^\circ\text{C}$, 0.5–0.9 $\text{g}\cdot\text{m}^{-3}$ for 10–20°C, and 1.0–3.0 $\text{g}\cdot\text{m}^{-3}$ for 30–40°C, which are close to values found in real clouds.

If the differences ΔT are 10–20°C, at T being between 20 and 40°C δ^* may be 5–10 $\text{g}\cdot\text{m}^{-3}$, and even more, up to 15–20 $\text{g}\cdot\text{m}^{-3}$. Such δ^* values can only be observed in very massive cumulonimbus clouds.

Values of δ^* given in Table II refer to the case of saturated (prior to mixing) air. From formulas (14) and (15) it follows that cloud LWC, δ^* , decreases with lowering dew point ($\tau_1 < T_1$ and $\tau_2 < T_2$).

The lowest τ_1^* and τ_2^* and relative humidities f_1^* and f_2^* at which air in the mixing zone is still saturated follow from Eqs. (14) and (15) with the condition $\delta^* = 0$:

$$E(\tau_1^*) = E(T) - B(T - T_1), \quad f_1^* = E(\tau_1^*)/E(T_1); \tag{16}$$

$$E(\tau_2^*) = E(T) - B(T_2 - T), \quad f_2^* = E(\tau_2^*)/E(T_2). \tag{17}$$

Calculations of f_1^* and f_2^* are illustrated in Table III, while τ_1^* and τ_2^* calculations are in Table IV. Values of $f_1 > f_1^*$ and $f_2 > f_2^*$ result in water vapor condensation giving rise to a cloud ($\delta^* > 0$). The ranges of f_1 ($f_1^* \leq f_1 \leq 100\%$) and f_2 ($f_2^* \leq f_2 \leq 100\%$) broaden as ΔT increases and T decreases in the mixing zone. Values of τ_1 favoring cloud formation ($\delta^* > 0$) have rather narrow ranges. For instance, when $T = 10^\circ\text{C}$ and $\Delta T = 6^\circ\text{C}$ (and hence $T_1 = 4^\circ\text{C}$), $\tau_1^* = 2.6^\circ\text{C}$ that is τ_1 varies between 2.6 and 4°C; when $T = 30^\circ\text{C}$ and $\Delta T = 6^\circ\text{C}$ ($T_1 = 24^\circ\text{C}$), $\tau_1^* = 22.9^\circ\text{C}$ that is $22.9 \leq \tau_1 \leq 24^\circ\text{C}$. As ΔT increases, possible range of τ_1 broadens. At the same $T = 10^\circ\text{C}$ and $\Delta T = 10^\circ\text{C}$ ($T_1 = 0^\circ\text{C}$) $\tau_1^* = -5.7^\circ\text{C}$ and $-5.7 \leq \tau_1 \leq 0^\circ\text{C}$ and at $T = 30^\circ\text{C}$ and $\Delta T = 10^\circ\text{C}$ ($T_1 = 20^\circ\text{C}$) $\tau_1^* = 16.0^\circ\text{C}$ and hence $16.0 \leq \tau_1 \leq 20^\circ\text{C}$.

TABLE II. Cloud LWC $10^2\delta^*$ (g/m^3): $\Delta T = T - T_1$ (numerator); $\Delta T = T_2 - T$ (denominator)

