

## MODEL OF THE AUTOMOBILE EMISSIONS DISPERSION IN THE ATMOSPHERE

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*The model discussed in this paper is based on the transfer equation, which can be solved under the boundary conditions near the ground surface and assuming the process to be stationary. The motor transport emissions are introduced into the model as linear low-altitude cold sources. The linear sources are simulated by a set of point sources. The examples are given of computation of the carbon oxide and nitrogen oxides concentration at some roads with heavy traffic such as that from Tomsk to Novosibirsk and from Novosibirsk to Omsk.*

Motor transport is one of the main sources of atmospheric pollution. In 1993 motor transport in Russia has emitted 19 million tons different pollutants<sup>8</sup> that is approximately 40% of all anthropogenic pollution.

The pollutants emitted by motor transport are: carbon monoxide (CO) – 14.7, nitrogen oxides (NO<sub>x</sub>) – 0.95, and hydrocarbons (C<sub>x</sub>H<sub>y</sub>) – 33.5 million tons. In addition, among them are highly toxic Pb compounds, sulfur dioxide (SO<sub>2</sub>) and different waste oil products.<sup>1,8</sup>

Motor transport contaminates the atmosphere in cities, where the emissions are mostly distributed along streets and roads.

There are different techniques for making computations of the concentration of the species emitted by motor transport (see, for example, Refs. 5 and 6). This paper deals with the pollution occurring on the straight line portions of roads (in cities along the streets about 1 km long, and outside the city along the roads up to 100 km long).

The peculiarity of the technique proposed is its simplicity in use for the specific portions of roads. The motor transport sources of emissions are considered to be “cold” and linear. Linear sources can be simulated by a set of point sources.

Different kinds of cars<sup>1</sup> emit gases to a distance of some tens centimeters above the road surface. So let us consider such emissions as the ground ones, i.e.  $z = 0$ . Since the emission are “cold”, let us assume that they are mixed along the vertical direction within the internal (near ground) boundary layer of the height  $h$ . This height can be calculated within the theory of the boundary layer.<sup>3</sup>

The technique proposed is based on the transfer equation for a component in the boundary layer. For the values averaged over the layer having thickness  $h$

$$\bar{s} = \frac{1}{h} \int_0^h s(z) dz, \quad \bar{u} = \frac{1}{h} \int_0^h u(z) dz$$

and for the direction along the  $x$ -axis this equation has the form<sup>3</sup>:

$$\frac{\partial \bar{s}}{\partial t} + \bar{u} \frac{\partial \bar{s}}{\partial x} + \sigma \bar{s} - k_y \frac{\partial^2 \bar{s}}{\partial y^2} = \bar{\varepsilon} + \frac{1}{h} f, \quad (1)$$

where  $\varepsilon$  is the component inflow to the layer  $h$ ;  $f = f(x)$  are the ground sources along the road;  $\sigma$  is the model parameter that will be described below.

Let us seek a solution for Eq. (1) in the stationary case ( $\partial s / \partial t = 0$ ) and in the absence of other impurity sources in addition to sources  $f$  for a preset model parameters  $\sigma$ ,  $u$ , and  $k$ . Let us write Eq. (1) under the above conditions in the form (averaging sign is omitted):

$$\frac{\partial s}{\partial x} + \frac{\sigma}{u} s - \frac{k_y}{u} \frac{\partial^2 s}{\partial y^2} = \varphi, \quad (2)$$

where  $\varphi = (1/h)f$ ;  $k_y = a^2 u x$  is the  $y$  component of the turbulence coefficient,  $a$  is the coefficient ( $\sim 10^{-2}$ );  $\sigma = \sigma_1 = \alpha_0 \beta / h$ ;  $\beta$  is the coefficient at the dry adsorption of an impurity by the surface; and,  $\alpha_0$  is the empirical coefficient.

Solution of the Eq. (2) is sought under conditions

$$s = s_0 \text{ at } x = y = 0, \quad s \rightarrow 0 \text{ at } x \rightarrow \infty \text{ and } y \rightarrow \infty. \quad (3)$$

Let us seek the solutions of Eq. (2) in three situations.

1) The wind blows across the road ( $x$ -axis), the sources are homogeneously distributed along the road portion parallel to the  $y$ -axis. As a result, we can assume that  $\partial^2 s / \partial y^2 = 0$ .