$T, ^\circ\text{C}$	$\Delta T, ^\circ\text{C}$										
	1	2	3	4	5	6	8	10	15	20	25
Water-droplet cloud											
40	<u>4.2</u>	<u>20</u>	<u>48</u>	<u>86</u>	<u>136</u>	<u>195</u>	<u>344</u>	<u>529</u>	<u>1134</u>	<u>1910</u>	<u>2821</u>
	8.3	30	66	115	179	257	462	735	1742	3286	5458
35	<u>3.3</u>	<u>17</u>	<u>41</u>	<u>73</u>	<u>114</u>	<u>165</u>	<u>289</u>	<u>444</u>	<u>945</u>	<u>1585</u>	<u>2335</u>
	6.5	25	54	95	148	214	386	615	1466	2785	4629
30	<u>3.2</u>	<u>15</u>	<u>34</u>	<u>62</u>	<u>96</u>	<u>138</u>	<u>240</u>	<u>368</u>	<u>781</u>	<u>1305</u>	<u>1916</u>
	5.5	20	44	78	122	177	320	511	1224	2328	3898
20	<u>2.4</u>	<u>10</u>	<u>24</u>	<u>42</u>	<u>65</u>	<u>93</u>	<u>161</u>	<u>245</u>	<u>516</u>	<u>856</u>	<u>1248</u>
	3.4	12	29	51	80	117	214	343	831	1591	2694
10	<u>1.7</u>	<u>7.0</u>	<u>16</u>	<u>27</u>	<u>42</u>	<u>60</u>	<u>103</u>	<u>156</u>	<u>326</u>	<u>536</u>	<u>778</u>
	2.0	8.0	18	31	51	75	137	222	399	1052	1795
0	<u>1.1</u>	<u>4.3</u>	<u>10</u>	<u>17</u>	<u>26</u>	<u>36</u>	<u>62</u>	<u>94</u>	<u>195</u>	<u>318</u>	<u>459</u>
	1.2	4.7	11	20	31	46	84	137	340	666	1148
-10	<u>0.66</u>	<u>2.5</u>	<u>5.6</u>	<u>9.8</u>	<u>15</u>	<u>21</u>	<u>36</u>	<u>54</u>	<u>110</u>	<u>179</u>	<u>256</u>
	0.65	2.7	6.2	11	18	27	49	81	203	402	702
-20	<u>0.37</u>	<u>1.4</u>	<u>3.1</u>	<u>5.3</u>	<u>8.1</u>	<u>11</u>	<u>19</u>	<u>29</u>	<u>59</u>	<u>94</u>	<u>134</u>
	0.34	1.5	3.4	6.2	9.9	15	27	45	114	230	392
Crystal cloud											
-10	<u>0.75</u>	<u>2.9</u>	<u>6.4</u>	<u>11</u>	<u>17</u>	<u>24</u>	<u>41</u>	<u>61</u>	<u>123</u>	<u>197</u>	-
	0.80	3.2	7.5	14	22	32	60	99	220	417	-
-20	<u>0.36</u>	<u>1.4</u>	<u>3.1</u>	<u>5.4</u>	<u>8.2</u>	<u>12</u>	<u>19</u>	<u>29</u>	<u>58</u>	<u>92</u>	-
	0.39	1.6	3.7	6.7	11	16	30	49	128	265	-
-30	<u>0.16</u>	<u>0.63</u>	<u>1.4</u>	<u>2.4</u>	<u>3.6</u>	<u>5.1</u>	<u>8.6</u>	<u>13</u>	<u>22</u>	<u>40</u>	-
	0.17	0.73	1.7	3.0	4.9	7.3	14	23	60	127	-
-40	<u>0.07</u>	<u>0.19</u>	<u>0.57</u>	<u>0.99</u>	<u>1.5</u>	<u>2.1</u>	<u>3.5</u>	<u>5.2</u>	<u>10</u>	<u>16</u>	-
	0.07	0.31	0.70	1.3	2.1	3.1	5.9	9.9	27	18	-
-50	<u>0.03</u>	<u>0.10</u>	<u>0.22</u>	<u>0.38</u>	<u>0.57</u>	<u>0.80</u>	<u>1.3</u>	<u>1.9</u>	<u>3.8</u>	<u>5.9</u>	-
	0.03	0.12	0.27	0.50	0.81	1.2	2.3	3.9	11	24	-

The parameter τ_2 has a wider range. At $T = 10^\circ\text{C}$ and $\Delta T = T_2 - T_1 = 4^\circ\text{C}$ ($T_2 = 16^\circ\text{C}$), $\tau_2^* = 2.6^\circ\text{C}$. However, τ_2 cannot be thought to be changing between 2.6 and 16°C . Formula (15) suggests that the dew point τ_2 has to be higher than T , because otherwise $\delta^* < 0$. Thus, in this example $10 < \tau_2 \leq 16^\circ\text{C}$. If $T = 10^\circ\text{C}$ and $\Delta T = 10^\circ\text{C}$ ($T_2 = 20^\circ\text{C}$), then $\tau_2^* = -5.4^\circ\text{C}$ and $10 < \tau_2 \leq 20^\circ\text{C}$.

For fixed T and dew point temperature (τ_1 or τ_2), the temperature of cold and warm fluids must be lower than T_1^* and T_2^* , which are found from the condition $\delta^* = 0$ in Eqs. (14) and (15).

$$T_1^* = T - \{[E(T) - E(\tau_1)]/B\}; \tag{18}$$

$$T_2^* = T + \{[E(\tau_2) - E(T)]/B\}. \tag{19}$$

The results of T_1^* and T_2^* calculation are presented in Table V. As formulas (14) and (15) state, a necessary and sufficient condition for cloud formation ($\delta^* < 0$) within the mixing zone is that the temperature of colder fluid be lower than T_1^* prior to mixing, while that of warmer fluid be lower than T_2^* .

The ranges of T_1 and T_2 , like those of τ_1 and τ_2 , are limited. For instance, at $T=10^\circ\text{C}$ and $\Delta\tau = T - \tau_1 = 6^\circ\text{C}$ (i.e., $\tau_1 = 4^\circ\text{C}$), $T_1^* = 5^\circ\text{C}$; and since $T_1 \geq \tau_1$, then T_1 is restricted to the range from 4 to 5°C only. When $T = 10^\circ\text{C}$ and $\Delta\tau = 10^\circ\text{C}$ (i.e., $\tau_1 = 0^\circ\text{C}$), $T_1^* = 2.5^\circ\text{C}$ and thus $0^\circ\text{C} \leq T_1 \leq 2.5^\circ\text{C}$.

At the same $T=10^\circ\text{C}$ and for $\Delta\tau = \tau_2 - T = 10^\circ\text{C}$ (i.e., $\tau_2 = 20^\circ\text{C}$), the range of T_2 is broader: $20 \leq T_2 \leq 23.6^\circ\text{C}$.

On the whole, the ranges of T_1 and T_2 , τ_1 and τ_2 favoring cloud formation within the mixing zone are too narrow (for normally observed differences $T_2 - T_1$ there ranges are typically a few degrees wide).

If the relative humidities f_1 and f_2 are lower than the values from Table III, while the temperatures T_1 and T_2 exceed values from Table IV, air in the mixing zone remains unsaturated and so no cloud is formed. Moreover, if one of the fluids is cloudy before mixing, it is totally scattered and somewhat dried after it.

On the whole, quantification of air mixing helps to answer the question: what conditions favor cloud formation and what cause its scattering.

No less interesting is the role of the mixing in developing such atmospheric formations as cyclones and fronts (depressions).

TABLE III. Minimum relative humidity f_1^* (%) (numerator, $\Delta T = T - T_1$), f_2^* (denominator $\Delta T = T_2 - T$)

$\Delta T, ^\circ\text{C}$	$T, ^\circ\text{C}$							
	40	35	30	20	10	0	-10	-20
1	99.9	99.9	99.9	99.9	99.8	99.7	99.7	99.6
	89.8	89.5	89.1	88.2	87.3	86.3	85.1	83.9
2	99.6	99.5	99.4	99.3	99.2	98.9	98.7	98.4
	80.4	79.8	79.1	77.6	75.8	74.0	72.0	69.7
3	98.9	98.8	98.7	98.4	98.0	97.6	97.6	98.3
	71.8	70.9	69.9	67.8	65.5	63.1	60.4	57.4
4	97.9	97.7	97.4	96.9	96.2	95.3	94.3	93.0
	63.9	62.8	61.6	59.0	56.3	53.3	50.1	46.6
5	96.6	96.2	95.8	94.8	93.7	92.3	90.5	88.0
	56.6	55.3	53.9	51.0	47.9	44.6	41.1	37.3
6	94.7	94.2	93.6	92.2	90.4	88.2	85.5	82.0
	49.9	48.5	47.0	43.8	40.4	36.8	33.1	29.1
7	92.4	91.6	90.8	88.7	86.2	82.9	79.1	74.3
	43.8	42.2	40.6	37.3	33.7	30.0	26.1	22.0
8	89.5	88.5	87.3	84.5	80.9	76.4	70.7	63.2
	38.1	36.5	34.8	31.3	27.7	23.9	19.9	15.9
9	86.3	84.7	83.1	79.3	74.5	68.4	60.0	50.1
	33.0	31.3	29.5	26.0	22.3	18.4	14.5	10.6
10	82.0	80.0	78.0	73.0	66.6	58.4	47.7	33.6
	28.2	26.5	24.7	21.1	17.4	13.6	9.8	6.0
11	77.1	74.7	72.0	65.5	57.2	46.5	32.3	13.5
	23.8	22.1	20.3	16.7	13.1	9.4	5.7	2.0
12	71.4	68.3	64.9	56.6	46.0	32.2	13.8	-
	19.8	18.1	16.3	12.8	9.2	5.6	2.1	-
13	64.7	60.9	56.6	46.2	32.8	15.2	-	-
	16.1	14.4	12.2	9.2	5.7	-	-	-
14	57.1	52.4	47.0	34.1	17.3	-	-	-
	12.8	11.1	9.4	6.0	2.6	-	-	-
15	48.2	42.5	35.9	20.0	-	-	-	-
	9.7	7.7	6.4	3.1	-	-	-	-
16	38.1	31.1	23.2	3.8	-	-	-	-
	6.9	5.3	3.7	0.5	-	-	-	-