2) The wind blows along the road, i.e. along the  $x$ -axis.

3) The wind blows at the angle  $\alpha$  to the road.

In the first case Eq. (2) can be reduced to a simpler form

$$\frac{ds}{dx} + \frac{\sigma}{u} s = \frac{1}{h} f. \tag{4}$$

The solution of this equation under the first of the conditions (3) can be written in the form:

$$s(x) = s_0 \exp[-(\sigma/u)x] \tag{5}$$

( $x$ -axis is directed along the wind and across the road), the initial concentration  $s_0$  is determined, for  $x = y = 0$  and the above parameters, by the relationship:

$$s_0 = f d_0 / uh, \tag{6}$$

where  $d_0$  is the road width ( $d_0 \sim 10\text{--}20$  m). For example, for  $f = 1$  mg/m<sup>2</sup>s,  $h = 40$  m,  $u = 7$  m/s,  $d_0 = 20$  m,  $s_0 = 71$  μg/m<sup>3</sup>. These values refer to carbon monoxide, for which  $\beta = 0.5$  cm/s. For nitrogen oxides  $f = 0.1$  mg/m<sup>2</sup>s,  $\beta = 1$  cm/s,  $s_0 = 14.4$  μg/m<sup>3</sup>.

Concentrations of carbon monoxide (CO) and nitrogen oxides (NO<sub>x</sub>) calculated by Eq. (5) for the above conditions and for different distances from the road  $x$  are given in the Table I. It follows from these data that the impurity concentration at the lee side quickly decreases with the distance from the road. It is 50–70% of its value at the road axis at a distance of 10 km being from 3 to 17% at 50 km distance, and only traces of the gases are observed at the distance of 100 km. The impurity concentration is equal to zero on the windward side.

TABLE I. Carbon monoxide (CO) and nitrogen oxides (NO<sub>x</sub>) concentrations  $s(x)$  as well as the values  $s(x)/s_0$  as functions of the distance  $x$  from the road at the wind blowing across the road, where  $s_0$  is the impurity concentrations on the road axis ( $u = 7$  m/s,  $d_0 = 20$  m,  $h = 40$  m,  $\alpha_0 = 2$ ).

$x$ , km	0	1	10	50	100
Carbon monoxide ( $\beta = 0.5$ cm/s, $\sigma_1 = 2.5 \cdot 10^{-4}$ s <sup>-1</sup> , MPC = 3000 μg/m <sup>3</sup> )					
$s(x)/s_0$ , %	100	97	70	17	3
$s(x)$ , μg/m <sup>3</sup>	142	138	100	24	4
Nitrogen oxide ( $\beta = 1$ cm/s, $\sigma_1 = 3 \cdot 10^{-4}$ s <sup>-1</sup> , MPC = 85 μg/m <sup>3</sup> )					
$s(x)/s_0$ , %	100	93	49	3	0.07
$s(x)$ , μg/g <sup>3</sup>	14.4	12.2	7.1	4.3	0.1

In the second case the wind blows along the road, and the solution of Eq. (2) on the  $x$ -axis is sought under the conditions (3). This solution found by separation of variables and the constant variation method<sup>7</sup> can be written in the form:

$$s(x, y) = P(x, y) (s_0 \exp[-(\sigma/u)x] + \int_0^x \exp[-(\sigma/u)(x-x')] \frac{f(x')}{uhP} dx'), \tag{7}$$

where the function

$$P(x, y) = \frac{1}{\sqrt{2\pi\sigma_y(x)}} \exp(-y^2/2\sigma_y^2(x)), \tag{8}$$

$s_0$  is the impurity amount at  $x = y = 0$  emitted from a linear source;  $\sigma_y^2 = \sigma_y^2(x) = (ax)^2$  is the variance of the impurity particle distances at the  $y$ -axis from the  $x$ -axis,  $a \sim 10^{-2}$  is the empirical coefficient.

When solving Eq. (7), it is revealed that the contribution of the term with  $s_0$  quickly decreases as the distance  $x$  increases, and it can be neglected at the distance  $x$  of several kilometers. The definite integral in the solution of Eq. (7) is calculated using the rectangle technique. To do that, let us introduce the points  $x_n = n\Delta x$  on the straight road portions of the length  $L$ , where  $\Delta x$  is the step along the  $x$ -axis. Let us then represent the definite integral in the solution of Eq. (7) for the impurity “plume” axis ( $y = 0$ ) in the form:

$$s(x_n) = \frac{\Delta x}{uh} \sum_{n'-1}^n \exp[(\sigma_1/u)(\Delta x/2)] f(x_{n'-1/2}), \tag{9}$$

$n = 1, 2, \dots$

Table II presents the results of calculations of the carbon monoxide and nitrogen oxides concentration for the straight line road portions of the length  $L$ , that simulates the Tomsk–Novosibirsk or Novosibirsk–Omsk road. The impurity flow distribution along the road can be presented in the form:

$$f(a) = f_{\min} + \frac{1}{2} (f_0 - f_{\min})(1 + \cos(2\pi/L)x), \tag{10}$$

where  $f_0$  is the value of  $f$  at the beginning and at the end of the road, i.e. at  $x = 0, L$ ;  $f_{\min}$  is the value of  $f$  at the middle of the road, i.e. for  $x = L/2$ . Such a distribution is accepted for the reason that at the beginning and at the end of the road, i.e. at  $x = 0, L$  (big cities), the motor transport emissions are maximum, and they are minimum at the middle of the road, where the traffic decreases because of furcations of a road. Then it was accepted that  $f = 1$  mg/m<sup>2</sup>s for CO and 0.1 mg/m<sup>2</sup>s for NO<sub>x</sub>.

Using the data of the table, one can follow how the impurity concentration changes as the distance along the wind increases.

Using the function  $P(x, y)$ , from Eq. (8) one can estimate the concentration variation across the road, i.e. along the  $y$ -axis, on both the lee and windward sides. The ratio of the concentration at the distances of 10, 20, and 30 m from the road to its value at the “plume” axis is 70, 25, and 4%, respectively.

TABLE II. Impurity flow from the road  $f(\text{mg}/\text{m}^2\cdot\text{s})$  and the impurity concentration  $s(x)$  ( $\mu\text{g}/\text{m}^3$ ) at the "plume" axis ( $y = 0$ ) at different distances from the road beginning ( $x$ , km) ( $u = 7$  m/s,  $d_0 = 20$  m,  $h = 40$  m,  $\alpha_0 = 2$ ).

$x$ , km	0	20	40	60	80	100	120
Carbon monoxide ( $\beta = 0.5$ cm/s, $\sigma_1 = 2.5 \cdot 10^{-4}$ s $^{-1}$ , MPC = 3000 $\mu\text{g}/\text{m}^3$ )							
$f(x)$ , mg/m $^2\cdot$ s	1.00	0.933	0.750	0.500	0.750	0.933	1.00
$s(x)$ , $\mu\text{g}/\text{m}^3$	–	47	84	108	160	193	240
Nitrogen oxides ( $\beta = 1$ cm/s, $\sigma_1 = 5 \cdot 10^{-4}$ s $^{-1}$ , MPC = 85 $\mu\text{g}/\text{m}^3$ )							
$f(x)$ , mg/m $^2\cdot$ s	0.100	0.093	0.075	0.0503	0.075	0.093	0.100
$s(x)$ , $\mu\text{g}/\text{m}^3$	–	3.3	6.0	7.6	10.3	13.5	20.3

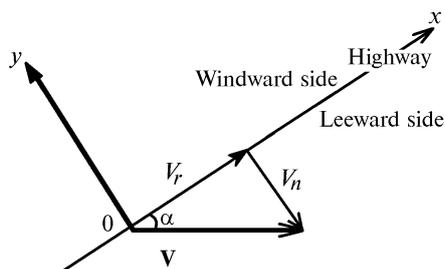


FIG. 1. Geometrical relationships between the wind and road directions:  $V$  is the wind vector,  $V_n$  and  $V_r$  are the projections of the wind vector to the road direction ( $x$ ) and to the direction normal to it ( $y$ -axis);  $\alpha$  is the angle between the road and the wind vector directions.

In the third case, when the wind is directed at an angle  $\alpha$  to the road (see Fig. 1), calculations were made twice: for the wind along and across the road. The values of concentration at any arbitrary point ( $x, y$ ) in the road vicinity can be obtained by adding.

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