TABLE IV. Minimum dew points τ_1^* and τ_2^* (in $^\circ\text{C}$): τ_1^* for $\Delta T = T - T_1$; τ_2^* for $\Delta T = T_2 - T$

$\Delta T, ^\circ\text{C}$	$T, ^\circ\text{C}$							
	40	35	30	20	10	0	-10	-20
1	39.0	4.0	29.0	19.0	8.9	-1.1	-11.0	-21.0
2	37.9	32.9	27.9	17.9	7.8	-2.1	-12.2	-22.1
3	36.8	31.8	26.8	16.7	6.7	-3.3	-13.4	-23.4
4	35.6	30.6	25.6	15.5	5.4	-4.7	-14.7	-24.8
5	34.4	29.3	24.3	14.2	4.1	-6.1	-16.2	-26.4
6	33.0	28.0	22.9	12.7	2.6	-7.6	-17.9	-28.2
7	31.6	26.5	21.4	11.2	0.9	-9.4	-19.8	-30.2
8	30.1	24.9	19.8	9.4	-0.9	-11.4	-22.0	-32.8
9	28.4	23.2	18.0	7.6	-3.0	-13.7	-24.8	-36.1
10	26.6	21.4	16.0	5.4	-5.7	-16.7	-28.2	-41.0
11	24.6	19.2	13.8	2.9	-8.4	-20.2	-33.3	-50.5
12	22.4	16.9	11.3	-0.1	-12.1	-25.4	-42.2	-
13	19.8	14.1	8.4	-3.8	-17.1	-	-	-
14	16.9	10.9	4.7	-8.6	-26.6	-	-	-
15	13.3	6.9	0.0	-16.1	-	-	-	-
16	8.9	1.6	-6.7	-35.5	-	-	-	-

TABLE V. Maximum temperatures ($^{\circ}\text{C}$) of colder fluid T_1^* (numerator, $\Delta\tau = T - \tau_1$) and warmer fluid T_2^* (denominator, $\Delta\tau = \tau_2 - T$), at which air in the mixing zone reaches saturation.

$T, ^{\circ}\text{C}$	$\Delta\tau$									
	2	3	4	5	6	8	10	15	20	25
40	<u>38.1</u>	<u>37.2</u>	<u>36.3</u>	<u>35.5</u>	<u>34.7</u>	<u>33.3</u>	<u>32.0</u>	<u>29.2</u>	<u>27.1</u>	<u>25.5</u>
	<u>42.1</u>	<u>43.2</u>	<u>44.4</u>	<u>45.7</u>	<u>47.0</u>	<u>49.7</u>	<u>52.7</u>	<u>61.5</u>	<u>72.2</u>	<u>85.2</u>
35	<u>33.1</u>	<u>32.2</u>	<u>31.3</u>	<u>30.5</u>	<u>29.8</u>	<u>28.3</u>	<u>27.0</u>	<u>24.4</u>	<u>22.3</u>	<u>20.8</u>
	<u>37.1</u>	<u>38.2</u>	<u>39.4</u>	<u>40.7</u>	<u>42.0</u>	<u>44.8</u>	<u>47.8</u>	<u>66.6</u>	<u>67.8</u>	<u>81.3</u>
30	<u>28.5</u>	<u>27.2</u>	<u>26.4</u>	<u>25.6</u>	<u>24.8</u>	<u>23.4</u>	<u>22.1</u>	<u>19.5</u>	<u>17.5</u>	<u>16.1</u>
	<u>32.1</u>	<u>33.2</u>	<u>34.4</u>	<u>35.7</u>	<u>37.0</u>	<u>39.9</u>	<u>43.0</u>	<u>52.1</u>	<u>63.5</u>	<u>77.5</u>
20	<u>18.3</u>	<u>17.2</u>	<u>16.4</u>	<u>15.6</u>	<u>14.9</u>	<u>13.5</u>	<u>12.3</u>	<u>9.8</u>	<u>8.0</u>	<u>6.7</u>
	<u>22.1</u>	<u>23.3</u>	<u>24.5</u>	<u>25.8</u>	<u>27.1</u>	<u>30.0</u>	<u>33.2</u>	<u>42.8</u>	<u>55.0</u>	<u>70.3</u>
10	<u>8.1</u>	<u>7.2</u>	<u>6.4</u>	<u>5.7</u>	<u>5.0</u>	<u>3.6</u>	<u>2.5</u>	<u>1.8</u>	<u>1.5</u>	<u>-2.6</u>
	<u>12.1</u>	<u>13.3</u>	<u>14.5</u>	<u>15.8</u>	<u>17.2</u>	<u>20.2</u>	<u>23.6</u>	<u>33.6</u>	<u>46.8</u>	<u>63.6</u>
0	<u>-1.9</u>	<u>-2.7</u>	<u>-3.5</u>	<u>-4.3</u>	<u>-5.0</u>	<u>-6.2</u>	<u>-7.3</u>	<u>-9.5</u>	<u>-10.9</u>	<u>-12.0</u>
	<u>2.1</u>	<u>3.3</u>	<u>4.6</u>	<u>5.9</u>	<u>7.3</u>	<u>10.4</u>	<u>13.9</u>	<u>24.7</u>	<u>38.9</u>	<u>57.6</u>
-10	<u>-11.9</u>	<u>-12.7</u>	<u>-13.5</u>	<u>-14.2</u>	<u>-14.9</u>	<u>-16.1</u>	<u>-17.1</u>	<u>-19.1</u>	<u>-20.4</u>	<u>-21.3</u>
	<u>-7.9</u>	<u>-6.7</u>	<u>-5.4</u>	<u>-4.0</u>	<u>-2.6</u>	<u>0.6</u>	<u>4.3</u>	<u>15.9</u>	<u>31.6</u>	<u>52.6</u>
-20	<u>-21.8</u>	<u>-22.2</u>	<u>-23.4</u>	<u>-24.1</u>	<u>-24.8</u>	<u>-25.9</u>	<u>-26.9</u>	<u>-28.7</u>	<u>-29.8</u>	<u>-30.6</u>
	<u>-17.8</u>	<u>-16.6</u>	<u>-15.3</u>	<u>-13.9</u>	<u>-12.4</u>	<u>-9.0</u>	<u>-5.1</u>	<u>7.3</u>	<u>24.8</u>	<u>48.9</u>

As is well known, an important contributor to the cyclone formation and development is the advection of cold: a cyclone continues developing until colder air from cyclone perimeter is no longer converged to its center. Naturally, this air when transported is mixed with the warmer one. The colder air entrainment to the cyclone (as well as to the depression) is favored by the convergence of air flows, which in these baric systems always takes place in the lower (and often even in the upper) troposphere.

For tropical cyclones, the temperature difference is particularly large (and so the role of mixing in cloud formation is appreciable). Such cyclones are known⁴ to form and move along warm oceanic streams (with water temperature exceeding 26–28°C), and so they acquire a large amount of heat from ocean.⁵

As estimates show, as the temperature of a 60 to 100-m thick ocean layer decreases by 1°C (this one is of true order of magnitude found by experiment), the temperature of the central part of tropical cyclone rises by 5–15°C (even considering that about 2/3 of

the energy acquired from ocean go to the evaporation of the sea water).

Satellite images of cloud cover (shaped as helix-like arms) suggest that a cyclone acquires air (through entrainment) from the area much more than the cyclone's own area. From Table II it follows that the entrainment of cold air and subsequent mixing with warm air, at high temperature typical of the tropical cyclones, give rise to a cloud with LWC on the order of 1–10 gm⁻³.

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